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AN ALGORITHM FOR TREATING FIXED POINTS OF ASYMPTOTICALLY QUASI-NONEXPANSIVE MAPPINGS

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Abstract. In this paper, we investigate fixed points of a finite family of non-self asymptotically quasi-nonexpansive mappings based on a multi-step iterative process in the framework of uniformly convex Banach spaces. Strong convergence theorems are established.

Keywords: asymptotically mapping; nonexpansive mapping; fixed point; multi-step iteration.

1. Introduction-Preliminaries

Let E be a real Banach spaces and K a nonempty subset of E. Denote by $F(T)$ the set of fixed points of T. A mapping $T: K \to K$ is said to be asymptotically nonexpansive if there exists a sequence $\{k_n\}$ of positive real numbers with $k_n \to 1$ as $n \to \infty$ such that

$$\|T^n x - T^n y\| \leq k_n \|x - y\|, \quad \forall x, y \in K.$$

This class of asymptotically nonexpansive mappings was introduced by Goebel and Kirk [1] in 1972. They proved that, if K is a nonempty bounded closed convex subset of a uniformly

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convex Banach space E, then every asymptotically nonexpansive self-mapping T of K has a fixed point. Moreover, the fixed points set $F(T)$ of T is closed and convex.

The mapping T is said to be asymptotically quasi-nonexpansive if $F(T) \neq \emptyset$ and there exists a sequence $\{k_n\} \subset [0,\infty)$ with $k_n \to 1$ as $n \to \infty$ such that

$$\|T^n x - p\| \leq k_n \|x - p\|$$

for all $x \in K,\ p \in F(T)$ and $n \geq 1$. The mapping T is said to be uniformly L-lipschitzian if there exists a positive constant L such that

$$\|T^n x - T^n y\| \leq L\|x - y\|$$

for all $x, y \in C$ and $n \geq 1$. The mapping T is said to be uniformly Hölder continuous if there exists a positive constants k and α such that

$$\|T^n x - T^n y\| \leq k\|x - y\|^{\alpha}$$

for all $x, y \in C$ and $n \geq 1$. The mapping T is said to be uniformly equi-continuous if, for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\|T^n x - T^n y\| \leq \varepsilon$$

whenever $\|x - y\| \leq \delta$ for all $x, y \in C$ and $n \geq 1$ or, equivalently, T is uniformly equi-continuous if and only if $\|T^n x_n - T^n y_n\| \to 0$ whenever $\|x_n - y_n\| \to 0$ as $n \to \infty$.

Recently concerning the convergence problems of an iterative process to a fixed point for asymptotically nonexpansive mappings have been considered by several authors, see [1-13] references therein. The concept of non-self asymptotically nonexpansive mappings was introduced by Chidume et al. [9] as an important generalization of asymptotically nonexpansive self mappings.

Definition 1.1. Let E be a real normed linear space, K a nonempty subset of E. Let $P : E \to K$ be the nonexpansive retraction of E onto K. A map $T : K \to E$ is said to be asymptotically nonexpansive mappings if there exists a sequence $\{k_n\} \subset [1,\infty)$ and $k_n \to 1$ as $n \to \infty$ such that the following inequality holds:

$$\|T(PT)^{n-1}x - T(PT)^{n-1}y\| \leq k_n\|x - y\|, \quad \forall x, y \in K,\ n \geq 1.$$

T is called uniformly L-lipschitzian if there $L > 0$ such that

$$\|T(PT)^{n-1}x - T(PT)^{n-1}y\| \leq L\|x-y\|, \quad \forall x,y \in K,\ n \geq 1.$$

In 2003, Chidume et al. [9] studied the iteration scheme defined by

$$x_1 \in K, \quad x_{n+1} = P((1-\alpha_n)x_n + \alpha_n T(PT)^{n-1}x_n), \quad n \geq 1,$$

in the framework of uniformly convex Banach space, where K is a nonempty closed convex nonexpansive retract of a real uniformly convex Banach space E with P as a nonexpansive retraction. $T : K \to E$ is an asymptotically nonexpansive non-self map with sequence $\{k_n\} \subset [1,\infty)$, $k_n \to 1$. $\{\alpha_n\}_{n=1}^{\infty}$ is a real sequence in [0, 1] satisfying the condition $\varepsilon \leq \alpha_n \leq 1-\varepsilon$ for all $n \geq 1$ and for some $\varepsilon > 0$. They proved strong and weak convergence theorems for asymptotically nonexpansive nonself-maps.

On the other hand, Shahzad [14] studied the sequence $\{x_n\}$ defined by

$$x_1 \in K, \quad x_{n+1} = P((1-\alpha_n)x_n + \alpha_n TP[(1-\beta_n)x_n + \beta_n Tx_n]),$$

where K is a nonempty closed convex nonexpansive retract of a real uniformly convex Banach space E with P as a nonexpansive retraction. He proved weak and strong convergence theorems for non-self nonexpansive mappings in Banach spaces.

Recently, Chidume and Ali [10] studied the following multi-step iterative process

$$\begin{cases} x_1 \in K, \\ x_{n+1} = P[(1-\alpha_{1n})x_n + \alpha_{1n}T_1(PT_1)^{n-1}y_{n+m-2}], \\ y_{n+m-2} = P[(1-\alpha_{2n})x_n + \alpha_{2n}T_2(PT_2)^{n-1}y_{n+m-3}], \\ \vdots \\ y_n = P[(1-\alpha_{mn})x_n + \alpha_{mn}T_m(PT_m)^{n-1}x_n], \end{cases}$$

where $\{T_i\}_{i=1}^{m}$ is a family of non-self asymptotically nonexpansive mappings. They proved strong and weak convergence theorems for the finite families of nonself asymptotically nonexpansive mappings in uniformly convex Banach spaces.

In this paper, we investigate the problem of approximating a common fixed point of uniformly equi-continuous and asymptotically quasi-nonexpansive nonself mappings.

Definition 1.2. Let E be a real normed linear space, K a nonempty subset of E. Let $P : E \to K$ be the nonexpansive retraction of E onto K. A map $T : K \to E$ is said to be asymptotically quasi-nonexpansive mappings if $F(T) \neq \emptyset$ and there exists a sequence $\{k_n\} \subset [1,\infty)$ and $k_n \to 1$ as $n \to \infty$ such that the following inequality holds:

$$\|(PT)^n x - p\| \leq k_n \|x - p\|, \quad \forall x \in K,\ p \in F(T) \text{ and } n \geq 1.$$

T is called uniformly equi-continuous, if there exists $\delta > 0$ such that

$$\|(PT)^n x - (PT)^n y\| \leq \varepsilon, \quad \text{whenever } \|x - y\| \leq \delta \text{ for } x, y \in K \text{ and } n \geq 1.$$

Let K be a nonempty closed convex subset of a real uniformly convex Banach space E. In this paper, the following iteration scheme is studied

$$\begin{cases} y_{n+m-2} = \alpha_{mn}(PT_m)^n x_n + \beta_{mn} x_n, \\ \vdots \\ y_{n+1} = \alpha_{3n}(PT_3)^n y_{n+2} + \beta_{3n} x_n, \\ y_n = \alpha_{2n}(PT_2)^n y_{n+1} + \beta_{2n} x_n, \\ x_{n+1} = \alpha_{1n}(PT_1)^n y_n + \beta_{1n} x_n, \quad n \geq 1 \end{cases} \tag{1.1}$$

where $\{\alpha_{in}\}$, and $\{\beta_{in}\}$ be real sequences in [0, 1] such that $\alpha_{in} + \beta_{in} = 1$.

In order to prove our main results, we shall make use of the following definitions and lemmas.

Let E be a real Banach space. A subset K of E is said to be a retract of E if there exists a continuous map $P : E \to E$ such that $Px = x$ for all $x \in K$. A map $P : E \to E$ is said to be a retraction if $P^2 = P$. It follows that if a map P is a retraction, then $Py = y$ for all y in the range of P.

A mapping T with domain $D(T)$ and range $R(T)$ in E is said to be demiclosed at p if whenever $\{x_n\}$ is a sequence in $D(T)$ such that $\{x_n\}$ converges weakly to $x^* \in D(T)$ and $\{Tx_n\}$ converges strongly to p, then $Tx^* = p$.

Recall that the mapping $T : K \to E$ with $F(T) \neq \emptyset$ where K is a subset of E, is said to satisfy condition A if there is a nondecreasing function $f : [0,\infty) \to [0,\infty)$ with $f(0) = 0$ and $f(r) > 0$

for all $r \in (0,\infty)$ such that for all $x \in K$

$$\|x - Tx\| \geq f(d(x,F(T)),$$

where $d(x,F(T)) = \inf\{\|x-p\| : p \in F(T)\}$.

Recall that finite families mappings $\{T_i\}_{i=1}^m : K \to E$ where K a subset of E, are said to satisfy condition (A') if there exists a nondecreasing function $f : [0,\infty) \to [0,\infty)$ with $f(0) = 0$, $f(r) > 0$ for all $r \in (0,\infty)$ such that $\sum_{i=1}^m a_i\|x - T_ix\| \geq f(d(x,F(T)))$ for all $x \in K$ where $d(x,F(T)) = \inf\{\|x-p\| : p \in \bigcap_{i=1}^m F(T_i)$ and a_i are m nonnegative real numbers such that $\sum_{i=1}^m a_i = 1$, $i = 1,2,\ldots,m$.

Note that condition (A$'$) reduces to condition (A) when $T_1 = T_2 = \cdots = T_m$.

Lemma 1.1. [15] *Suppose that E is a uniformly convex Banach space and $0 < p \leq t_n \leq q < 1$ for all $n \in N$. Suppose further that $\{x_n\}$ and $\{y_n\}$ are sequences of E such that*

$$\limsup_{n\to\infty} \|x_n\| \leq r, \limsup_{n\to\infty} \|y_n\| \leq r$$

and

$$\lim_{n\to\infty} \|t_nx_n + (1-t_n)y_n\| = r$$

hold for some $r \geq 0$, then $\lim_{n\to\infty}\|x_n - y_n\| = 0$.

Lemma 1.2. [10] *Let $\{r_n\}$, $\{s_n\}$ and $\{t_n\}$ be three nonnegative sequences satisfying the following conditions*

$$r_{n+1} \leq (1+s_n)r_n + t_n, \ \ \forall n \geq 1.$$

If $\sum_{n=1}^\infty s_n < \infty$ and $\sum_{n=1}^\infty t_n < \infty$, then $\lim_{n\to\infty} r_n$ exists.

2. Main results

Theorem 2.1. *Let E be a uniformly convex Banach space and K a nonempty closed convex subset which is also a nonexpansive retract of E. Let T_1, T_2,..., $T_m : K \to E$ be equi-continuous and asymptotically quasi-nonexpansive mappings with sequences $\{k_{in}\}_{n=1}^\infty$ as $n \to \infty$, $i = 1,2,\ldots,m$ such that $\sum_{n=1}^\infty (k_{in} - 1) < \infty$, respectively. Starting from arbitrary $x_1 \in K$, define the sequence*

$\{x_n\}$ by the recursion (1.1), then $\lim_{n\to\infty}\|x_n-p\|$ exists, for any $p\in F(T)$, where $F(T)$ denotes the nonempty common fixed points set of T_1, T_2,..., T_m and $\lim_{n\to\infty}\|x_n-T_ix_n\|=0,\quad i=1,2,\ldots,m.$.

Proof. For any given $p\in F(T)$, we see that

$$\begin{aligned}\|y_{n+m-2}-p\| &= \|\alpha_{mn}(PT_n)^n x_n+\beta_{mn}x_n-p\|\\ &\le \alpha_{mn}\|(PT_m)^n x_n-p\|+\beta_{mn}\|x_n-p\|\\ &\le \alpha_{mn}k_{mn}\|x_n-p\|+(1-\alpha_{mn})\|x_n-p\|\\ &\le k_{mn}\|x_n-p\|.\end{aligned}\tag{2.1}$$

From (1.1) and (2.1), we obtain

$$\begin{aligned}\|y_{n+m-3}-p\| &= \|\alpha_{(m-1)n}(PT_{m-1})^n y_{n+m-2}+\beta_{(m-1)n}x_n-p\|\\ &\le \alpha_{(m-1)n}\|(PT_{m-1})^n y_{n+m-2}-p\|+\beta_{(m-1)n}\|x_n-p\|\\ &\le \alpha_{(m-1)n}k_{(m-1)n}\|y_{n+m-2}-p\|+(1-\alpha_{(m-1)n})\|x_n-p\|\\ &\le \alpha_{(m-1)n}k_{(m-1)n}k_{mn}\|x_n-p\|+(1-\alpha_{(m-1)n})\|x_n-p\|\\ &\le k_{(m-1)n}k_{mn}\|x_n-p\|,\end{aligned}\tag{2.2}$$

from which it follows that

$$\begin{aligned}\|y_{n+m-4}-p\| &= \|\alpha_{(m-2)n}(PT_{m-2})^n y_{n+m-3}+\beta_{(m-2)n}x_n-p\|\\ &\le \alpha_{(m-2)n}\|(PT_{m-2})^n y_{n+m-3}-p\|+\beta_{(m-2)n}\|x_n-p\|\\ &\le \alpha_{(m-2)n}k_{(m-2)n}\|y_{n+m-3}-p\|+(1-\alpha_{(m-2)n})\|x_n-p\|\\ &\le \alpha_{(m-2)n}k_{(m-2)n}[k_{(m-1)n}k_{mn}\|x_n-p\|\\ &\quad +(1-\alpha_{(m-2)n})\|x_n-p\|\\ &\le k_{(m-2)n}k_{(m-1)n}k_{mn}\|x_n-p\|+k_{(m-2)n}\gamma_{(m-1)n}+\gamma_{(m-2)n}).\end{aligned}$$

Continue in this fashion, we obtain for $2\le h\le m$

$$\begin{aligned}&\|y_{n+m-h}-p\|\\ &\le k_{(m-h+2)n}k_{(m-h+3)n}\cdots k_{mn}\|x_n-p\|+k_{(m-h+2)n}\gamma_{(m-h+3)n}\ldots+\gamma_{(m-h+2)n}).\end{aligned}\tag{2.3}$$

It follows that

$$\begin{aligned}
\|x_{n+1}-p\| &= \|\alpha_{1n}(PT_1)^n y_n+\beta_{1n}x_n-p\| \\
&\le \alpha_{1n}\|(PT_1)^n y_n-p\|+\beta_{1n}\|x_n-p\| \\
&\le \alpha_{1n}k_{1n}\|y_n-p\|+(1-\alpha_{1n})\|x_n-p\| \qquad (2.4)\\
&\le k_{1n}k_{2n}k_{3n}\cdots k_{mn}\|x_n-p\| \\
&= [1+(k_{1n}k_{2n}k_{3n}\cdots k_{mn}-1)]\|x_n-p\|.
\end{aligned}$$

In view of Lemma 1.2, we find that $\lim_{n\to\infty}\|x_n-p\|$ exists for all $p\in F(T)$. Let $\lim_{n\to\infty}\|x_n-p\|=c$. If $c=0$, then by the continuity of T the conclusion follows. Now suppose $c>0$. It follows from (2.3) that $\limsup\limits_{n\to\infty}\|y_n-p\|\le c$. $\lim\limits_{n\to\infty}\|(PT_1)^n y_n-x_n\|=0$. Noticing that

$$\begin{aligned}
\|x_n-p\| &\le \|(PT_1)^n y_n-x_n\|+\|(PT_1)^n y_n-p\| \\
&\le \|(PT_1)^n y_n-x_n\|+k_{1n}\|y_n-p\|,
\end{aligned}$$

we obtain $c\le\liminf\limits_{n\to\infty}\|y_n-p\|\le\limsup\limits_{n\to\infty}\|y_n-p\|\le c$. That is, $\lim\limits_{n\to\infty}\|y_n-p\|=c$. Taking limsup on both the sides in the above inequality and using (2.3), we have

$$\limsup_{n\to\infty}\|(PT_2)^n y_{n+1}-p+\gamma_{2n}(u_{2n}-x_n)\|\le c.$$

It follows that $\lim\limits_{n\to\infty}\|(PT_2)^n y_{n+1}-x_n\|=0$. Observe that

$$\begin{aligned}
\|x_n-p\| \le&\|(PT_2)^n y_{n+1}-x_n\|+\|(PT_2)^n y_{n+1}-p\| \\
\le&\|(PT_2)^n y_{n+1}-x_n\|+k_{2n}\|y_{n+1}-p\|.
\end{aligned}$$

It follows from (2.3) and (2.15) that

$$c\le\liminf_{n\to\infty}\|y_{n+1}-p\|\le\limsup_{n\to\infty}\|y_{n+1}-p\|\le c.$$

That is, $\lim_{n\to\infty}\|y_{n+1}-p\|=c$. Continuing in this way, we observe that for $1\le h\le m-1$

$$\lim_{n\to\infty}\|(PT_h)^n y_{n+h-1}-x_n\|=0$$

and $\|y_{n+h-1}-p\|=c$. It follows that $\lim_{n\to\infty}\|x_{n+1}-x_n\|=0$. It follows from the uniform equi-continuity of T_i, $i=1,2,\ldots,m$ that $\lim\limits_{n\to\infty}\|(PT_1)^n x_{n+1}-(PT_1)^n x_n\|=0$ and $\lim\limits_{n\to\infty}\|(PT_1)^n x_n-$

$T_1x_n\| = 0$. On the other hand, we have

$$\begin{aligned}\|T_1x_n - x_n\| &\leq \|x_n - x_{n+1}\| + \|x_{n+1} - (PT_1)^n x_{n+1}\| \\ &\quad + \|(PT_1)^n x_{n+1} - (PT_1)^n x_n\| + \|(PT_1)^n x_n - T_1x_n\|.\end{aligned}$$

It follows that $\lim_{n\to\infty}\|T_1x_n - x_n\| = 0$. On the other hand, one has

$$\|y_{n+h-1} - x_n\| \leq \alpha_{(h+1)n}\|(PT_{h+1})^n y_{n+h} - x_n\| + \gamma_{(h+1)n}r.$$

In a similar way, we obtain $\lim_{n\to\infty}\|y_{n+h-1} - x_n\| = 0$. Observe that

$$\begin{aligned}&\|T_{h+1}x_n - x_n\| \\ &\leq \|x_n - (PT_{h+1})^n x_n\| + \|(PT_{h+1})^n x_n - (PT_{h+1})^n y_{n+h-1}\| \\ &\quad + \|(PT_{h+1})^n y_{n+h-1} - T_{h+1}x_n\| \\ &\leq \|x_n - (PT_{h+1})^n x_n\| + k_{(h+1)n}\|x_n - y_{n+h-1}\| \\ &\quad + \|(PT_{h+1})^n y_{n+h-1} - T_{h+1}x_n\|.\end{aligned}$$

It follows from the unform equi-continuity of T_i, $i = 1, 2, \ldots, m$ that $\lim_{n\to\infty}\|T_{h+1}x_n - x_n\| = 0$. It follows hat $\lim\limits_{n\to\infty}\|T_ix_n - x_n\| = 0.i = 1, 2, \ldots, m$. This completes the proof.

Theorem 2.3. *Suppose $\{T_i\}_{i=1}^m$ satisfies condition (A'). Under the same restrictions in Theorem 2.1, we hhen $\{x_n\}$ converges strongly to some common fixed point of T_i, $i = 1, 2, \ldots, m$.*

Proof. Let $\lim_{n\to\infty}\|x_n - p\| = c$ for some $c \geq 0$. If $c = 0$, there is nothing to prove. Suppose $c > 0$. By Theorem 2.2, $\lim_{n\to\infty}\|T_1x_n - x_n\| = \lim_{n\to\infty}\|T_2x_n - x_n\| = \lim_{n\to\infty}\|T_3x_n - x_n\| = 0$, gives that

$$\begin{aligned}&\inf_{p\in F}\|x_{n+1} - p\| \\ &\leq \inf_{p\in F} k_{1n}k_{2n}k_{3n}\cdots k_{mn}\|x_n - p\| \\ &\quad + M(k_{1n}\gamma_{2n} + \gamma_{3n}k_{1n}k_{2n} + \cdots + \gamma_{mn}k_{(m-1)n}k_{(m-2)n}\cdots k_{2n}k_{1n} + \gamma_{1n}).\end{aligned}$$

That is,

$$\begin{aligned}&d(x_{n+1}, F) \\ &\leq k_{1n}k_{2n}k_{3n}\cdots k_{mn}\|x_n - p\| \\ &\quad + M(k_{1n}\gamma_{2n} + \gamma_{3n}k_{1n}k_{2n} + \cdots + \gamma_{mn}k_{(m-1)n}k_{(m-2)n}\cdots k_{2n}k_{1n} + \gamma_{1n}).\end{aligned}$$

gives that $\lim_{n\to\infty} d(x_n, F)$ exists by virtue of Lemma 1.2. Now by condition (A$'$), $\lim_{n\to\infty} f(d(x_n, F)) = 0$. Since f is a nondecreasing function and $f(0) = 0$, therefore $\lim_{n\to\infty} d(x_n, F) = 0$. Now we can take a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ and sequence $\{y_j\} \subset F$ such that $\|x_{n_j} - y_j\| < 2^{-j}$. Then following the method of Tan and Xu [16], we get that $\{y_j\}$ is a Cauchy sequence in F and so it converges. Let $y_j \to y$. Since F is closed, therefore $y \in F$ and then $x_{n_j} \to y$. As $\lim_{n\to\infty} \|x_n - p\|$ exists, $x_n \to y \in F = F(T)$ thereby completing the proof.

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