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INFINITE AND FINITE-TIME STABILITY OF AN IMPULSIVE DYNAMICAL SYSTEM WITH FEEDBACK CONTROL

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Abstract. This paper deals with a pest control strategy integrated the disease, enemies, pesticides and competition factors, which is described by an impulsive dynamical system with static state feedback controller. It extends the finite-time stability to the problem designing a double control strategy. Compared with the ordinary differential system, the unstable coexistence equilibrium can be stabilized by impulsive control. Furthermore, the infinite time stability step by step of finite-time stable control through numerical simulations can be obtained. The results imply that the double control is a more effective and environmentally sensitive strategy than the unique impulsive control. That is, the few times spraying chemical pesticides under the double control which we only need.

Keywords. Integrated pest management; Finite-time stability; Impulsive control; Static state feedback control; Linear matrix inequalities.

1. Introduction

Compared to the traditional pest control approach by spreading chemical pesticides in the crops (it may has harmful effects on non-target organisms), Integrated pest management (IPM) is an effective, low-cost and environmentally sensitive strategy to limit pest damage for minimizing losses and maximizing returns in agroecosystems. Since IPM involves the integration

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of chemical control (pesticides) with some alternative pest strategies. An essential component of IPM is the biological control by releasing enemies, parasitoids or pathogen to control pests, insects, mites, weeds and plant diseases etc. [1, 2, 3, 4]. Thus, the objective of IPM is to achieve a level of pest control that is acceptable in economic terms to people while causing minimal disturbance to the environments of non target individuals. A good IPM should reduce a population to levels acceptable to the public. This implies that there is an economic threshold above which the financial damage is sufficient to justify using such measures [5, 6, 7].

In this paper, we will consider the disease, enemies, pesticides and competition factors to control the pest damage. Based on the above facts, we can describe the behavior of pest and predator populations by means of impulsive dynamical system (IDS), such as the following Lotka-Volterra pest-enemies system with disease in pest species (which will be considered in this paper)

$$\begin{cases} \dot{x_1}(t) = x_1(\gamma - \lambda x_2 - c_{13}y_1 - c_{14}y_2), \\ \dot{x_2}(t) = x_2(\lambda x_1 - c_{23}y_1 - c_{24}y_2), \\ \dot{y_1}(t) = y_1(-d_1 + k_{13}c_{13}x_1 + k_{23}c_{23}x_2 - ey_2), \ (t, x(t)) \notin \mathscr{S}, \\ \dot{y_2}(t) = y_2(-d_2 + k_{14}c_{14}x_1 + k_{24}c_{24}x_2 - ey_1), \\ \Delta x(t) = J[x^* - x(t)], \ (t, x(t)) \in \mathscr{S}, \end{cases} \tag{1}$$

where $x = (x_1, x_2, y_1, y_2)^T$; $x^* = (x_1^*, x_2^*, y_1^*, y_2^*)^T$ is the positive equilibrium of system (1). $J \in \mathbb{R}^{n\times n}$ is the matrix describing the resetting law of the system over the resetting set $\mathscr{S} \subset \mathbb{R}_0^+ * \mathbb{R}^{n*n}$. The following assumptions for the model are made:

(1) The variable $x_1(t)$ and $x_2(t)$ denote the density of the pest population susceptible and the infective individuals to the disease, respectively; $y_1(t)$ and $y_2(t)$ represent the density of two competitive enemies, respectively.

(2) In an unbounded habitat, the prey reproduce exponentially in absence of predators by the term γx_1. The disease made the infected preys not reproduce, and that spreads only among the preys with an incidence rated $\lambda x_1 x_2$.

(3) The predation rates of two competitive enemies upon the susceptible pest are $-c_{13}x_1y_1$ and $-c_{14}x_1y_2$, and upon the infective pest are $-c_{23}x_2y_1$ and $-c_{24}x_2y_2$, respectively. Thus, the growth rate of the both enemies y_1 and y_2, which is expressed by the terms $(k_{13}c_{13}x_1 + k_{23}c_{23}x_2)y_1$ and $(k_{14}c_{14}x_1 + k_{24}c_{14}x_2)y_2$, is proportional to number of (both susceptible and diseased) pests eaten by the two enemies. According to the realistic disease transmission that the healthy preys have a more nutritional value than the infected ones, i.e., $k_{13} > k_{23}$ and $k_{14} > k_{24}$.

(4) The natural death rate of the two class predators are described by the term $-d_1y_1$ and $-d_2y_2$, respectively. And the competitiveness between y_1 and y_2 is expressed by the term $-ex_3x_4$.

From the standpoint of biology, we only take into account the chemical pesticides when the pest population is above the threshold (the financial damage), that is, the state jumps $\Delta x(t) = x(t^+) - x(t^-)$ is initiated when the solution $(t, x(t)) \in \mathscr{S}$. Furthermore, we will treat the system (1) as an abstract IDS

$$\dot{x}(t) = Ax(t) + B(x),\ x(0) = x_0,\ (t, x(t)) \notin \mathscr{S}, \tag{2a}$$

$$x^+(t) = (I - J)x(t) + Jx^*,\ (t, x(t)) \in \mathscr{S}, \tag{2b}$$

where $A \in \mathbb{R}^{n\times n}$ and $B(x) = (x^T B_1^T x, x^T B_1^T x, \cdots, x^T B_n^T x)^T$, for $B_i \in \mathbb{R}^{n\times n}$ and $i = 1, 2, \cdots, n$, x^* is the positive equilibrium of system (1), I is identity matrix and $J \in \mathbb{R}^{n\times n}$ is the matrix describing the resetting law of the system over the resetting set $\mathscr{S} \subset \mathbb{R}_0^+ * \mathbb{R}^{n*n}$. The class of IDS is composed by a nonlinear continuous-time system (2a) and a linear time–invariant resetting law (2b). For the solution $x(\cdot)$ start from an initial condition $x_0 = x(0)$, we can define the corresponding resetting times and resetting states sets associated to the solution $x(\cdot)$ as follows:

$$\mathscr{T} = \{t \in \mathbb{R}_0^+ \mid (t, x(t)) \in \mathscr{S}\},\ \mathscr{D} = \{x \in \mathbb{R}^n \mid (t, x(t)) \in \mathscr{S}\}$$

Depending on the definition of the resetting set $\mathscr{S}$, IDS can be classified as follows:

(i) Time-dependent IDS (TD-IDS): in this case, given a set $\mathscr{T} := \{t_1, t_2, \cdots\}$, $\mathscr{S}$ is defined as $\mathscr{S} = \mathscr{T} * \mathscr{D}(x_0, \mathscr{T})$, where $\mathscr{D}(x_0, \mathscr{T}) = \{x(\bar{t}) : \bar{t} \in \mathscr{T}\} \subset \mathbb{R}^n$. The resetting set is defined by a prescribed sequence of time instants, which are independent of the state $x(\cdot)$. For our pest–enemies system, this kind of IDS means that we spray pesticides at some fixed times $t_1, t_2, \cdots$.

(ii) State-dependent IDS (SD-IDS): in this case, given a set $\mathscr{D} \subset \mathbb{R}^n$, $\mathscr{S}$ is defined as $\mathscr{S} = \mathscr{T}(x_0, \mathscr{D}) \times \mathscr{D}$, where $\mathscr{T}(x_0, \mathscr{D}) = \{\bar{t} : x(\bar{t}) \in \mathscr{D}\} \subset \mathbb{R}_0^+$. The resetting set is defined by a region in the state space, which does not depend on the time. This kind of IDS means that we spray pesticides when the state is above some levels $x \in \mathscr{D}$.

In this paper, we mainly investigate the finite-time stability to the ODE system with static state feedback control, and further discuss the infinite–time stability step by step of designing advisable control (the impulsive control or state feedback control). In biology, we can control the pest population level below the desired threshold under the double control and minimize the harmful effects of chemical pesticide on the non-target organisms. That is the goal of IPM can be obtained by the finite-time stability. The finite–time stability of a system means there is a bound on the initial condition such that its state does not exceed a certain threshold during a specified time interval. While the classical Lyapunov asymptotic stability means the behavior of a system within an infinite time interval. From the biological point of view, when the pest population is less than the threshold ($x \in \mathscr{S}$), our aim of IPM is preventing the species level from reaching the threshold. Thus, it is interesting to consider the finite–time stability. The concept of finite-time stability dates back to the 1950s, and then has been introduced in the control literature [8, 9, 10]. Recently, some authors in [11] consider the finite time stability of nonlinear impulsive sampled–data systems. In [12], the finite-time stability analysis of fractional-order neural networks delay system has been investigated. An explicit criterion for finite-time stability of linear nonautonomous systems with delays has been given in [13]. Our aim of this article is to consider its applications for the IDS combining with disease, enemies, pesticides and competition factors to control the pest, and illustrate the effectiveness by designing an appropriate resetting law and numerical simulations.

The structure of the paper is as follows. We first show that the coexistence equilibrium is unstable for the ODE system. Then discuss the finite-time stability for the ODE system with static state feedback control. Finally, the infinite time stability are illustrated for the IDS with static state feedback control by numerical simulations.

2. Stability analysis

For convenience of computation, we first study the stability of the coexistence equilibrium of the ordinary differential equations (without impulse). Since the other boundary equilibria

represent some steady-state which hazardous for an ecosystem, that is the disappearance of one or more species, and they are not feasible and useful to our IPM. In fact, our goal of IPM is not the eradication of some species. Thus, the only interesting equilibrium at which all species survive is the coexistence equilibrium. On account, we obtain the coexistence equilibrium $E^* = (x_1^*, x_2^*, x_3^*, x_4^*) = [\frac{A_{22}D_1 - A_{12}D_2}{A_{11}A_{22} - A_{12}A_{21}}, \frac{A_{21}D_1 - A_{11}D_2}{A_{12}A_{21} - A_{11}A_{22}}, \frac{A_{42}D_3 - A_{32}D_4}{A_{31}A_{42} - A_{32}A_{41}},$
$\frac{A_{41}D_3 - A_{31}D_4}{A_{32}A_{41} - A_{31}A_{42}}],$
where

$A_{11} = c_{13}c_{14}(k_{13} + k_{14}),\ A_{12} = e\lambda + c_{14}(c_{13}k_{24} + c_{23}k_{23}),\ D_1 = \gamma + c_{13}d_2 + c_{14}d_1,$

$A_{21} = e\lambda - c_{23}k_{14}c_{14} - c_{24}k_{13}c_{13},\ A_{22} = -c_{23}c_{24}(k_{23} + k_{24}),\ D_2 = -c_{23}d_2 - c_{24}d_1,$

$A_{31} = c_{13}c_{23}(k_{13} - k_{23}),\ A_{32} = k_{13}c_{13}c_{24} - k_{23}c_{23}c_{14} - e\lambda,\ D_3 = \lambda d_1 - \gamma k_{23}c_{23},$

$A_{41} = k_{14}c_{14}c_{23} - k_{24}c_{24}c_{13} - e\lambda,\ A_{42} = c_{14}c_{24}(k_{14} - k_{24}),\ D_4 = \lambda d_2 - \gamma k_{24}c_{24}.$

Let us now consider the stability analysis of the deterministic differential equations (ODE) governing the interior equilibrium E^*. The stability of the equilibrium is determined by the nature of eigenvalues of the Jacobian matrix

$$J_a(E^*) = \begin{bmatrix} 0 & -\lambda x_1^* & -c_{13}x_1^* & -c_{14}x_1^* \\ \lambda x_2^* & 0 & -c_{23}x_2^* & -c_{24}x_2^* \\ k_{13}c_{13}x_3^* & k_{23}c_{23}x_3^* & 0 & -ex_3^* \\ k_{14}c_{14}x_4^* & k_{24}c_{24}x_4^* & -ex_4^* & 0 \end{bmatrix}.$$

Thus, the eigenvalue equation can be obtained

$$\begin{vmatrix} \mu & \lambda x_1^* & c_{13}x_1^* & c_{14}x_1^* \\ -\lambda x_2^* & \mu & c_{23}x_2^* & c_{24}x_2^* \\ -k_{13}c_{13}x_3^* & -k_{23}c_{23}x_3^* & \mu & ex_3^* \\ -k_{14}c_{14}x_4^* & -k_{24}c_{24}x_4^* & ex_4^* & \mu \end{vmatrix} = 0,$$

where μ is the eigenvalue of $J_a(E^*)$. Simplify the above determinant as follows

$$a_0\mu^4 + a_1\mu^3 + a_2\mu^2 + a_3\mu + a_4 = 0,$$

where $a_0 = 1 > 0, \Delta_1 = a_1 = 0$. According to the assertion of Hurwitz criterion, at least a solution of the aforementioned function has the positive real part, then the positive equilibrium point E^* is unstable.

In this paper, the positive equilibrium E^* only has been considered. Next, we will discuss the Finite-time stability of IDS. Before discussion, we first give some preliminaries.

The definition of finite-time stability and two sufficient conditions for finite-time stability and stabilization are given in [14, 15].

Definition 1.1. (Finite-Time Stability) [15] Given the polytopes $\mathscr{P}_0, \mathscr{P}$ and a positive scalar, T, system (2a) is said to be FTS with respect to $(\mathscr{P}_0, \mathscr{P}, T)$ if

$$x(0) \in \mathscr{P}_0 \Rightarrow x(t) \in \mathscr{P},\ \forall t \in (0, T].$$

Recall that a polytope $\mathscr{P} \subset \mathbb{R}^{n*n}$ can be described as

$$\begin{aligned} \mathscr{P} &= conv\{x_{\mathscr{P}}^{(1)}, x_{\mathscr{P}}^{(2)}, \cdots, x_{\mathscr{P}}^{(p)}\}) \\ &= \{x \in \mathbb{R}^{n*n} : a_k^T x \leqslant 1,\ k = 1, 2, \cdots, q\}, \end{aligned} \tag{3}$$

where p and q are suitable integer numbers, $x_{\mathscr{P}}^{(i)}$ denotes the i-th vertex of the polytope $\mathscr{P}, a_k \in \mathbb{R}^{n*n}$ and conv$\{\cdot\}$ denotes the operation of taking the convex hull of the argument.

Lemma 1.1. [14] *Given two polytopes, $\mathscr{P}_0$ and $\mathscr{P}$, defined as in (3), and a positive scalar, T , system (2a) is FTS, with respect to $(\mathscr{P}_0, \mathscr{P}, T)$, if there exist a positive scalar α and a positive definite symmetric matrix $P \in \mathbb{R}^{n*n}$ such that*

$$\lambda_{max}(P) \max_i \|x_{\mathscr{P}_0}^{(i)}\|^2 e^{\alpha T} \leqslant 1$$

$$\begin{pmatrix} 1 & a_k^T \\ a_k & P \end{pmatrix} \geqslant 0,\ k = 1, 2, \cdots, q$$

$$x_{\mathscr{P}_0}^{(i)\ T} P x_{\mathscr{P}_0}^{(i)} \leqslant 1,\ i = 1, 2, \cdots, p$$

$$\begin{aligned} &[A^T + (B_1^T x_{\mathscr{P}}^{(i)}\ \ B_2^T x_{\mathscr{P}}^{(i)}\ \ \cdots\ \ B_n^T x_{\mathscr{P}}^{(i)})]P + P[A + (B_1^T x_{\mathscr{P}}^{(i)}\ \ B_2^T x_{\mathscr{P}}^{(i)}\ \ \cdots\ \ B_n^T x_{\mathscr{P}}^{(i)})^T] \\ &-\alpha P \leqslant 0,\ i = 1, 2, \cdots, p. \end{aligned}$$

Lemma 1.2. [14] *Given two polytopes, $\mathscr{P}_0$ and $\mathscr{P}$, and a positive scalar, T , system (9) is finite-time stablizable with respect to $(\mathscr{P}_0, \mathscr{P}, T)$, if there exist a positive scalar α and a positive symmetric matrix $P \in \mathbb{R}^{n*n}$ and a matrix $L \in \mathbb{R}^{m*n}$ such that*

$$P \geqslant \max_i \|x_{\mathscr{P}_0}^{(i)}\|^2 e^{\alpha T} I$$

$$\begin{pmatrix} 1 & a_k^T P \\ P a_k & P \end{pmatrix} \geqslant 0,\ k = 1, 2, \cdots, q$$

$$\begin{pmatrix} 1 & x_{\mathscr{P}_0}^{(i)\,T} \\ x_{\mathscr{P}_0}^{(i)} & P \end{pmatrix} \geqslant 0,\ i = 1, 2, \cdots, p$$

$$AP + PA^T + FL + L^T F^T + \begin{pmatrix} x_{\mathscr{P}}^{(i)^T} (B_1 P + N_1 L) \\ \vdots \\ x_{\mathscr{P}}^{(i)^T} (B_n P + N_n L) \end{pmatrix}$$
$$+ \left[\left(PB_1^T + L^T N_1^T\right) x_{\mathscr{P}}^{(i)} \ \cdots\ \left(PB_n^T + L^T N_n^T\right) x_{\mathscr{P}}^{(i)} \right] - \alpha P \leqslant 0,\ i = 1, 2, \cdots, p.$$

The gain matrix of the stabilizing state feedback controller (8) is given by $K = LP^{-1}$.

Now, we will discuss the main results of IDS by the above useful lemmas. The notation $\mathscr{D}(x_0, \mathscr{T})$ makes clear the dependence of the resetting states set on the initial state and on the set of resetting times in the case of TD-IDS; conversely in the case of SD-IDS, $\mathscr{T}(x_0, \mathscr{D})$ indicates that the resetting times depend on the initial state and on the resetting states set.

Note that according to the continuous-time dynamics (2a) and the resetting law (2b), an IDS presents a left-continuous trajectory with finite jump from $x(t_k)$ to $lim_{\varepsilon \to 0} x(t_k + \varepsilon)$ at each resetting time t_k. In this paper we will focus our attention on TD-IDS, since these class of IDS can be effectively exploited to model the behavior of pest and predator populations when an IPM control strategy is applied.

2.1. Time-dependent impulsive dynamical system

As mentioned in literatures [16, 17], the abstract IDS is composed by a nonlinear quadratic continuous-time system (2a) and a linear time-invariant resetting law (2b), and the above two lemmas can be directly used. Besides, we give the following assumption which is made to assure the well posedness of the resetting times.

Assumption : For all $t \in \mathbb{R}_0^+$ and $(t, x(t)) \in \mathscr{S}$, there is $\varepsilon > 0$ such that $(t + \delta, x(t + \delta)) \notin \mathscr{S}$, $\forall \delta \in [0, \varepsilon]$.

The continuous-time dynamics of the impulsive model is described by system (1), which can be rewritten in the form (2a) with

$$A=\begin{pmatrix} \gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -d_1 & 0 \\ 0 & 0 & 0 & -d_2 \end{pmatrix},$$

$$B_1=\begin{pmatrix} 0 & -\frac{\lambda}{2} & -\frac{c_{13}}{2} & -\frac{c_{14}}{2} \\ -\frac{\lambda}{2} & 0 & 0 & 0 \\ -\frac{c_{13}}{2} & 0 & 0 & 0 \\ -\frac{c_{14}}{2} & 0 & 0 & 0 \end{pmatrix},\ B_2=\begin{pmatrix} 0 & \frac{\lambda}{2} & 0 & 0 \\ \frac{\lambda}{2} & 0 & -\frac{c_{23}}{2} & -\frac{c_{24}}{2} \\ 0 & -\frac{c_{23}}{2} & 0 & 0 \\ 0 & -\frac{c_{24}}{2} & 0 & 0 \end{pmatrix},$$

$$B_3=\begin{pmatrix} 0 & 0 & \frac{k_{13}c_{13}}{2} & 0 \\ 0 & 0 & \frac{k_{23}c_{23}}{2} & 0 \\ \frac{k_{13}c_{13}}{2} & \frac{k_{23}c_{23}}{2} & 0 & -\frac{e}{2} \\ 0 & 0 & -\frac{e}{2} & 0 \end{pmatrix},\ B_4=\begin{pmatrix} 0 & 0 & 0 & \frac{k_{14}c_{14}}{2} \\ 0 & 0 & 0 & \frac{k_{24}c_{24}}{2} \\ 0 & 0 & 0 & -\frac{e}{2} \\ \frac{k_{14}c_{14}}{2} & \frac{k_{24}c_{24}}{2} & -\frac{e}{2} & 0 \end{pmatrix}.$$

The periodic spraying of pesticides, which involves the chemical control, is taken into account by considering a time-dependent resetting law as the form (2b) with

$$J=\begin{pmatrix} \delta_1 & 0 & 0 & 0 \\ 0 & \delta_2 & 0 & 0 \\ 0 & 0 & \delta_3 & 0 \\ 0 & 0 & 0 & \delta_4 \end{pmatrix}, \tag{4}$$

where $\delta_1,\delta_2,\delta_3,\delta_4$ model the poisoning effects of the pesticide release $(0<\delta_i<1, i=1,2,3,4)$. It is assumed that also non-target individuals (i.e. the predators) are affected by the harmful effects of the pesticides.

In agriculture ecosystems environment, it is impossible to continuously spray pesticides. If they have been continuously sprayed, the pesticides will create enormous endanger. Then, we have to formulate an appropriate period to release pesticides by means of IPM strategies. That is, once the population density of the pest exceed the control threshold, the impulsive action should be taken. Thereby a reasonable time-dependent resetting law will be designed. Lemma

1, which has been firstly introduced in literature [14], gives a sufficient condition for finite-time stability of system (2a). This condition will be exploited to design the IPM strategy in subsection 2.4.

2.2. Static state feedback controller

The static state feedback controller will be introduced the IDS. In fact, a suitable static state feedback controller can be designed through measuring and calculating the experimental data. So let us consider the following system subject to a control input

$$\begin{cases} \dot{x_1}(t) = x_1(\gamma - \lambda x_2 - c_{13}x_3 - c_{14}x_4) + u_1(x_1,x_2,x_3,x_4) \\ \dot{x_2}(t) = x_2(\lambda x_1 - c_{23}x_3 - c_{24}x_4) + u_2(x_1,x_2,x_3,x_4) \\ \dot{x_3}(t) = x_3(-d_1 + k_{13}c_{13}x_1 + k_{23}c_{23}x_2 - ex_4) + u_3(x_1,x_2,x_3,x_4) \\ \dot{x_4}(t) = x_4(-d_2 + k_{14}c_{14}x_1 + k_{24}c_{24}x_2 - ex_3) + u_4(x_1,x_2,x_3,x_4), \end{cases} \tag{5}$$

where $u_i(x_1,x_2,x_3,x_4)$, $i = 1,2,3,4$, is the control input.

Then the system(5) can be rephrase as

$$\dot{x}(t) = Ax(t) + B(x) + U(x), \tag{6}$$

If there exist the control input $u(t) \in \mathbb{R}^m$ and

$$N(x,u) = \begin{pmatrix} x^T N_1 \\ x^T N_2 \\ \vdots \\ x^T N_n \end{pmatrix} u, \quad N_i \in \mathbb{R}^{n*m} \;, \quad i = 1,2,\cdots,n, \tag{7}$$

then the linear state feedback controller can be defined as

$$u = Kx, \tag{8}$$

where$K \in \mathbb{R}^{m*n}$, and

$$U(x) = Fu + N(x,u).$$

Thereby the system (6) can be written equally as

$$\dot{x}(t) = Ax(t) + B(x) + Fu + N(x,u). \tag{9}$$

The closed-loop connection between system(9) and controller(8) yields

$$\dot{x} = (A + FK)x + \begin{pmatrix} x^T(B_1 + N_1K) \\ \vdots \\ x^T(B_n + N_nK) \end{pmatrix} x. \tag{10}$$

Lemma 2 provides a sufficient condition for finite-time stabilization of the closed loop system (9).

2.3. The IDS with static state feedback controller

The IDS and the dynamical systems via static state feedback control all have been stated above, respectively. Now we put them together taking into consideration. That is, the problem can be extended an IDS with static state feedback controller

$$\begin{cases} \dot{x}(t) = Ax(t) + B(x) + Fu + N(x,u),\ x(0) = x_0, & (t, x(t)) \notin \mathscr{S}, \\ x^+(t) = (I - J)x(t) + JE^{**}, & (t, x(t)) \in \mathscr{S}, \end{cases} \tag{11}$$

where E^{**} is the coexistence equilibrium of system with static state feedback controller.

As mentioned above, the coexistence equilibrium is unstable for the ODE system, however, we can prove that it is finite–time stable according to the above system. Furthermore, we will show it is infinite–time stable under the impulsive control, which is interesting and our main results. Besides, the time of finite-time stabilization can be extend by means of designing appropriate static state back and impulsive control. That is, the periodic of spraying pesticides can be shorted, the times will decrease, the IPM strategies are optimized to go a step further.

2.4. Pest control with double controller

Our focus so far has been on the finite and infinite–time stability of the pest control with impulsive control and static state feedback control. To facilitate the interpretation of our mathematical results in the system, we proceed to investigate it step by step using numerical simulations.

The IPM problem defined above can be now rephrased in finite–time stability or finite-time stabilization terms. Let $\mathscr{P}_0$ be a set of initial conditions sufficiently close to the unstable equilibrium E^*, while the set $\mathscr{P}$ identifies a bounded region of the state space defined on the basis of considerations on the impulsive threshold. Let assume that the jumps described by the impulse are able to steer the state trajectories starting from the boundary of $\mathscr{P}$ at least to the boundary of $\mathscr{P}_0$. The state trajectories of (1) starting from $\mathscr{P}_0$ do not violate the bounds defined by P in a maximum time T_{max} which can be estimated from the application of Lemma 1. Whenever the period occurring between two consecutive jumps does not exceed T_{max}, the state trajectories starting from $\mathscr{P}_0$ can never exit $\mathscr{P}$ and, at resetting time, the state jumps again into $\mathscr{P}_0$.

The nominal values of the model parameters used in the example are reported in Table 1.

The corresponding coexistence equilibrium is $E^* = [0.4729\ 2.3213\ 0.5688\ 0.6375]^T$. The assigned polytope $\mathscr{P}$, expressed in terms of variations of the state variables with respect to the nominal condition, E^*, is

$$\mathscr{P} := [0.1, 0.7] \times [2.0, 2.6] \times [0.2, 0.7] \times [0.3, 0.9].$$

From Definition 1, the polytope $\mathscr{P}_0$ can be defined by scaling the dimensions of $\mathscr{P}$ by δ_i. In this way, it is possible to ensure that, at resetting time, the state can jump from the boundary of $\mathscr{P}$ to the boundary of $\mathscr{P}_0$. The resulting polytope is

$$\mathscr{P}_0 := [0.3, 0.5] \times [2.2, 2.4] \times [0.4, 0.6] \times [0.5, 0.7].$$

For the considered example, the conditions of Lemma 1 are verified through the feasibility problem solver of the Matlab LMI Control Toolbox [18]. By a dichotomous search, it possible to find T_{max}. An admissible solution is $T_{max} = 0.12,\ \alpha = 1.5815$ and

$$P = \begin{pmatrix} 0.0154 & -0.0048 & 0.0039 & 0.0034 \\ -0.0048 & 0.0142 & -0.0517 & 0.0260 \\ 0.0039 & -0.0517 & 0.5793 & 0.0351 \\ 0.0034 & -0.0260 & 0.0351 & 0.2867 \end{pmatrix}.$$

Now consider the closed loop system (9), $E^{**} = [0.7360\ 2.0675\ 0.4183\ 0.5469]^T$, with the same matrix A and matrices B_1, B_2, B_3, B_4 given in above, and

$$F = \begin{pmatrix} 0.7 & 2.2 & 0.6 \\ 1.1 & 0.6 & 1.5 \\ 4.5 & 0.6 & 2.2 \\ 2.8 & 0.7 & 3.1 \end{pmatrix}, \ N_1 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 0.8 & 0.3 \\ 0 & -1 & 0 \end{pmatrix},$$

$$N_2 = \begin{pmatrix} 0 & -0.9 & 0 \\ 0 & 0 & 1.2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N_3 = \begin{pmatrix} 0 & 1 & 1.5 \\ -1 & -0.5 & -2 \\ 0.7 & 0 & -1.2 \\ -3.2 & -0.4 & 2.1 \end{pmatrix}, N_4 = \begin{pmatrix} 0 & -1.2 & -3.1 \\ 1 & -0.5 & -1.1 \\ 0 & 4 & 0 \\ 2 & -0.7 & 0.4 \end{pmatrix}.$$

The LMIs optimization problem defined in Lemma 2 turns out to be feasible with

$$T_{max} = 0.22,\ \alpha = 0.3760,$$

$$P = \begin{pmatrix} 18.2904 & 0.2912 & 1.6068 & -2.5914 \\ 0.2912 & 17.1825 & 1.5699 & -0.6256 \\ 1.6068 & 1.5699 & 24.7171 & -1.5571 \\ -2.5914 & -0.6256 & -1.5571 & 23.7725 \end{pmatrix},$$

$$L = \begin{pmatrix} -0.8374 & -0.0508 & 0.9535 & -0.3779 \\ -4.5222 & -0.1413 & -0.0428 & 2.4859 \\ -0.9311 & -1.1036 & 0.6716 & 0.3504 \end{pmatrix}.$$

By letting $K = LP^{-1}$, a suitable state feedback gain matrix which makes the closed loop system FTS is

$$K = \begin{pmatrix} -0.0520 & -0.0065 & 0.0412 & -0.0190 \\ -0.2376 & -0.0030 & 0.0189 & 0.0798 \\ -0.0516 & -0.0662 & 0.0353 & 0.0097 \end{pmatrix}.$$

Now we give some explanations on the simulation. Under the same system parameters, we find that the coexistence equilibrium E^* is unstable, while the coexistence equilibrium E^* is stable which is shown by take advisable impulsive control in Figure 1. This implies the pest population level can be controled below the desired threshold i.e., the impulsive control is feasible. However, compared to the only impulsive control, the state jumps are more frequent than the system with double control (the impulsive control and the static state feedback control) which is shown in Figure 2. That is we need spray pesticides many times in a fixed time interval. Thus, there are more harmful effects on the biological environment or the non-target organisms. In this sense, the unique impulsive control strategy is not feasible, while the double control is a more effective and environmentally sensitive strategy.

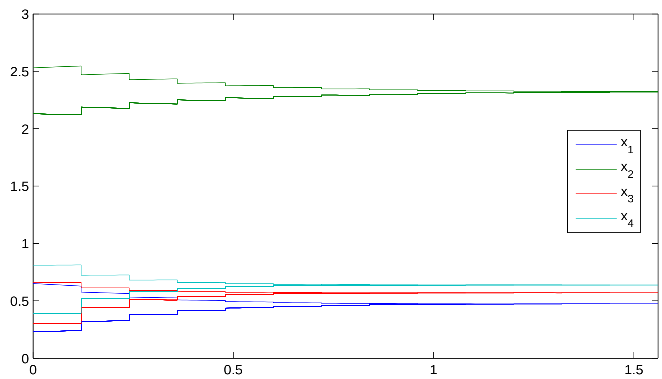

FIGURE 1. State response of the impulsive system from perturbed initial conditions.

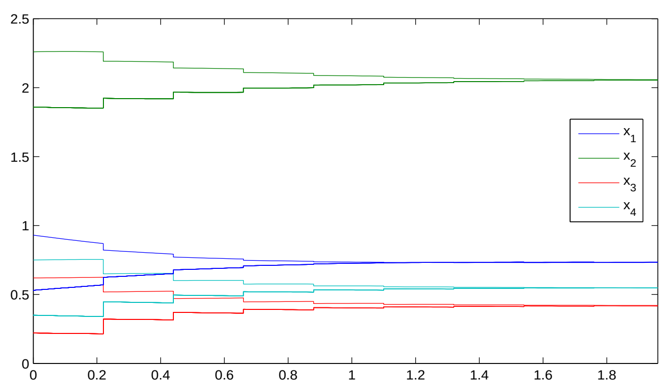

FIGURE 2. State response of the impulsive system with static state feedback from perturbed initial conditions.

3. Conclusions

TABLE 1. Parameters of the impulsive IPM model.

Parameter	γ	λ	e	d_1	d_2	c_{13}	c_{14}	c_{23}	c_{24}	k_{13}	k_{23}	k_{14}
Value	2.5	0.9	0.41	0.1	0.15	0.33	0.35	0.3	0.4	0.62	0.38	0.52
	k_{24}	δ_1	δ_2	δ_3	δ_4							
	0.32	0.34	0.34	0.52	0.51							

IPM strategy to deal with finite-time stability of quadratic systems is an extremely effectively mode. An impulsive resetting will be taken whenever some prescribed state bounds are reached. The utilize of IPM can save manpower and economical resources. That subjected control Lotka-Volterra systems impose the impulse can prevent the systems to exceed the EIL over a long period of time. And then, given a bound time of the growth of crops, the periodic spraying of pesticides are extend; that is, as a whole, the spraying times of pesticides are reduced. Therefore they accord with the biological significance of IPM strategy the better.

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