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CONVERGENCE THEOREMS FOR COMMON SOLUTIONS OF A SYSTEM OF EQUILIBRIUM PROBLEMS AND VARIATIONAL INEQUALITIES

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Abstract. In this paper, we studied a system of equilibrium problems and a monotone variational inequality. Weak convergence theorems of common solutions are established in the framework of Hilbert spaces.

Keywords. Equilibrium problem; Fixed point; Monotone mapping; Nonexpansive mapping; Variational inequality.

1. Introduction-preliminaries

In what follows, we always assume that H is a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\|\cdot\|$, and C is a nonempty, closed, and convex subset of H.

Let $\mathbb{R}$ denote the set of real numbers, and F a bifunction of $C \times C$ into $\mathbb{R}$. Recall the bifunction equilibrium problem is to find an x such that

$$F(x,y) \geq 0, \quad \forall y \in C. \tag{1.1}$$

In this paper, the solution set of the equilibrium problem is denoted by $EP(F)$. To study equilibrium problems (1.1), we may assume that F satisfies the following conditions:

(A1) $F(x,x) = 0$ for all $x \in C$;

(A2) F is monotone, i.e., $F(x,y) + F(y,x) \leq 0$ for all $x, y \in C$;

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(A3) for each $x,y,z \in C$,

$$\limsup_{t \downarrow 0} F(tz+(1-t)x,y) \le F(x,y);$$

(A4) for each $x \in C$, $y \mapsto F(x,y)$ is convex and lower semi-continuous.

Let $S: C \to C$ be a mapping. In this paper, we use $F(S)$ to stand for the set of fixed points. Recall that the mapping S is said to be nonexpansive if

$$\|Sx - Sy\| \le \|x-y\|, \quad \forall x,y \in C.$$

Let $A: C \to H$ be a mapping. Recall that A is said to be monotone if

$$\langle Ax - Ay, x-y \rangle \ge 0, \quad \forall x,y \in C.$$

A set-valued mapping $T: H \to 2^H$ is said to be monotone if, for all $x,y \in H$, $f \in Tx$ and $g \in Ty$ imply $\langle x-y, f-g \rangle > 0$. A monotone mapping $T: H \to 2^H$ is maximal if the graph $G(T)$ of T is not properly contained in the graph of any other monotone mapping. It is known that a monotone mapping T is maximal if and only if, for any $(x,f) \in H \times H$, $\langle x-y, f-g \rangle \ge 0$ for all $(y,g) \in G(T)$ implies $f \in Tx$. The class of monotone operators is one of the most important classes of operators. Within the past several decades, many authors have been devoting to the studies on the existence and convergence of zero points for maximal monotone operators.

Let $F(x,y) = \langle Ax, y-x \rangle$, $\forall x,y \in C$. we see that the problem (1.1) is reduced to the following classical variational inequality. Find $x \in C$ such that

$$\langle Ax, y-x \rangle \ge 0, \quad \forall y \in C. \tag{1.2}$$

It is known that $x \in C$ is a solution to (1.2) if and only if x is a fixed point of the mapping $P_C(I - \rho A)$, where $\rho > 0$ is a constant, and I is the identity mapping.

In this paper, we investigate a system of equilibrium problems and a monotone variational inequality based on a man valued iterative algorithm. Weak convergence theorems of common solutions are established in Hilbert spaces. Our results improve many corresponding results announced recently []

The following lemmas paly an important role in this paper.

Lemma 1.1. *Let C be a nonempty, closed, and convex subset of H. Then the following inequality holds*

$$\|x-\mathrm{Proj}_C x\|^2+\|y-\mathrm{Proj}_C\|^2 \leq \|x-y\|^2, \quad \forall x\in H,\, y\in C.$$

Lemma 1.2. ([6],[7]) *Let C be a nonempty closed convex subset of H, and $F: C\times C\to\mathbb{R}$ a bifunction satisfying $(A1)$-$(A4)$. Then, for any $r>0$ and $x\in H$, there exists $z\in C$ such that*

$$F(z,y)+\frac{1}{r}\langle y-z, z-x\rangle \geq 0, \quad \forall y\in C.$$

Further, define

$$T_r x=\{z\in C: F(z,y)+\frac{1}{r}\langle y-z, z-x\rangle \geq 0, \quad \forall y\in C\}$$

for all $r>0$ and $x\in H$. Then, the following hold:

(a) *T_r is single-valued;*

(b) *T_r is firmly nonexpansive, i.e., for any $x,y\in H$,*

$$\|T_r x-T_r y\|^2 \leq \langle T_r x-T_r y, x-y\rangle;$$

(c) *$F(T_r)=EP(F)$;*

(d) *$EP(F)$ is closed and convex.*

Lemma 1.3. [8] *Let A be a monotone mapping of C into H and $N_C v$ the normal cone to C at $v\in C$, i.e.,*

$$N_C v=\{w\in H: \langle v-u, w\rangle \geq 0, \quad \forall u\in C\}$$

and define a mapping T on C by

$$Tv=\begin{cases} Av+N_C v, & v\in C\\ \emptyset, & v\notin C.\end{cases}$$

Then T is maximal monotone and $0\in Tv$ if and only if $\langle Av, u-v\rangle \geq 0$ for all $u\in C$.

Lemma 1.4 [9] *Let $\{a_n\}_{n=1}^N$ be real numbers in $[0,1]$ such that $\sum_{n=1}^N a_n=1$. Then we have the following.*

$$\|\sum_{i=1}^N a_i x_i\|^2 \leq \sum_{i=1}^N a_i\|x_i\|^2,$$

for any given bounded sequence $\{x_n\}_{n=1}^N$ in H.

Lemma 1.5 [10] *Let $0 < p \le t_n \le q < 1$ for all $n \ge 1$. Suppose that $\{x_n\}$, and $\{y_n\}$ are sequences in H such that*

$$\limsup_{n\to\infty} \|x_n\| \le d, \quad \limsup_{n\to\infty} \|y_n\| \le d$$

and

$$\lim_{n\to\infty} \|t_n x_n + (1-t_n)y_n\| = d$$

hold for some $r \ge 0$. Then $\lim_{n\to\infty} \|x_n - y_n\| = 0$.

Lemma 1.6 [11] *Let $\{a_n\}$, $\{b_n\}$, and $\{c_n\}$ be three nonnegative sequences satisfying the following condition:*

$$a_{n+1} \le (1+b_n)a_n + c_n, \quad \forall n \ge n_0,$$

where n_0 is some nonnegative integer, $\sum_{n=1}^{\infty} b_n < \infty$ and $\sum_{n=1}^{\infty} c_n < \infty$. Then the limit $\lim_{n\to\infty} a_n$ exists.

2. Main results

Theorem 2.1. *Let C be a nonempty closed convex subset of H, and $A : C \to H$ a L-Lipschitz continuous and monotone mapping. Let F_m be a bifunction from $C\times C$ to $\mathbb{R}$ which satisfies (A1)-(A4). Let $N \ge 1$ denote some positive integer. Assume that $\mathscr{F} := \cap_{m=1}^{N} EP(F_m) \cap VI(C,A)$ is not empty. Let $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\delta_{n,1}\}, \ldots, \{\delta_{n,N}\}$ be real number sequences in $(0,1)$. Let $\{\lambda_n\}$, $\{r_{n,1}\}, \ldots,$ and $\{r_{n,N}\}$ be positive real number sequences. Let $\{e_n\}$ be a bounded sequence in H. Let $\{x_n\}$ be a sequence generated in the following manner:*

$$\begin{cases} x_1 \in H, \\ x_{n+1} = \alpha_n x_n + \beta_n \mathrm{Proj}_C(\sum_{m=1}^{N} \delta_{n,m} z_{n,m} - \lambda_n A y_n), \quad n \ge 1, \\ y_n = \mathrm{Proj}_C(\sum_{m=1}^{N} \delta_{n,m} z_{n,m} - \lambda_n A \sum_{m=1}^{N} \delta_{n,m} z_{n,m}), \end{cases}$$

where $z_{n,m}$ is such that

$$F_m(z_{n,m}, z) + \frac{1}{r_{n,m}} \langle z - z_{n,m}, z_{n,m} - x_n \rangle \ge 0, \quad \forall z \in C, \forall m \in \{1, 2 \ldots, N\}.$$

Assume that $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\delta_{n,1}\}, \ldots, \{\delta_{n,N}\}$, $\{\lambda_n\}$, $\{r_{n,1}\}, \ldots,$ and $\{r_{n,N}\}$ satisfy the following restrictions:

(a) $\alpha_n+\beta_n+\gamma_n=1$,

(b) $0<a\le\beta_n\le b<1$, *and* $\sum_{n=1}^{\infty}\gamma_n<\infty$;

(c) $\sum_{m=1}^{N}\delta_{n,m}=1$, *and* $0<c\le\delta_{n,m}\le 1$;

(d) $\liminf_{n\to\infty} r_{n,m}>0$, *and* $d\le\lambda_n\le e$, *where* $d,e\in(0,1/L)$.

Then the sequence $\{x_n\}$ *weakly converges to some point* $\bar{x}\in\mathscr{F}$.

Proof. Put $u_n=\mathrm{Proj}_C(\sum_{m=1}^{N}\delta_{n,m}z_{n,m}-\lambda_nAy_n)$, and $v_n=\sum_{m=1}^{N}\delta_{n,m}z_{n,m}$. Letting $p\in\mathscr{F}$, we see from Lemma 1.1 that

$$\begin{aligned}\|u_n-p\|^2&\le\|v_n-\lambda_nAy_n-p\|^2-\|v_n-\lambda_nAy_n-u_n\|^2\\&=\|v_n-p\|^2-\|v_n-u_n\|^2+2\lambda_n(\langle Ay_n-Ap,p-y_n\rangle+\langle Ap,p-y_n\rangle\\&\quad+\langle Ay_n,y_n-u_n\rangle)\\&\le\|v_n-p\|^2-\|v_n-y_n\|^2-\|y_n-u_n\|^2+2\langle v_n-\lambda_nAy_n-y_n,u_n-y_n\rangle.\end{aligned}\tag{2.1}$$

Notice that A is L-Lipschitz continuous, and $y_n=\mathrm{Proj}_C(v_n-\lambda_nAv_n)$. It follows that

$$\begin{aligned}&\langle v_n-\lambda_nAy_n-y_n,u_n-y_n\rangle\\&=\langle v_n-\lambda_nAv_n-y_n,u_n-y_n\rangle+\langle\lambda_nAv_n-\lambda_nAy_n,u_n-y_n\rangle\\&\le\lambda_nL\|v_n-y_n\|\|u_n-y_n\|.\end{aligned}\tag{2.2}$$

Substituting (2.2) into (2.1), we obtain that

$$\begin{aligned}\|u_n-p\|^2&\le\|v_n-p\|^2-\|v_n-y_n\|^2-\|y_n-u_n\|^2+2\lambda_nL\|v_n-y_n\|\|u_n-y_n\|\\&\le\|v_n-p\|^2+(\lambda_n^2L^2-1)\|v_n-y_n\|^2.\end{aligned}\tag{2.3}$$

On the other hand, we have obtain from the restriction (c) that

$$\begin{aligned}\|v_n-p\|^2&\le\|\sum_{m=1}^{N}\delta_{n,m}z_{n,m}-p\|^2\\&\le\sum_{m=1}^{N}\delta_{n,m}\|T_{r_{n,m}}x_n-p\|^2\\&\le\|x_n-p\|^2.\end{aligned}\tag{2.4}$$

Substituting (2.4) into (2.3), we obtain that

$$\|u_n-p\|^2\le\|x_n-p\|^2+(\lambda_n^2L^2-1)\|v_n-y_n\|^2.\tag{2.5}$$

This in turn implies the restriction (d) that

$$\begin{aligned}
\|x_{n+1}-p\|^2 &\le \alpha_n\|x_n-p\|^2+\beta_n\|u_n-p\|^2+\gamma_n\|e_n-p\|^2\\
&\le \alpha_n\|x_n-p\|^2+\beta_n(\|x_n-p\|^2+(\lambda_n^2L^2-1)\|v_n-y_n\|^2)+\gamma_n\|e_n-p\|^2\\
&\le \|x_n-p\|^2+\beta_n(\lambda_n^2L^2-1)\|v_n-y_n\|^2+\gamma_n\|e_n-p\|^2\\
&\le \|x_n-p\|^2+\gamma_n\|e_n-p\|^2.
\end{aligned} \tag{2.6}$$

It follows from Lemma 1.7 that the $\lim_{n\to\infty}\|x_n-p\|$ exists. This in turn shows that $\{x_n\}$ is bounded. It follows from (2.6) that

$$\beta_n(1-\lambda_n^2L^2)\|v_n-y_n\|^2 \le \|x_n-p\|^2-\|x_{n+1}-p\|^2+\gamma_n\|e_n-p\|^2$$

This implies from the restrictions (b), and (d) that

$$\lim_{n\to\infty}\|v_n-y_n\|=0. \tag{2.7}$$

Notice that $\|y_n-u_n\|\le\lambda L\|v_n-y_n\|$. It follows from (2.7) that

$$\lim_{n\to\infty}\|y_n-u_n\|=0. \tag{2.8}$$

We see from (2.7) and (23.8) that

$$\lim_{n\to\infty}\|v_n-u_n\|=0. \tag{2.9}$$

Notice that

$$\begin{aligned}
\|z_{n,m}-p\|^2 &\le \langle T_{r_{n,m}}x_n-T_{r_{n,m}}p, x_n-p\rangle\\
&=\frac{1}{2}(\|z_{n,m}-p\|^2+\|x_n-p\|^2-\|z_{n,m}-x_n\|^2), \quad \forall 1\le m\le N.
\end{aligned}$$

This implies that

$$\|z_{n,m}-p\|^2\le\|x_n-p\|^2-\|z_{n,m}-x_n\|^2, \quad \forall 1\le m\le N. \tag{2.10}$$

In view of (2.10) and $v_n = \sum_{m=1}^{N} \delta_{n,m} z_{n,m}$, where $\sum_{m=1}^{N} \delta_{n,m} = 1$, we see from Lemma 1.4 that

$$\begin{aligned}\|v_n - p\|^2 &\leq \sum_{m=1}^{N} \delta_{n,m} \|z_{n,m} - p\|^2 \\ &\leq \sum_{m=1}^{N} \delta_{n,m} (\|x_n - p\|^2 - \|z_{n,m} - x_n\|^2) \\ &= \|x_n - p\|^2 - \sum_{m=1}^{N} \delta_{n,m} \|z_{n,m} - x_n\|^2.\end{aligned} \tag{2.11}$$

In view of (2.3), we obtain from restriction (d) that

$$\begin{aligned}\|x_{n+1} - p\|^2 &\leq \alpha_n \|x_n - p\|^2 + \beta_n \|u_n - p\|^2 + \gamma_n \|e_n - p\|^2 \\ &\leq \alpha_n \|x_n - p\|^2 + \beta_n \|v_n - p\|^2 + \gamma_n \|e_n - p\|^2 \\ &\leq \|x_n - p\|^2 - \beta_n \sum_{m=1}^{N} \delta_{n,m} \|z_{n,m} - x_n\|^2 + \gamma_n \|e_n - p\|^2.\end{aligned}$$

It follows that $\beta_n \delta_{n,m} \|z_{n,m} - x_n\|^2 \leq \|x_n - p\|^2 - \|x_{n+1} - p\|^2 + \gamma_n \|e_n - p\|^2$. In view of the restrictions (b), and (c), we find that

$$\lim_{n\to\infty} \|z_{n,m} - x_n\| = 0. \tag{2.12}$$

Since $\{x_n\}$ is bounded, we may assume that a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ converges weakly to ξ. It follows from (2.12) that $\{z_{n_i,m}\}$ converges weakly to ξ for each $1 \leq m \leq N$. Next, we show that $\xi \in EP(F_m)$ for each $1 \leq m \leq N$. Since $z_{n,m} = T_{r_{n,m}} x_n$, we have $F_m(z_{n,m}, z) + \frac{1}{r_{n,m}} \langle z - z_{n,m}, z_{n,m} - x_n \rangle \geq 0$, From the assumption (A2), we see that $\frac{1}{r_{n,m}} \langle z - z_{n,m}, z_{n,m} - x_n \rangle \geq F_m(z, z_{n,m})$, Replacing n by n_i, we arrive at $\langle z - z_{n_i,m}, \frac{z_{n_i,m} - x_{n_i}}{r_{n_i,m}} \rangle \geq F_m(z, z_{n_i,m})$ In view of the assumption (A4), we get from (2.12) that $F_m(z, \xi) \leq 0$,. For t_m with $0 < t_m \leq 1$, and $z \in C$, let $z_{t_m} = t_m z + (1 - t_m)\xi$, for each $1 \leq m \leq N$. Since $z \in C$, and $\xi \in C$, we have $z_{t_m} \in C$, for each $1 \leq m \leq N$. It follows that $F_m(z_{t_m}, \xi) \leq 0$ for each $1 \leq m \leq N$. Notice that

$$0 = F_m(z_{t_m}, z_{t_m}) \leq t_m F_m(z_{t_m}, z) + (1 - t_m) F_m(z_{t_m}, \xi) \leq t_m F_m(z_{t_m}, z), \quad \forall 1 \leq m \leq N,$$

which yields that $F_m(z_{t_m}, z) \geq 0$, $\forall z \in C$. Letting $t_m \downarrow 0$ for each $1 \leq m \leq N$, we obtain from the assumption (A3) that $F_m(\xi, z) \geq 0$, $\forall z \in C$. This implies that $\xi \in EP(F_m)$ for each $1 \leq m \leq N$. This proves that $\xi \in \cap_{m=1}^{N} EP(F_m)$.

Next, we show that $\xi \in VI(C,A)$. In fact, let T be the maximal monotone mapping defined by:

$$Tx = \begin{cases} Ax + N_C x, & x \in C, \\ \emptyset, & x \notin C. \end{cases}$$

For any given $(x,y) \in G(T)$, we have $y - Ax \in N_C x$. So, we have $\langle x - m, y - Ax \rangle \geq 0$, for all $m \in C$. On the other hand, we have $u_n = \mathrm{Proj}_C(v_n - \lambda_n A y_n)$. we obtain that $\langle v_n - \lambda_n A y_n - u_n, u_n - x \rangle \geq 0$ and hence $\langle x - u_n, \frac{u_n - v_n}{\lambda_n} + A y_n \rangle \geq 0$. In view the monotonicity of A, we see that

$$\begin{aligned} \langle x - u_{n_i}, y \rangle &\geq \langle x - u_{n_i}, Ax \rangle \\ &\geq \langle x - u_{n_i}, Ax \rangle - \langle x - u_{n_i}, \frac{u_{n_i} - v_{n_i}}{\lambda_{n_i}} + A y_{n_i} \rangle \\ &\geq \langle x - u_{n_i}, A u_{n_i} - A y_{n_i} \rangle - \langle x - u_{n_i}, \frac{u_{n_i} - v_{n_i}}{\lambda_{n_i}} \rangle \end{aligned} \tag{2.13}$$

It follows from (2.12) that

$$\lim_{n \to \infty} \| v_n - x_n \| = 0. \tag{2.14}$$

Combining (2.9) with (2.14), we arrive at

$$\lim_{n \to \infty} \| u_n - x_n \| = 0. \tag{2.15}$$

This in turn implies that $u_{n_i} \rightharpoonup \xi$. It follows from (2.13) that $\langle x - \xi, y \rangle \geq 0$. Notice that T is maximal monotone and hence $0 \in T\xi$. This shows from Lemma 1.3 that $\xi \in VI(C,A)$.

Finally, we show that the whole sequence $\{x_n\}$ weakly converges to ξ. Let $\{x_{n_j}\}$ be another subsequence of $\{x_n\}$ converging weakly to ξ', where $\xi' \neq \xi$. In the same way, we can show that $\xi' \in \mathscr{F}$. Since the space H enjoys Opial's condition, we, therefore, obtain that

$$\begin{aligned} d &= \liminf_{i \to \infty} \| x_{n_i} - \xi \| < \liminf_{i \to \infty} \| x_{n_i} - \xi' \| \\ &= \liminf_{j \to \infty} \| x_j - \xi' \| < \liminf_{j \to \infty} \| x_j - \xi \| = d. \end{aligned}$$

This is a contradiction. Hence $\xi = \xi'$. This completes the proof.

If $N = 1$, then Theorem 2.1 is reduced to the following.

Corollary 2.2. *Let C be a nonempty closed convex subset of H, and $A : C \to H$ a L-Lipschitz continuous and monotone mapping. Let F be a bifunction from $C \times C$ to $\mathbb{R}$ which satisfies $(A1)$-$(A4)$. Assume that $\mathscr{F} := EP(F) \cap VI(C,A)$ is not empty. Let $\{\alpha_n\}$, $\{\beta_n\}$, and $\{\gamma_n\}$ be real* *number sequences in* $(0,1)$*. Let* $\{\lambda_n\}$*,* $\{r_n\}$ *be positive real number sequences. Let* $\{e_n\}$ *be a bounded sequence in* H*. Let* $\{x_n\}$ *be a sequence generated in the following manner:*

$$\begin{cases} x_1 \in H, \\ F(z_n, z) + \frac{1}{r_n}\langle z - z_n, z_n - x_n\rangle \geq 0, \quad \forall z \in C, \\ x_{n+1} = \alpha_n x_n + \beta_n \mathrm{Proj}_C(z_n - \lambda_n A y_n) + \gamma_n e_n, \quad n \geq 1, \\ y_n = \mathrm{Proj}_C(z_n - \lambda_n A z_n), \end{cases}$$

Assume that $\{\alpha_n\}$*,* $\{\beta_n\}$*,* $\{\gamma_n\}$*,* $\{\delta_n\}$*,* $\{\lambda_n\}$*,* $\{r_n\}$ *satisfy the following restrictions:*

(a) $\alpha_n + \beta_n + \gamma_n = 1$*,*

(b) $0 < a \leq \beta_n \leq b < 1$*, and* $\sum_{n=1}^{\infty} \gamma_n < \infty$*;*

(c) $\liminf_{n\to\infty} r_n > 0$*, and* $c \leq \lambda_n \leq d$*, where* $c, d \in (0, 1/L)$*.*

Then the sequence $\{x_n\}$ *weakly converges to some point* $\bar{x} \in \mathscr{F}$*.*

If $F_m(x,y) \equiv 0$, for all $x, y \in C$, and $r_{n,m} \equiv 1$, then Theorem 3.1 is reduced to the following.

Corollary 2.3. *Let* C *be a nonempty closed convex subset of* H*, and* $A : C \to H$ *a* L*-Lipschitz continuous and monotone mapping. Assume that* $VI(C,A)$ *is not empty. Let* $\{\alpha_n\}$*,* $\{\beta_n\}$*, and* $\{\gamma_n\}$ *be real number sequences in* $(0,1)$*. Let* $\{\lambda_n\}$ *be a positive real number sequence. Let* $\{e_n\}$ *be a bounded sequence in* H*. Let* $\{x_n\}$ *be a sequence generated in the following manner:*

$$\begin{cases} x_1 \in H, \\ x_{n+1} = \alpha_n x_n + \beta_n \mathrm{Proj}_C(\mathrm{Proj}_C x_n - \lambda_n A y_n) + \gamma_n e_n, \quad n \geq 1, \\ y_n = \mathrm{Proj}_C(\mathrm{Proj}_C x_n - \lambda_n A \mathrm{Proj}_C x_n). \end{cases}$$

Assume that $\{\alpha_n\}$*,* $\{\beta_n\}$*,* $\{\gamma_n\}$*, and* $\{\lambda_n\}$*, satisfy the following restrictions:*

(a) $\alpha_n + \beta_n + \gamma_n = 1$*,*

(b) $0 < a \leq \beta_n \leq b < 1$*, and* $\sum_{n=1}^{\infty} \gamma_n < \infty$*;*

(c) $c \leq \lambda_n \leq d$*, where* $c, d \in (0, 1/L)$*.*

Then the sequence $\{x_n\}$ *weakly converges to some point* $\bar{x} \in \mathscr{F}$*.*

References

[1] S. Lv, Strong convergence of a general iterative algorithm in Hilbert spaces, J. Inequal. Appl. 2013 (2013), Article ID 19.

[2] P. Cheng, H. Wu, Strong convergence theorems for common solutions of a family of nonexpansive mappings and an accretive operator, Fixed Point Theory Appl. 2013 (2013), Article ID 172.

[3] Y. Hao, Some results on a modified Mann iterative scheme in a reflexive Banach space, Fixed Point Theory Appl. 2013 (2013), Article ID 227.

[4] C. Wu, A. Liu, Strong convergence of a hybrid projection iterative algorithm for common solutions of operator equations and of inclusion problems, Fixed Point Theory Appl. 2012 (2012), Article ID 90.

[5] S. Wang, Monotone variational inequalities, generalized equilibrium problems and fixed point methods, Fixed Point Theory Appl. 2014 (2014), Article ID 236.

[6] E. Blum, W. Oettli, From optimization and variational inequalities to equilibrium problems, Math. Student 63 (1994), 123-145.

[7] P.L. Combettes, S.A. Hirstoaga, Equilibrium programming in Hilbert spaces, J. Nonlinear Convex Anal. 6 (2005), 117-136.

[8] R.T. Rockafellar, On the maximality of sums of nonlinear monotone operators, Trans. Amer. Math. Soc. 149 (1970), 75-88.

[9] Y. Hao, S.Y. Cho, X. Qin, Some weak convergence theorems for a family of asymptotically nonexpansive nonself mappings, Fixed Point Theory Appl. 2010 (2010), Article ID 218573.

[10] J. Schu, Weak and strong convergence of fixed points of asymptotically nonexpansive mappings, Bull. Austral. Math. Soc. 43 (1991), 153-159.

[11] K.K. Tan, H.K. Xu, Approximating fixed points of nonexpansive mappings by the Ishikawa iterative process, J. Math. Anal. Appl. 178 (1993) 301-308.