J. Nonlinear Funct. Anal. 2015 (2015), Article ID 20

MATHRES
Journal of Nonlinear Functional Analysis
Available online at http://jnfa.mathres.org

FIXED POINTS ON INTUITIONISTIC FUZZY METRIC SPACES USING THE E.A. PROPERTY

ISMAT BEG[1]*, VISHAL GUPTA[2], ASHIMA KANWAR[2]

[1]Centre for Mathematics and Statistical Sciences, Lahore School of Economics, Lahore-53200, Pakistan

[2]Department of Mathematics, Maharishi Markandeshwar University, Mullana, Ambala, Haryana, India

Abstract. The aim of this paper is to prove new common fixed point theorems on intuitionistic fuzzy metric spaces. Our main result, bring into play the concept of the E.A. property, complete subspaces and weakly compatible mappings. An example is furnished which demonstrates the validity of our main results. We also apply our results to integral types contraction conditions to identify the unique fixed point of four self mappings on intuitionistic fuzzy metric spaces.

Keywords. Fixed point; Fuzzy set; Intuitionistic fuzzy metric space; Weakly compatible mapping; Contractive condition of integral type.

1. Introduction

The contraction mapping principle on complete metric spaces first appeared in Banach thesis [1]. While dealing with natural world with uncertainty, we find that classical techniques do not suffice and thus some techniques with some specific logic are necessitated. In his seminal paper, Zadeh [2] introduced the notion of fuzzy sets for broad applications. Fuzzy set theory is one of the uncertainty approaches which help to build up mathematical models well suited to concrete real life situations. Fuzziness is further explored as an alternative to randomness for describing uncertainty. Shortly after the appearance of fuzzy set, Kramosil and Michalek [3]

*Corresponding author

E-mail address: ibeg@lahoreschool.edu.pk (I. Beg)

Received July 20, 2015

introduced the concept of fuzzy metric space. The relationships between the fixed point theory and the geometry of fuzzy metric spaces have been very close and cohesive. Following this concept, the notion of the continuous t-norm is pertains to George and Veeramani [4]. Seesa [5] introduced the notion of weakly commuting maps in metric spaces. Jungck [6] further generalized it to compatible maps.

Afterward, Jungck and Rhoades [7] introduced weakly compatible maps and proved that compatible maps are weakly compatible but the converse is not true. Later on, many researches contributed towards the development of fixed point theory on metric spaces. Aamri and Moutawakil [8] generalized the concept of non compatibility by defining the E.A. property for self mappings. It contained the class of non-compatible mappings in metric spaces. Subsequently, a number of fixed point results were proved for contraction mappings satisfying the E.A. property.

In 2009, Vijayaraju and Sajath [9] proved following theorem on two self mappings using the E.A. property:

Theorem 1.1. [9] *Let S and T be two weakly compatible self mappings of a fuzzy metric space $(X,M,*)$ with $t*t \geq t$ such that for each $x \neq y$, $0<q<1$,*

(1) *S and T satisfy the E.A. property,*
(2) $M(Tx,Ty,qt) \geq \min\{M(Sx,Sy,t), M(Sx,Ty,t), M(Tx,Sy,t), M(Tx,Sx,t), M(Ty,Sy,t)\}$,
(3) $T(X) \supset S(X)$,
(4) *$T(X)$ or $S(X)$ is a complete subspace of X.*

Then S and T have a unique common fixed point.

The idea of an intuitionistic fuzzy set is initiated by Atanassov [10]. Using this theory, Alaca *et al.* [11] defined the notion of intuitionistic fuzzy metric spaces with the help of the continuous t-norm and the continuous t-conorm. Further, they introduced the notion of Cauchy sequences in intuitionistic fuzzy metric spaces.

The following result on intuitionistic fuzzy metric spaces for two self mappings satisfying contractive condition is proved by Turkoglu *et al.* [12]:

Theorem 1.2. [12] *Let* $(X,M,N,*,\diamond)$ *be a complete intuitionistic fuzzy metric spaces. Let* $f,g : X \to X$ *be self mapping satisfying the following conditions:*

(1) $g(X) \subset f(X)$,

(2) f *is continuous,*

(3) there exists $k \in (0,1)$ *such that for every* $x,y \in X, t > 0$,

$$\begin{cases} M(gx,gy,kt) \geq M(fx,fy,t), \\ N(gx,gy,kt) \leq N(fx,fy,t). \end{cases}$$

Then f *and* g *have a unique common fixed point in* X *provided* f *and* g *commute.*

In 2009, Sharma *et al.* [13] proved common fixed point theorems on intuitionistic fuzzy metric spaces without using the continuity and compatibility. Some significant results on fuzzy and intuitionistic fuzzy metric spaces were proved by Beg *et al.* [14, 15].

In 2002, Branciari [16] gave an analogue of Banach contraction principle by defining Lebesgue-integrable function and proved a fixed point theorem satisfying contractive conditions of integral type.

Lemma 1.3. [16] **(Branciari-Integral contractive type condition)** *Let* (X,d) *be a complete metric space,* $c \in (0,1)$*, and let* $f : X \to X$ *be a mapping such that for each* $x,y \in X$

$$\int_0^{d(fx,fy)} g(t)dt \leq c \int_0^{d(x,y)} g(t)dt,$$

where $g : [0,\infty) \to [0,\infty)$ *is a Lebesgue-integrable mapping which is summable on each compact subset of* $[0,\infty)$*, non-negative and such that for each* $\varepsilon > 0$*,* $\int_0^{\varepsilon} g(t)dt > 0$*, then* f *has a unique fixed point* $a \in X$ *such that for each* $x \in X$*,* $\lim_{n\to\infty} f^n x = a$.

Thus, Branciari-Integral contractive type condition is a generalization of Banach contraction condition if $g(t) = 1$, $\forall t \geq 0$.

Some other useful fixed point theorems satisfying the E.A. property and generalized weak con-tractive condition of integral type on fuzzy metric space are proved by Gupta *et al.* [17,18,19].

2. Preliminaries

Definition 2.1. [20] *A binary operation* $*: [0,1] \times [0,1] \to [0,1]$ is called continuous t-norm if $*$ satisfies following conditions:

(1) $*$ is commutative and associative,

(2) $*$ is continuous,

(3) $a * 1 = a$, $\forall a \in [0,1]$,

(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition 2.2. [20] *A binary operation* $\diamond : [0,1] \times [0,1] \to [0,1]$ is called continuous t-conorm if $\diamond$ satisfies following conditions:

(1) $\diamond$ is commutative and associative,

(2) $\diamond$ is continuous,

(3) $a \diamond 0 = a$, $\forall a \in [0,1]$,

(4) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition 2.3. [11] *A 5-tuple* $(X, M, N, *, \diamond)$ *is said to be intuitionistic fuzzy metric space(IFM space) if* X *is an arbitrary set,* $*$ *is a continuous t -norms,* $\diamond$ *is a continuous t-conorm.* M *and* N *are fuzzy sets in* $X \times X \times [0, \infty)$ *satisfying following conditions for all* $x, y, z \in X$ *and* $s, t > 0$,

(1) $M(x,y,t) + N(x,y,t) \leq 1$,

(2) $M(x,y,0) = 0$,

(3) $M(x,y,t) = 1$ *for all* $t > 0$ *if and only if* $x = y$,

(4) $M(x,y,t) = M(y,x,t)$

(5) $M(x,y,t) = 1$ *as* $t \to \infty$,

(6) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$,

(7) $M(x,y,\cdot) : [0,\infty) \to [0,1]$ *is left continuous,*

(8) $N(x,y,0) = 1$,

(9) $N(x,y,t) = 0$ *for all* $t > 0$ *if and only if* $x = y$,

(10) $N(x,y,t) = N(y,x,t)$

(11) $N(x,y,t) = 0$ *as* $t \to \infty$,

(12) $N(x,y,t) \diamond N(y,z,s) \geq N(x,z,t+s)$,

(13) $M(x,y,\cdot) : [0,\infty) \to [0,1]$ *is right continuous.*

Here, $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non near-ness between x and y with respect t, respectively. In intuitionistic fuzzy metric space, $M(x,y,t)$ is non-decreasing and $N(x,y,t)$ is non-increasing.

Definition 2.4. [11] *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space(IFM space) then*

(1) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if
$\lim_{n\to\infty} M(x_n,x,t) = 1$, $\lim_{n\to\infty} N(x_n,x,t) = 0, \forall\, t > 0$.

(2) A sequence $\{x_n\}$ in X is said to be a Cauchy sequence if
$\lim_{n\to\infty} M(x_{n+p},x_n,t) = 1$, $\lim_{n\to\infty} N(x_{n+p},x_n,t) = 0, \forall\, t > 0$ *and* $p > 0$.

Definition 2.5. [11] *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space in which every Cauchy sequence is convergent, then $(X,M,N,*,\diamond)$ is said to be a complete fuzzy metric space.*

Definition 2.6. [21] *Two self maps P and Q from an intuitionistic fuzzy metric space $(X,M,N,*,\diamond)$ into itself are said to be compatible maps if*

$$\lim_{n\to\infty} M(PQx_n, QPx_n, t) = 1,\ \lim_{n\to\infty} N(PQx_n, QPx_n, t) = 0,$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = z$ *for some $z \in X$.*

Definition 2.7. [7] *Two self maps P and Q on a set X are said to be weakly compatible if they commute at coincidence points.*

Definition 2.8. [8] *Two self maps P and Q on a set X are said to satisfy the E.A. property if there exists a sequence $\{x_n\}$ in X such that* $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = x$ *for some $x \in X$.*

Lemma 2.9. [22] *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space and for a number $k \in (0,1)$ and for all $x,y \in X, t > 0$, $M(x,y,kt) \geq M(x,y,t)$ and $N(x,y,kt) \leq N(x,y,t)$, then $x = y$.*

The above definitions and results motivated us to prove new common fixed point theorems for six self mappings on intuitionistic fuzzy metric spaces by using the E.A. property. In this paper, we have given an example, which justifies our result.

3. Main results

Theorem 3.1. *Let P,Q,R,S,T and U be self mappings on an intuitionistic fuzzy metric spaces $(X,M,N,*,\diamond)$ with t-norm $a*b=\min\{a,b\}$ and t-conorm $a\diamond b=\max\{a,b\}$ such that*

T1: $P(X)\subset SU(X)$, $Q(X)\subset RT(X)$,

T2: the pair (P,RT) or (Q,SU) satisfies E.A. property,

T3: there exists $k\in(0,1)$ such that for every $x,y\in X,t>0$,

$$\begin{cases} M(Px,Qy,kt)\geq M(RTx,SUy,t)*M(Px,RTx,t)*M(Qy,SUy,t)*M(Px,SUy,t), \\ N(Px,Qy,kt)\leq N(RTx,SUy,t)\diamond N(Px,RTx,t)\diamond N(Qy,SUy,t)\diamond N(Px,SUy,t), \end{cases}$$

if one of $P(X),Q(X),RT(X),ST(X)$ is a complete subspace of X, then (P,RT) and (Q,SU) have a coincident point. Further, if (P,RT) and (Q,SU) are weakly compatible, then P, Q, RT and SU have a unique common fixed point in X.

Proof. Suppose (Q,SU) satisfies the E.A. property. Then there exists a sequence $\{x_n\}$ such that $x_n\in X$, such that

$$\lim_{n\to\infty}Qx_n=\lim_{n\to\infty}SUx_n=l,l\in X. \tag{3.1}$$

Also $Q(X)\subset RT(X)$, then there exists $\{y_n\}$ in X such that

$$Qx_n=RTy_n. \tag{3.2}$$

We get, $\lim_{n\to\infty}RTy_n=l$.

Step - I: We claim that $\lim_{n\to\infty}Py_n=l$.

For this, put $x=y_n$, $y=x_n$ in (T3) and from (3.1),(3.2), we get

$$\begin{cases} M(Py_n,Qx_n,kt)\geq M(RTy_n,SUx_n,t)*M(Py_n,RTy_n,t)*M(Qx_n,SUx_n,t)*M(Py_n,SUx_n,t), \\ N(Py_n,Qx_n,kt)\leq N(RTy_n,SUx_n,t)\diamond N(Py_n,RTy_n,t)\diamond N(Qx_n,SUx_n,t)\diamond N(Py_n,SUx_n,t). \end{cases}$$

Taking $n\to\infty$, we get

$$\begin{cases} M(Py_n,l,kt)\geq M(l,l,t)*M(Py_n,l,t)*M(l,l,t)*M(Py_n,l,t), \\ N(Py_n,l,kt)\leq N(l,l,t)\diamond N(Py_n,l,t)\diamond N(l,l,t)\diamond N(Py_n,l,t). \end{cases}$$

Then

$$\lim_{n\to\infty}Py_n=l=\lim_{n\to\infty}SUy_n. \tag{3.3}$$

Step - II: Let $SU(X)$ is a complete subspace of X, then $l = SU(m)$ for some $m \in X$.

This gives

$$\lim_{n\to\infty} Py_n = \lim_{n\to\infty} RTy_n = \lim_{n\to\infty} Qx_n = \lim_{n\to\infty} SUx_n = l = RT(m). \tag{3.4}$$

Now we show that, $P(m) = RT(m)$. Taking $x = m$, $y = x_n$ in (T3), we have

$$\begin{cases} M(Pm, Qx_n, kt) \geq M(RTm, SUx_n, t) * M(Pm, RTm, t) * M(Qx_n, SUx_n, t) * M(Pm, SUx_n, t), \\ N(Pm, Qx_n, kt) \leq N(RTm, SUx_n, t) \diamond N(Pm, RTm, t) \diamond N(Qx_n, SUx_n, t) \diamond N(Pm, SUx_n, t). \end{cases}$$

By considering $n \to \infty$ and using (3.4), we get

$$P(m) = RT(m). \tag{3.5}$$

This implies (P, RT) have coincident point $m \in X$.

Step-III: The weak compatibility of (P, RT) implies that $PRT(m) = RTP(m)$.

Thus

$$PP(m) = PRT(m) = RTP(m) = RTRT(m). \tag{3.6}$$

As $P(X) \subset SU(X)$, there exists $p \in X$ such that

$$P(m) = SU(p). \tag{3.7}$$

We claim that $SU(p) = Q(p)$. Taking $x = m, y = p$ in (T3), and from (3.5) and (3.7), we obtain

$$\begin{cases} M(Pm, Qp, kt) \geq M(RTm, SUp, t) * M(Pm, RTm, t) * M(Qp, SUp, t) * M(Pm, SUp, t), \\ N(Pm, Qp, kt) \leq N(RTm, SUp, t) \diamond N(Pm, RTm, t) \diamond N(Qp, SUp, t) \diamond N(Pm, SUp, t). \end{cases}$$

So we have

$$\begin{cases} M(Pm, Qp, kt) \geq M(Pm, Qp, t), \\ N(Pm, Qp, kt) \leq N(Pm, Qp, t). \end{cases}$$

Hence, one can get $Pm = Qp$. Thus we have

$$Pm = RTm = SUp = Qp. \tag{3.8}$$

The weak compatibility of (Q, SU) implies that $QSUp = SUQp$. This gives, $QSUp = SUQp = QQp = SUSUp$.

Step -IV: Claim that Pm is the common fixed point of P,Q,RT,SU. Again taking $x=Pm, y=p$ in (T3)

$$\begin{cases} M(PPm,Qp,kt) \geq M(RTPm,SUp,t) * M(PPm,RTPm,t) * M(Qp,SUp,t) * M(PPm,SUp,t), \\ N(PPm,Qp,kt) \leq N(RTPm,SUp,t) \diamond N(PPm,RTPm,t) \diamond N(Qp,SUp,t) \diamond N(PPm,SUp,t). \end{cases}$$

Therefore, one get

$$\begin{cases} M(PPm,Qp,kt) \geq M(PPm,Qp,t), \\ N(PPm,Qp,kt) \leq N(PPm,Qp,t). \end{cases}$$

This gives $PPm = Qp = Pm$. Therefore,

$$Pm = PPm = RTPm \tag{3.9}$$

is common fixed point of P and RT. Similarly, we can prove that Qp is the common fixed point of SU and Q. Since $Pm = Qp$. So, Pm is the fixed point of P,Q,RT,SU.

Step- V: Finally, we show the uniqueness of the common fixed point. If possible, let x_0 and y_0 be two different fixed point of P,Q,RT,SU. By taking $x=x_0, y=y_0$ in (T3), one has

$$\begin{cases} M(Px_0,Qy_0,kt) \geq M(RTx_0,SUy_0,t) * M(Px_0,RTx_0,t) * M(Qy_0,SUy_0,t) * M(Px_0,SUy_0,t), \\ N(Px_0,Qy_0,kt) \leq N(RTx_0,SUy_0,t) \diamond N(Px_0,RTx_0,t) \diamond N(Qy_0,SUy_0,t) \diamond N(Px_0,SUy_0,t). \end{cases}$$

By using the definition of fixed points and fuzzy metric spaces, we get $x_0 = y_0$. Thus, the mappings P,Q,RT,SU have a unique common fixed point.

Theorem 3.2. *Let P,Q,R and S be self mappings on an intuitionistic fuzzy metric spaces $(X,M,N,*,\diamond)$ with t-norm $a*b = \min\{a,b\}$ and t-conorm $a \diamond b = \max\{a,b\}$ such that*

i: $P(X) \subset S(X)$, $Q(X) \subset R(X)$,

ii: the pair (P,R) or (Q,S) satisfies the E.A. property,

iii: there exists $k \in (0,1)$ such that for every $x,y \in X, t > 0$,

$$\begin{cases} M(Px,Qy,kt) \geq M(Rx,Sy,t) * M(Px,Rx,t) * M(Qy,Sy,t) * M(Px,Sy,t), \\ N(Px,Qy,kt) \leq N(Rx,Sy,t) \diamond N(Px,Rx,t) \diamond N(Qy,Sy,t) \diamond N(Px,Sy,t), \end{cases}$$

if one of $P(X), Q(X), R(X), S(X)$ is a complete subspace of X, then (P,R) and (Q,S) have a coincident point. Further, if (P,R) and (Q,S) are weakly compatible, then P, Q, R and S have a unique common fixed point in X.

Proof. If we put $T = U = I_X$ (the identity map on X) in Theorem 3.1, then we have above result.

Example 3.3. *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric spaces $X = [1,7]$ with t-norm $a*b = \min\{a,b\}$ and t-conorm $a\diamond b = \max\{a,b\}$. M and N are fuzzy set on $X^2\times(0,\infty)$ defined by*

$$M(x,y,t) = \frac{t}{t+d(x,y)},\ N(x,y,t) = \frac{d(x,y)}{t+d(x,y)},$$

where $d(x,y) = |x-y|$, $\forall x,y \in X$, $t > 0$. Define $P,Q,R,S : X \to X$ by

$$P(x) = \begin{cases} 1 & if \quad x = 1, \\ 3 & if \quad 1 < x < 7, \end{cases} \qquad Q(x) = \begin{cases} 1 & if \quad x = 1, \\ 4 & if \quad 1 < x < 7, \end{cases}$$

$$R(x) = \begin{cases} 1 & if \quad x = 1, \\ 4 & if \quad 1 < x \le 5, \\ x-5, & if \quad 5 < x \le 7, \end{cases} \qquad S(x) = \begin{cases} 1 & if \quad x = 1, \\ x-1 & if \quad 1 < x \le 5, \\ 3 & if \quad 5 < x \le 7. \end{cases}$$

Clearly, (P,R) and (Q,S) are weakly compatible. Let sequence $\{x_n\}$ be defined as

$$x_n = 5 + \frac{1}{n}, n \ge 1.$$

Then, we have $\lim_{n\to\infty} Qx_n = \lim_{n\to\infty} Sx_n = 4$. Hence (Q,S) satisfy the E.A. property. Thus P,Q,R and S satisfies all the conditions of Theorem 3.2, then $x = 1$ is unique common fixed point of P,Q,R and S in X.

Corollary 3.4. *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric spaces. Let P,Q,R and S be mappings from X into itself satisfying (i) - (ii) of Theorem 3.2 such that*

$$\begin{cases} M(Px,Qy,kt) \ge M(Rx,Sy,t)*M(Px,Rx,t)*M(Qy,Sy,t)*M(Qy,Rx,2t)*M(Px,Sy,t) \\ N(Px,Qy,kt) \le N(Rx,Sy,t)\diamond N(Px,Rx,t)\diamond N(Qy,Sy,t)\diamond N(Qy,Rx,2t)\diamond N(Px,Sy,t) \end{cases}$$

for every $x, y \in X, t > 0$. If one of $P(X), Q(X), R(X), S(X)$ is a complete subspace of X, then (P,R) and (Q,S) have a coincident point. Further, if (P,R) and (Q,S) are weakly compatible, then P, Q, R and S have a unique common fixed point in X.

Proof. Since

$$\begin{cases} M(Px,Qy,kt) \geq M(Rx,Sy,t) * M(Px,Rx,t) * M(Qy,Sy,t) * M(Qy,Rx,2t) * M(Px,Sy,t), \\ N(Px,Qy,kt) \leq N(Rx,Sy,t) \diamond N(Px,Rx,t) \diamond N(Qy,Sy,t) \diamond N(Qy,Rx,2t) \diamond N(Px,Sy,t). \end{cases}$$

By using definition of an intuitionistic fuzzy metric spaces, on has $M(Px,Qy,kt) \geq M(Rx,Sy,t) *$ $M(Px,Rx,t) * M(Qy,Sy,t) * M(Qy,Rx,2t) * M(Px,Sy,t)$

$\geq M(Rx,Sy,t) * M(Px,Rx,t) * M(Qy,Sy,t) * M(Rx,Sy,t) * M(Sy,Qy,t) * M(Px,Sy,t)$,

$N(Px,Qy,kt) \leq N(Rx,Sy,t) \diamond N(Px,Rx,t) \diamond N(Qy,Sy,t) \diamond N(Qy,Rx,2t) \diamond N(Px,Sy,t)$

$\leq N(Rx,Sy,t) \diamond N(Px,Rx,t) \diamond N(Qy,Sy,t) \diamond M(Rx,Sy,t) \diamond M(Sy,Qy,t) \diamond N(Px,Sy,t)$.

Thus by using Theorem 3.2 one can show that P, Q, R and S have a unique fixed point in X.

Corollary 3.5. *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric spaces. Let P,Q,R and S be mappings from X into itself satisfying (i)- (ii) of Theorem 3.2 such that*

$M(Px,Qy,kt) \geq M(Rx,Sy,t)$ and $N(Px,Qy,kt) \leq N(Rx,Sy,t)$ for every $x,y \in X, t > 0$.

If one of $P(X), Q(X), R(X), S(X)$ is a complete subspace of X, then (P,R) and (Q,S) have a coincident point. Further, if (P,R) and (Q,S) are weakly compatible, then P, Q, R and S have a unique common fixed point in X.

Proof. Here, we have

$$\begin{cases} M(Rx,Sy,t) = M(Rx,Sy,t) * 1 = M(Rx,Sy,t) * M(Px,Px,5t), \\ N(Rx,Sy,t) = N(Rx,Sy,t) \diamond 0 = N(Rx,Sy,t) \diamond M(Px,Px,5t). \end{cases}$$

By using the definition of intuitionistic fuzzy metric spaces, we have

$$\begin{cases} M(Rx,Sy,t) * M(Px,Px,5t) & \geq M(Rx,Sy,t) * M(Px,Sy,t) * M(Qy,Sy,t) * M(Qy,Rx,2t), \\ & *M(Px,Rx,t), \\ N(Rx,Sy,t) \diamond M(Px,Px,5t) & \leq N(Rx,Sy,t) \diamond N(Px,Sy,t) \diamond N(Qy,Sy,t) \diamond N(Qy,Rx,2t) \\ & \diamond N(Px,Rx,t). \end{cases}$$

From Corollary 3.4, we obtained the result immediately.

Let S and R be identity mappings on X in Corollary 3.5. Then we get the following results.

Corollary 3.6. *Let $(X,M,N,*,\diamond)$ be a complete intuitionistic fuzzy metric spaces. Let P and Q be mappings from X into itself satisfying (i)- (ii) of Theorem 3.2 such that $M(Px,Qy,kt) \geq M(x,y,t)$ and $N(Px,Qy,kt) \leq N(x,y,t)$ for every $x,y \in X, t > 0$. Then P and Q have aunique fixed point in X.*

Remark 3.7. *In Corollary 3.6, if we take $P = Q$, then following result becomes the Banach contraction theorem on intuitionistic fuzzy metric space.*

Corollary 3.8. *Let $(X,M,N,*,\diamond)$ be a complete intuitionistic fuzzy metric spaces. Let P be mapping from X into itself such that*
$M(Px,Py,kt) \geq M(x,y,t)$ and $N(Px,Py,kt) \leq N(x,y,t)$ for every $x,y \in X, t > 0$. Then P has a unique fixed point in X.

Now we extend Theorem 3.1 for finite number of mappings in the following way:

Theorem 3.9. *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric spaces. Let $R_1,R_2,R_3,\cdots,R_z$, $S_1,S_2,S_3,\cdots,S_z$, P and Q be mappings from X into itself such that*

- *i: $P(X) \subset S_1S_2S_3\cdots S_z(X)$, $Q(X) \subset R_1R_2R_3\cdots R_z(X)$,*
- *ii: the pair $(P,R_1R_2R_3\cdots R_z)$ or $(Q,S_1S_2S_3\cdots S_z)$ satisfies the E.A. property,*
- *iii: there exists $k \in (0,1)$ such that for every $x,y \in X, t > 0$,*

$$\begin{cases} M(Px,Qy,kt) & \geq M(R_1R_2R_3\cdots R_zx, S_1S_2S_3\cdots S_zy,t) * M(Px,R_1R_2R_3\cdots R_zx,t) \\ & * M(Qy,S_1S_2S_3\cdots S_zy,t) * M(Px,S_1S_2S_3\cdots S_zy,t), \\ N(Px,Qy,kt) & \leq N(R_1R_2R_3\cdots R_zx, S_1S_2S_3\cdots S_zy,t) \diamond N(Px,R_1R_2R_3\cdots R_zx,t) \\ & \diamond N(Qy,S_1S_2S_3\cdots S_zy,t) \diamond N(Px,S_1S_2S_3\cdots S_zy,t), \end{cases}$$

if one of $P(X),Q(X),R_1R_2R_3\cdots R_z(X)$ and $S_1S_2S_3\cdots S_z(X)$ is a complete subspace of X, then $(P,R_1R_2R_3\cdots R_z)$ and $(Q,S_1S_2S_3\cdots S_z)$ have a coincident point. Further, if $(P,R_1R_2R_3\cdots R_z)$ and $(Q,S_1S_2S_3\cdots S_z)$ are weakly compatible, then P, Q, $R_1R_2R_3\cdots R_z$ and $S_1S_2S_3\cdots S_z$ have a unique common fixed point in X.

Proof. Since $(Q, S_1S_2S_3\cdots S_z)$ satisfies the E.A. property, there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Qx_n = \lim_{n\to\infty} S_1S_2S_3\cdots S_zx_n = l \in X$. Also, $Q(X) \subset R_1R_2R_3\cdots R_z(X)$, there exists $\{y_n\}$ in X such that $Qx_n = R_1R_2R_3\cdots R_zy_n$. By using the method of proof of Theorem 3.1, one can easily get the result.

Now, we give a fixed point theorem on fuzzy metric spaces which satisfies the integral types contractive condition for four self mappings.

Theorem 3.10. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric spaces. Let P, Q, R and S be mappings from X into itself such that*

i: $P(X) \subset S(X)$, $Q(X) \subset R(X)$,

ii: *the pair (P, R) or (Q, S) satisfies E.A. property,*

iii: *there exists $k \in (0, 1)$ such that for every $x, y \in X, t > 0$,*

$$\begin{cases} \int_0^{M(Px,Qy,kt)} \psi(t)dt \geq \int_0^{U(x,y,t)} \psi(t)dt, \\ \int_0^{N(Px,Qy,kt)} \psi(t)dt \leq \int_0^{V(x,y,t)} \psi(t)dt, \end{cases}$$

where, $\psi : R^+ \to R$ is Lebesgue- integrable mapping which is summable and non-negative and $U(x,y,t) = M(Rx,Sy,t) * M(Px,Rx,t) * M(Qy,Sy,t) * M(Px,Sy,t)$, $V(x,y,t) = N(Rx,Sy,t) \diamond N(Px,Rx,t) \diamond N(Qy,Sy,t) \diamond N(Px,Sy,t)$.

If one of $P(X), Q(X), R(X), S(X)$ is a complete subspace of X, then (P, R) and (Q, S) have a coincident point.
Further, if (P, R) and (Q, S) are weakly compatible, then P, Q, R and S have a unique common fixed point in X.

Proof. Suppose (Q, S) satisfies the E.A. property. Then there exists a sequence $\{x_n\}$ in X, such that

$$\lim_{n\to\infty} Qx_n = \lim_{n\to\infty} Sx_n = l, l \in X. \tag{3.10}$$

Also $Q(X) \subset R(X)$, then there exists $\{y_n\}$ in X such that

$$Qx_n = Ry_n. \tag{3.11}$$

We get

$$\lim_{n\to\infty} Ry_n = l. \tag{3.12}$$

We show that $\lim_{n\to\infty} Py_n = l$. For this, put $x = y_n$, $y = x_n$ in (iii), from (3.11) and (3.12), this implies

$$\begin{cases} \int_0^{M(Py_n,Qx_n,kt)} \psi(t)dt \geq \int_0^{U(y_n,x_n,t)} \psi(t)dt, \\ \int_0^{N(Py_n,Qx_n,kt)} \psi(t)dt \leq \int_0^{V(y_n,x_n,t)} \psi(t)dt, \end{cases}$$

where, $U(y_n,x_n,t) = M(Ry_n,Sx_n,t) * M(Py_n,Ry_n,t) * M(Qx_n,Sx_n,t) * M(Py_n,Sx_n,t)$, $V(y_n,x_n,t) = N(Ry_n,Sx_n,t) \diamond N(Py_n,Ry_n,t) \diamond N(Qx_n,Sx_n,t) \diamond N(Py_n,Sx_n,t)$. By using (3.10), (3.11), (3.12) and Lemma 1.3 and Lemma 2.9, we have

$$\lim_{n\to\infty} Py_n = l = \lim_{n\to\infty} Sy_n.$$

Suppose $S(X)$ is a complete subspace of X. Then $l = S(m)$ for some $m \in X$.

$$\lim_{n\to\infty} Py_n = \lim_{n\to\infty} Ry_n = \lim_{n\to\infty} Qx_n = \lim_{n\to\infty} Sx_n = l = R(m). \tag{3.13}$$

Now we show that, $P(m) = R(m)$. Taking $x = m$, $y = x_n$ in (iii), by using (3.13), we have

$$\begin{cases} \int_0^{M(Pm,Qx_n,kt)} \psi(t)dt \geq \int_0^{U(m,x_n,t)} \psi(t)dt, \\ \int_0^{N(Pm,Qx_n,kt)} \psi(t)dt \leq \int_0^{V(m,x_n,t)} \psi(t)dt, \end{cases}$$

where $U(m,x_n,t) = M(Rm,Sx_n,t) * M(Pm,Rm,t) * M(Qx_n,Sx_n,t) * M(Pm,Sx_n,t)$, $V(m,x_n,t) = N(Rm,Sx_n,t) \diamond N(Pm,Rm,t) \diamond N(Qx_n,Sx_n,t) \diamond N(Pm,Sx_n,t)$. As $n \to \infty$, using Lemma 1.3 and Lemma 2.9, we get $P(m) = R(m)$. This implies (P,R) have coincident point $m \in X$. The weak compatibility of (P,R) implies that $PR(m) = RP(m)$. Thus

$$PP(m) = PR(m) = RP(m) = RR(m). \tag{3.14}$$

As $P(X) \subset S(X)$, there exists $p \in X$ such that

$$P(m) = S(p). \tag{3.15}$$

Next, we claim that $S(p) = Q(p)$. Taking $x = m, y = p$ in (iii), we get

$$\begin{cases} \int_0^{M(Pm,Qp,kt)} \psi(t)dt \geq \int_0^{U(m,p,t)} \psi(t)dt, \\ \int_0^{N(Pm,Qp,kt)} \psi(t)dt \leq \int_0^{V(m,p,t)} \psi(t)dt, \end{cases}$$

where $U(m,p,t)=M(Rm,Sp,t)*M(Pm,Rm,t)*M(Qp,Sp,t)*M(Pm,Sp,t)$, $V(m,p,t)=N(Rm,Sp,t)\diamond N(Pm,Rm,t)\diamond N(Qp,Sp,t)\diamond N(Pm,Sp,t)$. By considering (3.15) and using Lemma 1.3 and Lemma 2.9, we obtain $Pm=Qp$. Thus we have

$$Pm=Rm=Sp=Qp. \tag{3.16}$$

The weak compatibility of (Q,S) implies that $QSp=SQp$. Thus,

$$QSp=SQp=QQp=SSp. \tag{3.17}$$

Next, we prove that Pm is the common fixed point of P,Q,R and S. Again taking $x=Pm, y=p$ in (iii)

$$\begin{cases} \int_0^{M(PPm,Qp,kt)}\psi(t)dt \geq \int_0^{U(Pm,p,t)}\psi(t)dt, \\ \int_0^{N(PPm,Qp,kt)}\psi(t)dt \leq \int_0^{V(Pm,p,t)}\psi(t)dt, \end{cases}$$

where $U(Pm,p,t)=M(RPm,Sp,t)*M(PPm,RPm,t)*M(Qp,Sp,t)*M(PPm,Sp,t)$, $V(Pm,p,t)=N(RPm,Sp,t)\diamond N(PPm,RPm,t)\diamond N(Qp,Sp,t)\diamond N(PPm,Sp,t)$. From (3.14), (3.17) and by using Lemma 1.3 and Lemma 2.9 one can get $Pm=PPm=RPm$ is common fixed point of P and R. Similarly, we can prove that Qp is the common fixed point of S and Q. Since $Pm=Qp$, therefore, Pm is the fixed point of P,Q,R and S. Finally, we can easily show the uniqueness of the common fixed point by using condition (iii) of Theorem 3.10.

REFERENCES

[1] S. Banach, Sur les operations dans les ensembles abstraits et leur application aux equations integrales, Fund. Math. 3 (1922), 133-181.

[2] L.A. Zadeh, Fuzzy Sets, Information and Control 8 (1965), 338-353.

[3] I. Kramosil, J. Michalek, Fuzzy metric and Statistical metric spaces, Kybernetica 11 (1975), 326-334.

[4] A. George, P. Veeramani, On some results in fuzzy metric spaces, Fuzzy Sets and Systems 64 (1994), 336-344.

[5] S. Sessa, On a weak commutative condition in fixed point consideration, Publ. Inst. Math. 32 (1982), 146-153.

[6] G. Jungck, Compatible mappings and common fixed points, Int. J. Math Sci. 9 (1986), 771-779.

[7] G. Jungck, B.E. Rhoades, Fixed point for set valued functions without continuity, Indian J. Pure Appl. Math. 20 (1998), 227-238.

[8] M. Aamri, El. D. Moutawakil, Some new common fixed point theorems under strict contractive conditions, J. Math. Anal. Appl. 270 (2002), 181-188.

[9] P. Vijayaraju, Z. M. I. Sajath, Some common fixed point theorems in fuzzy metric spaces, Int. J. of Math. Anal. Ruse 3 (2009), 701-710.

[10] K. Atanassov, Intuitionistic fuzzy set, Fuzzy Sets and Systems 20 (1986), 87-96.

[11] C. Alaca, D. Turkoglu, C. Yildiz, Fixed points in intuitionistic fuzzy metric spaces, Chaos, Solitons Fractals, 29 (2006), 1073-1078.

[12] D. Turkoglu, C. Alaca, Y.J. Cho, C. Yildiz, Common fixed points in intuitionistic fuzzy metric spaces, J. Appl. Math. Comput. 22 (2006), 411- 424.

[13] S. Sharma, B. Deshpande, Common fixed point theorems for finite number of mappings without continuity and compatibility on intuitionistic fuzzy metric spaces, Chaos, Solitons Fractals, 40 (2009), 2242-2256.

[14] I. Beg, S. Sedghi, N. Shobe, Fixed point theorems in fuzzy metric spaces, Int. J. Anal. 2013 (2013), Article ID 934145.

[15] I. Beg, C. Vetro, D. Gopal, M. Imdad, (ϕ,ψ)-weak contractions in intuitionistic fuzzy metric spaces, J. Intelligent Fuzzy Sys. 26 (2014), 2497-2504.

[16] A. Branciari, A fixed point theorem for mappings satisfying a general contractive condition of integral type, Int. J. Math. Math. Sci. 29 (2002), 531-536.

[17] V. Gupta, A. Kanwar, Fixed point theorem in fuzzy metric spaces satisfying E.A. property, Indian J. Sci. Tech. 5 (2012), 3767-3769.

[18] V. Gupta, N. Mani, Existence and uniqueness of fixed point in fuzzy metric spaces and its applications, Advances in Intelligent Systems and Computing, Springer, 236 (2014), 217-224.

[19] V. Gupta, N. Mani, Fixed point theorems using control function in fuzzy metric spaces, Cogent Math. 2 (2015), Article ID 1053173.

[20] B. Schweizer, A. Sklar, Statistical metric spaces, Pacific J. Math. 10 (1960), 314-334.

[21] D. Turkoglu, C. Alaca, C. Yildiz, Compatible maps and Compatible maps of type (α) and (β) in intuitionistic fuzzy metric spaces, Demonstration Math. 39 (2006), 671-684.

[22] C. Alaca, I. Altun, D. Turkoglu, On compatible mappings of type (I) and (II) in IFM-spaces, Commun. Korean Math. Soc. 23 (2008), 427-446.