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SOME RESULTS ON A TWO-STEP ITERATIVE ALGORITHM IN HILBERT SPACES

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Abstract. In this article, a two-step iterative algorithm is investigated for a fixed point problem of a strict pseudocontraction and an equilibrium problem of a bifunction. Strong convergence theorems of common solutions are established in the framework of Hilbert spaces.

Keywords. Strictly pseudocontractive mapping; Nonexpansive mapping; Inverse-strongly monotone mapping; Equilibrium problem.

1. Introduction

Throughout this paper, we always assume that H is a real Hilbert space with the inner product $\langle \cdot, \cdot \rangle$ and the norm $\|\cdot\|$. Let C a nonempty closed convex subset of H and let P_C be the metric projection from H onto C.

Let $A : C \to H$ be a mapping. Recall that A is said to be monotone if

$$\langle Ax - Ay, x - y \rangle \geq 0, \quad \forall x, y \in C.$$

A is said to be strongly monotone if there exists a constant $\alpha > 0$ such that

$$\langle Ax - Ay, x - y \rangle \geq \alpha \|x - y\|^2, \quad \forall x, y \in C.$$

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For such a case, A is also called an α-strongly monotone mapping. A is said to be inverse-strongly monotone if there exists a constant $\alpha > 0$ such that

$$\langle Ax - Ay, x - y\rangle \geq \alpha\|Ax - Ay\|^2, \quad \forall x, y \in C.$$

For such a case, A is also called an α-inverse-strongly monotone mapping. From the definition, we see that A is inverse-strongly if and only if the inverse of A is strongly monotone. From the definition, we also have A is Lipschitz continuous.

Recall that a set-valued mapping $T : H \to 2^H$ is said to be monotone if for all $x, y \in H$, $f \in Tx$ and $g \in Ty$ imply $\langle x - y, f - g\rangle > 0$. A monotone mapping $T : H \to 2^H$ is maximal if the graph $Graph(T)$ of R is not properly contained in the graph of any other monotone mapping. It is known that a monotone mapping T is maximal if and only if, for any $(x, f) \in H \times H$, $\langle x - y, f - g\rangle \geq 0$ for all $(y, g) \in Graph(T)$ implies $f \in Tx$. Let A be a monotone mapping of C into H and let $N_C v$ be the normal cone to C at $v \in C$, i.e.,

$$N_C v = \{w \in H : \langle w, v - u\rangle \geq 0, \quad \forall u \in C\}$$

and define a mapping R on C by

$$Tv = \begin{cases} \emptyset, & v \notin C, \\ Av + N_C v, & v \in C. \end{cases}$$

Then T is maximal monotone and $0 \in Rv$ if and only if $\langle Av, u - v\rangle \geq 0$ for all $u \in C$; see [1] and the references therein. Gradient methods are popular and efficient to study zero points of monotone operators.

Let $S : C \to C$ be a mapping. In this paper, we use $F(S)$ to denote the fixed point set of S. Recall that the mapping S is said to be nonexpansive if

$$\|Sx - Sy\| \leq \|x - y\|, \quad \forall x, y \in C.$$

S is said to be k-strictly pseudocontractive if there exists a constant $k \in [0, 1)$ such that

$$\|Sx - Sy\|^2 \leq \|x - y\|^2 + k\|(x - Sx) - (y - Sy)\|^2, \quad \forall x, y \in C.$$

The class of strictly pseudocontractive mappings was introduced by Browder and Petryshyn [2] in 1967. It is easy to see that the class of strictly pseudocontractive mappings includes the class of nonexpansive mappings as a special case. If $k = 1$, then it called a pseudocontractive mapping. It is also easy to see that if A is an inverse-strongly monotone mapping, then the mapping $I - A$ is a strictly pseudocontractive mapping. Let I denote the identity operator on H and $A : H \to 2^H$ be a maximal monotone operator. Then we can define, for each $r > 0$, a nonexpansive single valued mapping $J_r : H \to H$ by $J_r = (I + rA)^{-1}$. It is called the resolvent of A. We know that $A^{-1}0 = F(J_r)$ for all $r > 0$ and J_r is firmly nonexpansive.

The classical variational inequality problem is to find $u \in C$ such that

$$\langle Au, v - u\rangle \geq 0, \quad \forall v \in C.$$

We denoted by $VI(C,A)$ the set of solutions of the variational inequality. For a given $z \in H, u \in C$ satisfies the inequality $\langle u - z, v - u\rangle \geq 0, \quad \forall v \in C$, if and only if $u = Proj_C z$. It is known that projection operator P_C is firmly nonexpansive. It is also known that $Proj_C x$ is characterized by the property: $Proj_C x \in C$ and $\langle x - Proj_C x, Proj_C x - y\rangle \geq 0$ for all $y \in C$. One can see that the variational inequality problem is equivalent to a fixed point problem, that is, an element $u \in C$ is a solution of the variational inequality if and only if $u \in C$ is a fixed point of the mapping $Proj_C(I - \lambda A)$, where $\lambda > 0$ is a constant and I is the identity mapping.

Let $T : C \to H$ be monotone mapping and let F be a bifunction of $C \times C$ into $\mathbb{R}$, where $\mathbb{R}$ denotes the set of real numbers. In this paper, we consider the following generalized equilibrium problem.

$$\text{Find } x \in C \text{ such that } F(x,y) + \langle Tx, y - x\rangle \geq 0, \quad \forall y \in C. \tag{1.1}$$

In this paper, we use $EP(F,T)$ to denote the solution set of the problem (1.1).

Next, we give two special cases of the problem (1.1).

(a) If $F \equiv 0$, then the problem (1.1) is reduced to the classical variational inequality.

(a) If $T \equiv 0$, then the generalized equilibrium problem (1.1) is reduced to the following equilibrium problem:

$$\text{Find } x \in C \text{ such that } F(x,y) \geq 0, \quad \forall y \in C. \tag{1.2}$$

In this paper, we use $EP(F)$ to denote the solution set of problem (1.2). We remark here that problem (1.2) is first introduced by Fan [3].

To study the equilibrium problems, we may assume that F satisfies the following conditions:

(A1) $F(x,x) = 0$ for all $x \in C$;

(A2) F is monotone, i.e., $F(x,y) + F(y,x) \leq 0$ for all $x, y \in C$;

(A3) for each $x, y, z \in C$,

$$\limsup_{t \downarrow 0} F(tz + (1-t)x, y) \leq F(x,y);$$

(A4) for each $x \in C$, $y \mapsto F(x,y)$ is convex and weakly lower semi-continuous.

Recently, many authors based on iterative methods investigated the problems (1.1), (1.2); see [4-17] and the references therein. In this paper, motivated by the above results, we investigated fixed points of strictly pseudocontractive mappings and solutions of equilibrium problem (1.1). Weak convergence theorems are established in Hilbert spaces.

Lemma 1.1. [18] *Let C be a nonempty closed convex subset of H and let $F : C \times C \to \mathbb{R}$ be a bifunction satisfying (A1)-(A4). Then, for any $r > 0$ and $x \in H$, there exists $z \in C$ such that*

$$\langle y - z, z - x \rangle + rF(z,y) \geq 0, \quad \forall y \in C.$$

Further, define $T_r x = \{z \in C : \langle y - z, z - x \rangle + rF(z,y) \geq 0, \quad \forall y \in C\}$ for all $r > 0$ and $x \in H$. Then, the following hold:

(a) *T_r is single-valued;*

(b) $\|T_r x - T_r y\|^2 \leq \langle T_r x - T_r y, x - y \rangle$;

(c) *$F(T_r) = EP(F)$ is closed and convex.*

Lemma 1.2. [1] *Let C be a nonempty closed convex subset of a real Hilbert space H and $S : C \to C$ a k-strict pseudo-contraction with a fixed point. Define $S : C \to C$ by $S_a x = ax + (1-a)Sx$ for each $x \in C$. If $a \in [k, 1)$, then S_a is nonexpansive with $F(S_a) = F(S)$.*

Lemma 1.3. [19] *Let H be a Hilbert space and $0 < p \leq t_n \leq q < 1$ for all $n \geq 1$. Suppose that $\{x_n\}$ and $\{y_n\}$ are sequences in H such that $\limsup_{n\to\infty} \|x_n\| \leq r$, $\limsup_{n\to\infty} \|y_n\| \leq r$ and $\lim_{n\to\infty} \|t_n x_n + (1-t_n) y_n\| = r$ hold for some $r \geq 0$. Then $\lim_{n\to\infty} \|x_n - y_n\| = 0$.*

Lemma 1.4. [20] *Let C be a nonempty closed convex subset of a Hilbert space H and $S : C \to C$ a k-strict pseudocontraction. Then S is $\frac{1+k}{1-k}$-Lipschitz. $I - S$ is demi-closed, this is, if $\{x_n\}$ is a sequence in C with $x_n \rightharpoonup x$ and $x_n - Sx_n \to 0$, then $x \in F(S)$.*

2. Main results

Now, we are in a position to show the main results of the article.

Theorem 2.1. *Let C be a nonempty closed convex subset of a real Hilbert space H. Let $T : C \to H$ be a λ-inverse strongly monotone mapping and let F be a bifunction from $C \times C$ to $\mathbb{R}$ which satisfies (A1)-(A4). Let $S : C \to C$ be a κ-strict pseudocontraction. Assume that $\mathscr{F} := EP(F,T) \cap F(S)$ is not empty. Let $\{\alpha_n\}$, $\{\beta_n\}$, and $\{\delta_n\}$ be sequences in $(0,1)$. Let $\{r_n\}$ be a sequence in $(0,2\lambda)$. Let $\{x_n\}$ be a sequence generated in the following manner:*

$$\begin{cases} r_nF(u_n,u) + \langle u - u_n, u_n - x_n\rangle + r_n\langle Tx_n, u - u_n\rangle \geq 0, \quad \forall u \in C, \\ x_{n+1} = \beta_n\delta_n u_n + \alpha_n x_n + (1-\delta_n)\beta_n S u_n, \quad \forall n \geq 1. \end{cases}$$

Assume that the sequences $\{\alpha_n\}$, $\{\beta_n\}$, $\{\delta_n\}$, and $\{r_n\}$ satisfy the following restrictions: $0 < a \leq \alpha_n \leq a' < 1$, $0 \leq k \leq \delta_n \leq b < 1$, $0 < c \leq r_n \leq d < 2\lambda$. Then the sequence $\{x_n\}$ converges weakly to some point $\bar{x} \in \mathscr{F}$, where $\bar{x} = \lim_{n\to\infty} P_{\mathscr{F}} x_n$, where P is the metric projection.

Proof. Set $S_n = \delta_n I + (1-\delta_n)S$. It follows from Lemma 1.2 that S_n is nonexpansive and $F(S_n) = F(S)$. Note that

$$\begin{aligned} \|(I - r_nT)x - (I - r_nT)y\|^2 &= \|x-y\|^2 - 2r_n\langle x-y, Tx-Ty\rangle + r_n^2\|Tx-Ty\|^2 \\ &\leq \|x-y\|^2 - r_n(2\lambda - r_n)\|Tx-Ty\|^2 \\ &\leq \|x-y\|^2, \quad \forall x,y \in C. \end{aligned}$$

Fixing $p \in \mathscr{F}$, we find from Lemma 1.1 that $p = Sp = T_{r_n}(I - r_nT)p$. Since

$$\|u_n - p\| \leq \|T_{r_n}(I-r_nT)x_n - T_{r_n}(I-r_nT)p\| \leq \|x_n - p\|,$$

we find that

$$\begin{aligned} \|x_{n+1} - p\| &\leq \alpha_n\|x_n - p\| + \beta_n\|S_nu_n - p\| \\ &\leq \|x_n - p\|. \end{aligned} \tag{2.1}$$

This implies that $\lim_{n\to\infty}\|x_n-p\|$ exists. This shows that $\{x_n\}$ is bounded, so is $\{u_n\}$. Since $\|\cdot\|^2$ is convex, we find that

$$\begin{aligned}\|x_{n+1}-p\|^2 &\le \alpha_n\|x_n-p\|^2+\beta_n\|u_n-p\|^2\\ &\le \alpha_n\|x_n-p\|^2+\beta_n\|(I-r_nT)x_n-p\|^2\\ &\le \|x_n-p\|^2-r_n(2\lambda-r_n)\beta_n\|Tx_n-Tp\|^2.\end{aligned}$$

It follows that

$$r_n(2\lambda-r_n)\beta_n\|Tx_n-Tp\|^2\le(1-\gamma_n)\|x_n-p\|^2-\|x_{n+1}-p\|^2.$$

This yields that

$$\lim_{n\to\infty}\|Tx_n-Tp\|=0. \tag{2.2}$$

Using Lemma 1.2, we see that

$$\begin{aligned}\|u_n-p\|^2 &\le \langle (I-r_nT)x_n-(I-r_nT)p,u_n-p\rangle\\ &=\frac{1}{2}\big(\|(I-r_nT)x_n-(I-r_nT)p\|^2+\|u_n-p\|^2\\ &\quad-\|(I-r_nT)x_n-(I-r_nT)p-(u_n-p)\|^2\big)\\ &\le\frac{1}{2}\big(\|x_n-p\|^2+\|u_n-p\|^2-\|x_n-u_n-r_n(Tx_n-Tp)\|^2\big)\\ &=\frac{1}{2}\Big(\|x_n-p\|^2+\|u_n-p\|^2-\big(\|x_n-u_n\|^2\\ &\quad-2r_n\langle x_n-u_n,Tx_n-Tp\rangle+r_n^2\|Tx_n-Tp\|^2\big)\Big).\end{aligned}$$

This implies that

$$\|u_n-p\|^2\le\|x_n-p\|^2-\|x_n-u_n\|^2+2r_n\|x_n-u_n\|\|Tx_n-Tp\|.$$

Since $\|\cdot\|^2$ is convex, we find that

$$\begin{aligned}\|x_{n+1}-p\|^2 &\le \alpha_n\|x_n-p\|^2+\beta_n\|S_nu_n-p\|^2\\ &\le \alpha_n\|x_n-p\|^2+\beta_n\|u_n-p\|^2\\ &\le \|x_n-p\|^2-\beta_n\|x_n-u_n\|^2+2r_n\beta_n\|x_n-u_n\|\|Tx_n-Tp\|.\end{aligned}$$

It follows that that

$$\beta_n\|x_n-u_n\|^2\le\|x_n-p\|^2-\|x_{n+1}-p\|^2+2r_n\|x_n-u_n\|\|Tx_n-Tp\|.$$

Using the restrictions imposed on the sequences, we obtain from (2.2) that

$$\lim_{n\to\infty} \|x_n - u_n\| = 0. \tag{2.3}$$

Since $\{x_n\}$ is bounded, we see that there exits a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ which converges weakly to $\bar{x}$. Using (2.3), we also find that $\{u_{n_i}\}$ converges weakly to $\bar{x}$. Note that

$$F(u_n,u) + \langle Tx_n, u-u_n\rangle + \frac{1}{r_n}\langle u-u_n, u_n-x_n\rangle \geq 0, \quad \forall u \in C.$$

From (A2), we see that

$$\langle Tx_n, u-u_n\rangle + \frac{1}{r_n}\langle u-u_n, u_n-x_n\rangle \geq F(u,u_n), \quad \forall u \in C.$$

Replacing n by n_i, we arrive at

$$\langle Tx_{n_i}, u-u_{n_i}\rangle + \frac{1}{r_{n_i}}\langle u-u_{n_i}, u_{n_i}-x_{n_i}\rangle \geq F(u,u_{n_i}), \quad \forall u \in C. \tag{2.4}$$

For t with $0<t\leq 1$ and $u\in C$, let $u_t = tu+(1-t)\bar{x}$. Since $u\in C$ and $\bar{x}\in C$, we have $u_t\in C$. It follows from (2.4) that

$$\begin{aligned}\langle u_t-u_{n_i}, Tu_t\rangle &\geq \langle u_t-u_{n_i}, Tu_t\rangle - \langle Tx_{n_i}, u_t-u_{n_i}\rangle - \langle u_t-u_{n_i}, \frac{u_{n_i}-x_{n_i}}{r_{n_i}}\rangle + F(u_t,u_{n_i})\\ &= \langle u_t-u_{n_i}, Tu_t-Tu_{n_i}\rangle + \langle u_t-u_{n_i}, Tu_{n_i}-Tx_{n_i}\rangle \\ &\quad - \langle u_t-u_{n_i}, \frac{u_{n_i}-x_{n_i}}{r_{n_i}}\rangle + F(u_t,u_{n_i}).\end{aligned} \tag{2.5}$$

Using (2.3), we have $Tu_{n_i}-Tx_{n_i}\to 0$ as $i\to\infty$. Using the monotonicity of T, we see that $\langle u_t-u_{n_i}, Tu_t-Tu_{n_i}\rangle \geq 0$. It follows from (A4) that

$$\langle u_t-\bar{x}, Tu_t\rangle \geq F(u_t,\bar{x}). \tag{2.6}$$

Using (A1) and (A4), we see from (2.6) that

$$\begin{aligned}0 = F(u_t,u_t) &\leq tF(u_t,u) + (1-t)F(u_t,\bar{x})\\ &\leq tF(u_t,u) + (1-t)\langle u_t-\bar{x}, Tu_t\rangle\\ &= tF(u_t,u) + (1-t)t\langle u-\bar{x}, Tu_t\rangle.\end{aligned}$$

It follows that $F(u_t,u)+(1-t)\langle u-\bar{x}, Tu_t\rangle \geq 0$. Letting $t\to 0$ in the above inequality, we arrive at $F(\bar{x},u)+\langle u-\bar{x}, T\bar{x}\rangle \geq 0$. Hence, $\bar{x}\in EP(F,T)$.

Next, we are in a position to show that $\bar{x} \in F(S)$. Note that $\lim_{n\to\infty} \|x_n - p\|$ exists. We may assume that $\lim_{n\to\infty} \|x_n - p\| = d > 0$. Note that

$$\lim_{n\to\infty} \|x_{n+1} - p\| = \lim_{n\to\infty} \|\beta_n\big(S_n u_n - p\big) + (1-\beta_n)\big(x_n - p\big)\| = d.$$

Note that $\lim_{n\to\infty} \|S_n x_n - p\| \le d$ and $\lim_{n\to\infty} \|x_n - p\| \le d$. Using Lemma Lemma 1.3, we obtain that $\lim_{n\to\infty} \|S_n u_n - x_n\| = 0$. In view of

$$Su_n - x_n = \frac{S_n u_n - x_n}{1-\delta_n} + \frac{\delta_n(x_n - u_n)}{1-\delta_n}.$$

It follows that $\lim_{n\to\infty} \|Su_n - x_n\| = 0$. Note that $\|Sx_n - x_n\| \le \|Sx_n - Su_n\| + \|Su_n - x_n\|$. Using Lemma 1.4, we find that $\lim_{n\to\infty} \|Sx_n - x_n\| = 0$. It follows from Lemma 1.4 that $\bar{x} \in F(S)$. This proves that $\bar{x} \in \mathscr{F}$. Assume that there exits another subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $\{x_{n_j}\}$ converges weakly to x'. We can find that $x' \in \mathscr{F}$. If $\bar{x} \neq x'$, we get from Opial condition that

$$\begin{aligned} \lim_{n\to\infty} \|x_n - \bar{x}\| &= \liminf_{i\to\infty} \|x_{n_i} - \bar{x}\| < \liminf_{i\to\infty} \|x_{n_i} - x'\| \\ &= \lim_{n\to\infty} \|x_n - x'\| = \liminf_{j\to\infty} \|x_{n_j} - x'\| \\ &< \liminf_{j\to\infty} \|x_{n_j} - \bar{x}\| = \lim_{n\to\infty} \|x_n - \bar{x}\|. \end{aligned}$$

This derives a contradiction. Hence, we have $\bar{x} = x'$. This implies that $x_n \rightharpoonup \bar{x} \in \mathscr{F}$. The proof is completed.

From Theorem 2.1, we have the following common fixed point problem.

Theorem 2.2. *Let C be a nonempty closed convex subset of a real Hilbert space H. Let $S_m : C \to C$ be a k_m-strict pseudocontraction for each $1 \le m \le N$, where N is some positive integer. Assume that $\mathscr{F} := \cap_{m=1}^{\infty} F(S)$ is not empty. Let $\{\alpha_n\}$, $\{\beta_n\}$ be sequences in $(0,1)$. Let $\{e_n\}$ is a bounded sequence in C. Let $\{x_n\}$ be a sequence generated in the following manner:*

$$x_1 \in C, \quad x_{n+1} = \alpha_n x_n + \beta_n\big(\delta_n x_n + (1-\delta_n)\sum_{i=1}^{N} \mu_i S_i x_n\big), \quad \forall n \ge 1.$$

Assume that the sequences $\{\alpha_n\}$, $\{\beta_n\}$, $\{\delta_n\}$, and $\{r_n\}$ satisfy the following restrictions: $0 < a \le \alpha_n \le a' < 1$, $0 \le k \le \delta_n \le b < 1$, $0 < c \le r_n \le d < 2\lambda$ and $\sum_{n=1}^{\infty} \gamma_n < \infty$. Then the sequence $\{x_n\}$ converges weakly to some point $\bar{x} \in \mathscr{F}$, where $\bar{x} = \lim_{n\to\infty} P_{\mathscr{F}} x_n$.

Proof. Using the definition of strict pseudocontractions, we see that a mapping T is said to be a k-strict pseudocontraction iff

$$2\langle x-y,(I-T)x-(I-T)y\rangle \geq (1-k)\|(I-T)x-(I-T)y\|^2.$$

Define a mapping $S: C \to C$ by $S=\sum_{m=1}^{N}\mu_m S_m$. Next, we prove that $F(S)=\cap_{m=1}^{N}F(S_m)$ and S is a k-strict pseudocontraction, where $k=\max\{k_m : 1\leq m\leq N\}$. It follows that S is a k-strict pseudocontraction, where $k=\max\{k_m : 1\leq m\leq N\}$. Next, we show that $F(S)=\cap_{m=1}^{N}F(S_m)$. It is clear to see that $F(S)\supseteq \cap_{m=1}^{N}F(S_m)$. It suffices to prove that $\cap_{m=1}^{N}F(S_m)\supseteq F(S)$. Let $x\in F(S)$ and write $T_m=I-S_m$. Let $y\in \cap_{m=1}^{N}F(S_m)$. For any $i,j\in\{1,2,\cdots,N\}$ and $i\neq j$, we have

$$\begin{aligned}
\|x-y\|^2 &= \|\sum_{m=1}^{N}\mu_m(y-S_m x)\|^2\\
&\leq \sum_{m=1}^{N}\mu_m\|y-S_m x\|^2-\mu_i\mu_j\|S_i x-S_j x\|^2\\
&\leq \sum_{m=1}^{N}\mu_m\big(\|y-x\|^2+k_m\|T_m x\|^2\big)-\mu_i\mu_j\|S_i x-S_j x\|^2\\
&\leq \|y-x\|^2+k\sum_{i=1}^{N}\mu_m\|T_m x\|^2-\mu_i\mu_j\|S_i x-S_j x\|^2.
\end{aligned}$$

This shows that

$$\mu_i\mu_j\|S_i x-S_j x\|^2\leq k\sum_{m=1}^{N}\mu_m\|T_m x\|^2.$$

Since $\sum_{i=1}^{N}\mu_m T_m x=0$, we find that $\|S_i x-S_j x\|=0$. This proves that $S_i x=S_j x$. Since x is a fixed point of S, we obtain $\cap_{m=1}^{N}F(S_m)\supseteq F(S)$. This proves that $F(S)=\cap_{m=1}^{N}F(S_m)$. Putting $T=0$, $F=0$ and $r_n=1$, we find from Theorem 2.1 the desired conclusion immediately.

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