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DYNAMIC BEHAVIORS OF A LOTKA-VOLTERRA COMPETITIVE SYSTEM WITH INFINITE DELAYS AND SINGLE FEEDBACK CONTROL

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Abstract. A two species Lotka-Volterra competitive system with infinite delays and single feedback control variable is studied. Sufficient conditions which ensure the extinction and global stability of the system are obtained, respectively. Our extinction result shows that for the traditional Lotka-Volterra competitive system, in any one of the cases: (i) globally stable, (ii) extinct and (iii) bistable, by choosing the suitable feedback control variable, one of the species will be driven to extinction, while the other one will stabilize at a positive equilibrium. Specially, by choosing the suitable feedback control variable, the extinct species in original system could become permanent, while the permanent species in original system will be driven to extinction. Our stability result shows that under certain conditions, the extinct species can become globally stable and the stable species still keep the property of stability.

Keywords. Extinction; Global stability; Lotka-Volterra competitive system; Infinite delay; Single feedback control.

1. Introduction

Traditional two-species autonomous Lotka-Volterra competition system takes the form [1]:

$$\begin{aligned} x_1'(t) &= x_1(t)(a_1 - b_{11}x_1(t) - b_{12}x_2(t)),\\ x_2'(t) &= x_2(t)(a_2 - b_{21}x_1(t) - b_{22}x_2(t)). \end{aligned} \tag{1.1}$$

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There are four possible relationship of the coefficients, consequently, the system admits four kinds of dynamic behaviors:

(I) If the coefficients of system (1.1) satisfy

$$\frac{b_{11}}{b_{21}} > \frac{a_1}{a_2} > \frac{b_{12}}{b_{22}}, \tag{D_1}$$

then system (1.1) admits a unique positive equilibrium $(\bar{x}_1, \bar{x}_2)$, which is globally attractive.

(II) If the coefficients of the system (1.1) satisfy

$$\frac{b_{11}}{b_{21}} < \frac{a_1}{a_2} < \frac{b_{12}}{b_{22}}, \tag{D_2}$$

then system (1.1) is bistable, which of the two species survives depends on the initial conditions.

(III) If the coefficients of the system (1.1) satisfy

$$\frac{a_1}{a_2} < \frac{b_{11}}{b_{21}}, \ \frac{a_1}{a_2} < \frac{b_{12}}{b_{22}}, \tag{D_3}$$

then the first species will be driven to extinction while the other one will stabilize at a positive equilibrium of a logistic equation.

(IV) If the coefficients of the system (1.1) satisfy

$$\frac{a_1}{a_2} > \frac{b_{11}}{b_{21}}, \ \frac{a_1}{a_2} > \frac{b_{12}}{b_{22}}, \tag{D_4}$$

then the second species will be driven to extinction while the other one will be stable.

During the last decades, many scholars proposed and studied the dynamic behaviors of the ecosystem with feedback controls (see [2]-[25] and the references cited therein), such topics as the persistent, permanent and partial survival of the system, the existence of the positive periodic solution, almost periodic solution have been extensively investigated. However, seldom did scholars investigate the autonomous system. K. Gopalsamy and P. X. Weng [10] argued that in some case, it is likely that the equilibrium level $(\bar{x}_1, \bar{x}_2)$ in (1.1) may not be sustainable or even desirable, and it may be necessary to maintain and stabilize the competition system at a lower level. To achieve this aim, they introduced the following two species competitive system with feedback controls:

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= x_1(t)(b_1 - a_{11}x_1(t) - a_{12}x_2(t) - \alpha_1 u_1(t-\tau)),\\
\frac{dx_2(t)}{dt} &= x_2(t)(b_2 - a_{21}x_1(t) - a_{22}x_2(t) - \alpha_2 u_2(t-\tau)),\\
\frac{du_1(t)}{dt} &= -\eta_1 u_1(t) + a_1 x_1(t-\tau),\\
\frac{du_2(t)}{dt} &= -\eta_2 u_2(t) + a_2 x_2(t-\tau),
\end{aligned} \tag{1.2}$$

where $b_i, a_{ij}, \alpha_i, \eta_i, a_i (i, j = 1, 2), \tau$ are positive constants, τ is a non-negative constant. By constructing a suitable Lyapunov function and using the iterative method, they obtained a set of sufficient conditions which ensure the existence of a globally attractive positive equilibrium for the case $\tau = 0$ and $\tau \neq 0$, respectively. However, they did not investigate the extinction property of the system. Recently, Li, Han and Chen [11] considered an autonomous Lotka-Volterra competitive system with infinite delays and feedback controls

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= x_1(t)(b_1 - a_{11}x_1(t) - a_{12}\int_0^{+\infty} K_1(s)x_2(t-s)ds - c_1 u_1(t)),\\
\frac{dx_2(t)}{dt} &= x_2(t)(b_2 - a_{21}\int_0^{+\infty} K_2(s)x_1(t-s)ds - a_{22}x_2(t) - c_2 u_2(t)),\\
\frac{du_1(t)}{dt} &= -e_1 u_1(t) + d_1 x_1(t),\\
\frac{du_2(t)}{dt} &= -e_2 u_2(t) + d_2 x_2(t),
\end{aligned} \tag{1.3}$$

where $b_i, a_{ij}, e_i, d_i (i, j = 1, 2)$ are positive constants; $x_i(t)$ denotes the density of the population; $u_i(t)$ denotes the feedback control variable. By constructing a suitable Lyapunov functional, they showed that the feedback controls only change the position of the unique positive equilibrium if the Lotka-Volterra competitive system is globally stable; However, for the extinct case, by choosing suitable control variables, extinct species can become globally stable, or still keep the property of extinction. Their findings show that for the extinction case, feedback control variables may change the stability property of the system.

As we can see, model (1.2) and (1.3) contains two feedback control variables, which means that for the different species, different control strategy is adopted. However, in some cases, one may choose a single control strategy, and such a strategy has influence on both species. For instance, in the medical system when we take chemotherapeutic drugs for cancer patients, cancer cells will decrease rapidly, but at the same time, drugs do harm to normal cells and body's immune function. F. X. Yao, *et al.* [12] investigated the effect of chemotherapeutic drugs on cellular immunity in patients with lung cancer, they found that cell immunity is inhibited in patients of lung cancer, moreover, it is impaired considerably by chemotherapy. In the agricultural system, spraying pesticide can reduce the number of weeds, but pesticide can also have a negative impact on crops' growth [13]. This motivated us to propose and study the following two species competitive system with single feedback control variable:

$$x_1'(t) = x_1(t)\Big(a_1 - b_{11}x_1(t) - b_{12}\int_0^{+\infty} K_1(s)x_2(t-s)ds - d_1u(t)\Big),$$

$$\begin{aligned} x_2'(t) &= x_2(t)\Big(a_2 - b_{21}\int_0^{+\infty} K_2(s)x_1(t-s)ds - b_{22}x_2(t) - d_2u(t)\Big), \\ u'(t) &= -e_1u(t) + f_1x_1(t) + f_2x_2(t), \end{aligned} \tag{1.4}$$

where e_1 and a_i, b_{ij}, d_i, f_i, $i, j = 1,2$ are all positive constants; $x_i(t), i = 1,2$ denotes the density of the population x_i at time t; $u(t)$ denotes the feedback control variable.

The kernel $K_i : [0,+\infty) \to [0,+\infty), i = 1,2$ are continuous functions such that

$$\int_0^{+\infty} K_i(s)ds = 1, \ \ \sigma_i = \int_0^{+\infty} sK_i(s)ds < +\infty, \ \ i = 1,2.$$

System (1.4) satisfy the initial conditions

$$\begin{aligned} x_i(\theta) = \varphi_i(\theta), \ \ \theta \in (-\infty,0], \ \ \varphi_i(0) > 0, \ \ i = 1,2, \\ u(\theta) = \psi(\theta), \ \ \theta \in (-\infty,0], \ \ \psi(0) > 0, \end{aligned} \tag{1.5}$$

where φ_i, ψ, $i = 1,2$ are non-negative and bounded continuous functions on $(-\infty,0]$.

It is well known that depending on the relationship of the coefficients, the dynamic behaviors of system (1.1) can be classified as global stability, bistable and extinction. The aim of this paper is to study the influence of feedback control variable for these three cases, respectively. The organization of this paper is as follows. We study the extinction and stability properties of the system (1.4) in Section 2 and analyze the influence of the feedback control on the system in Section 3. In Section 4, several numeric examples are presented to show the feasibility of the main results. We end this paper by a briefly discussion.

2. Main results

Lemma 2.1. *Let* $(x_1(t),x_2(t),u(t))^T$ *be a solution of system* (1.4) *with initial condition(1.5). Then for all* $t \geq 0$, $(x_1(t),x_2(t),u(t))^T$ *is positive and bounded.*

Proof. Obviously, the solution $(x_1(t),x_2(t),u(t))^T$ of the system (1.4) with initial condition (1.5) is positive for all $t \geq 0$. It follows from the first and the second equations of the system (1.4) that

$$x_i{}'(t) \leq x_i(t)(a_i - b_{ii}x_i(t)), \ \ i = 1,2.$$

By applying the standard comparison principle, it follows that

$$\limsup_{t\to+\infty} x_i(t) \leq \frac{a_i}{b_{ii}}, \ \ i = 1,2.$$

Then for any $\varepsilon > 0$, there exists a $T > 0$ such that $x_i(t) \leq \frac{a_i}{b_{ii}} + \varepsilon$, $i = 1,2$, $t > T$. And so, from the third equation of the system (1.4), we have

$$u'(t) \leq -e_1u(t) + f_1(\frac{a_1}{b_{11}} + \varepsilon) + f_2(\frac{a_2}{b_{22}} + \varepsilon), \ \ i = 1,2, \ t > T.$$

Thus

$$\limsup_{t\to+\infty} u(t) \leq \frac{f_1}{e_1}(\frac{a_1}{b_{11}} + \varepsilon) + \frac{f_2}{e_1}(\frac{a_2}{b_{22}} + \varepsilon), \ \ i = 1,2.$$

Setting $\varepsilon \to 0$, we see that

$$\limsup_{t\to+\infty} u(t) \leq \frac{f_1a_1}{e_1b_{11}} + \frac{f_2a_2}{e_1b_{22}}, \ \ i = 1,2.$$

This completes the proof of Lemma 2.1.

Lemma 2.2. *Let* $(x_1(t),x_2(t),u(t))^T$ *be any solution of system* (1.4) *with initial condition (1.5). Assume that inequalities*

$$\frac{a_1}{a_2} > \frac{b_{11}e_1 + d_1f_1}{b_{21}e_1 + d_2f_1}, \ \ \frac{a_1}{a_2} > \frac{b_{12}e_1 + d_1f_2}{b_{22}e_1 + d_2f_2} \tag{A_1}$$

hold. Then

$$\lim_{t\to+\infty} x_2(t) = 0, \ \ \int_0^{+\infty} x_2(t)dt < +\infty.$$

Proof. We will adapt the idea of Li, Han, Chen [11] and Hu, Teng, Gao [15] to prove this lemma. By the above inequalities (A_1), we can choose positive constants α,β such that

$$\frac{a_1}{a_2}>\frac{\beta}{\alpha}>\frac{b_{11}e_1+d_1f_1}{b_{21}e_1+d_2f_1},\ \ \frac{a_1}{a_2}>\frac{\beta}{\alpha}>\frac{b_{12}e_1+d_1f_2}{b_{22}e_1+d_2f_2}.$$

Thus, there exists a $\delta>0$ such that

$$\begin{aligned}\beta a_2-\alpha a_1<-\delta<0,\\ (\alpha b_{11}-\beta b_{21})+\frac{\alpha d_1-\beta d_2}{e_1}f_1<0,\\ (\alpha b_{12}-\beta b_{22})+\frac{\alpha d_1-\beta d_2}{e_1}f_2<0.\end{aligned}\tag{2.1}$$

From the continuity of function $y=a+bx$, there exists a constant γ such that

$$\frac{\alpha d_1-\beta d_2}{e_1}<\gamma\tag{2.2}$$

and

$$\begin{aligned}(\alpha b_{11}-\beta b_{21})+\frac{\alpha d_1-\beta d_2}{e_1}f_1<(\alpha b_{11}-\beta b_{21})+\gamma f_1<0,\\ (\alpha b_{12}-\beta b_{22})+\frac{\alpha d_1-\beta d_2}{e_1}f_2<(\alpha b_{12}-\beta b_{22})+\gamma f_2<0\end{aligned}\tag{2.3}$$

hold. Consider the following Lyapunov functional

$$\begin{aligned}V(t)=x_1^{-\alpha}(t)x_2{}^{\beta}(t)\exp\Big(\gamma u(t)+\alpha b_{12}\int_0^{+\infty}K_1(s)\int_{t-s}^{t}x_2(u)duds\\ -\beta b_{21}\int_0^{+\infty}K_2(s)\int_{t-s}^{t}x_1(u)duds\Big).\end{aligned}$$

Calculating the derivative of $V(t)$ along the solution of the system (1.4), we have

$$\begin{aligned}V'(t)&=V(t)\Big[-\alpha\Big(a_1-b_{11}x_1(t)-b_{12}\int_0^{+\infty}K_1(s)x_2(t-s)ds-d_1u(t)\Big)\\ &\quad+\beta\Big(a_2-b_{21}\int_0^{+\infty}K_2(s)x_1(t-s)ds-b_{22}x_2(t)-d_2u(t)\Big)\\ &\quad+\gamma\Big(-e_1u(t)+f_1x_1(t)+f_2x_2(t)\Big)\\ &\quad+\alpha b_{12}x_2(t)-\alpha b_{12}\int_0^{+\infty}K_1(s)x_2(t-s)ds\\ &\quad-\beta b_{21}x_1(t)+\beta b_{21}\int_0^{+\infty}K_2(s)x_1(t-s)ds\Big]\\ &=V(t)\Big[(\beta a_2-\alpha a_1)+(\alpha b_{11}-\beta b_{21}+\gamma f_1)x_1(t)\\ &\quad+(\alpha b_{12}-\beta b_{22}+\gamma f_2)x_2(t)\\ &\quad+(\alpha d_1-\beta d_2-\gamma e_1)u(t)\Big].\end{aligned}$$

From the first inequalities of (2.1), (2.2) and (2.3), we obtain

$$V'(t)\le-\delta V(t).$$

Integrating this equality from 0 to t, we have

$$V(t) \leq V(0)\exp(-\delta t). \tag{2.4}$$

From Lemma 2.1, there exists a $M > 0$ such that $0 < x_i(t) < M, 0 < u(t) < M, i = 1,2, t \in R$. Therefore

$$\begin{aligned} V(0) &= x_1^{-\alpha}(0)x_2{}^{\beta}(0)\exp\Big(\gamma u(0) + \alpha b_{12}\int_0^{+\infty} K_1(s)\int_{-s}^{0} x_2(u)duds \\ &\qquad -\beta b_{21}\int_0^{+\infty} K_2(s)\int_{-s}^{0} x_1(u)duds\Big). \\ &\leq x_1^{-\alpha}(0)x_2{}^{\beta}(0)\exp\Big(|\gamma|u(0) + \alpha b_{12}M\int_0^{+\infty} K_1(s)\int_{-s}^{0} duds\Big) \\ &= x_1^{-\alpha}(0)x_2{}^{\beta}(0)\exp\Big(|\gamma|u(0) + \alpha b_{12}M\sigma_1\Big) \\ &< +\infty. \end{aligned}$$

On the other hand, we have

$$\begin{aligned} V(t) &\geq x_1^{-\alpha}(t)x_2{}^{\beta}(t)\exp\Big(-|\gamma|u(t) - \beta b_{21}\int_0^{+\infty} K_2(s)\int_{t-s}^{t} x_1(u)duds\Big) \\ &\geq M^{-\alpha}x_2{}^{\beta}(t)\exp(-|\gamma|M - \beta b_{21}M\int_0^{+\infty} K_2(s)\int_{t-s}^{t} duds) \\ &= M^{-\alpha}x_2{}^{\beta}(t)\exp(-|\gamma|M - \beta b_{21}M\sigma_2). \end{aligned} \tag{2.5}$$

Combining inequalities (2.4) and (2.5), we have

$$x_2{}^{\beta}(t) \leq M^{\alpha}\exp(|\gamma|M + \beta b_{21}M\sigma_2)V(0)\exp(-\delta t).$$

Then

$$x_2(t) \leq \Big(M^{\alpha}\exp(|\gamma|M + \beta b_{21}M\sigma_2)V(0)\Big)^{\frac{1}{\beta}}\exp(-\frac{\delta}{\beta}t).$$

Hence we obtain that

$$\lim_{t\to+\infty} x_2(t) = 0, \quad \int_0^{+\infty} x_2(t)dt < +\infty.$$

This completes the proof of Lemma 2.2.

Theorem 2.1. *Let $(x_1(t), x_2(t), u(t))^T$ be any solution of system (1.4) with initial condition (1.5). If condition (A_1) holds, then the equilibrium $(x_1^*, 0, u^*)$ of system (1.4) is globally asymptotically stable, that is,*

$$\lim_{t\to+\infty} x_1(t) = x_1^*, \quad \lim_{t\to+\infty} x_2(t) = 0, \quad \lim_{t\to+\infty} u(t) = u^*,$$

where $x_1^* = \dfrac{a_1 e_1}{b_{11} e_1 + d_1 f_1}$, $u^* = \dfrac{f_1}{e_1} x_1^*$.

Proof. Under assumption (A_1), it follows from Lemma 2.2 that

$$\lim_{t \to +\infty} x_2(t) = 0.$$

To end the proof of Theorem 2.1, it suffices to show that

$$\lim_{t \to +\infty} x_1(t) = x_1^*, \quad \lim_{t \to +\infty} u(t) = u^*.$$

Let $(x_1(t), x_2(t), u(t))^T$ be a solution of system (1.4)-(1.5). We define a Lyapunov functional as follows:

$$V(t) = V_1(t) + V_2(t),$$

where

$$\begin{aligned} V_1(t) &= \left(x_1 - x_1{}^* - x_1{}^* \ln \frac{x_1}{x_1{}^*} \right) + \frac{d_1}{2 f_1} (u - u^*)^2, \\ V_2(t) &= b_{12}(M + x_1{}^*) \int_0^{+\infty} K_1(s) \int_{t-s}^{t} x_2(u) du ds. \end{aligned}$$

Calculating the derivative of $V(t)$ along the positive solution of system (1.4), and from Lemma 2.1 there exists a $M > 0$ such that $0 < x_i(t), u(t) < M, i = 1, 2$, then we have

$$\begin{aligned} V_1{}'(t) &\leq -b_{11}(x_1(t) - x_1{}^*)^2 - \frac{d_1 e_1}{f_1}(u(t) - u^*)^2 \\ &\quad + b_{12}(x_1(t) - x_1{}^*) \int_0^{+\infty} K_1(s) x_2(t-s) ds + \frac{d_1 f_2}{f_1}(u(t) - u^*) x_2(t) \\ &\leq -b_{11}(x_1(t) - x_1{}^*)^2 - \frac{d_1 e_1}{f_1}(u(t) - u^*)^2 \\ &\quad + b_{12}(M + x_1{}^*) \int_0^{+\infty} K_1(s) x_2(t-s) ds + \frac{d_1 f_2}{f_1}(M + u^*) x_2(t). \\ V_2{}'(t) &= b_{12}(M + x_1{}^*) x_2(t) - b_{12}(M + x_1{}^*) \int_0^{+\infty} K_1(s) x_2(t-s) ds, \end{aligned}$$

and so

$$\begin{aligned} V'(t) &= V_1{}'(t) + V_2{}'(t) \\ &\leq -b_{11}(x_1(t) - x_1{}^*)^2 - \frac{d_1 e_1}{f_1}(u(t) - u^*)^2 \\ &\quad + \frac{d_1 f_2}{f_1}(M + u^*) x_2(t) + b_{12}(M + x_1{}^*) x_2(t). \end{aligned} \tag{2.6}$$

Integrating both sides of (2.6) on the interval [T,t), we have

$$\begin{aligned} & V(t)+b_{11}\int_T^t (x_1(s)-x_1{}^*)^2ds+\frac{d_1e_1}{f_1}\int_T^t(u(s)-u^*)^2ds \\ < \; & V(T)+\Big(b_{12}(M+x_1{}^*)+\frac{d_1f_2}{f_1}(M+u^*)\Big)\int_T^t x_2(s)ds. \end{aligned} \tag{2.7}$$

Obvious $V_1(T)$ is bounded, also

$$\begin{aligned} V_2(T) &= b_{12}(M+x_1{}^*)\int_0^{+\infty}K_1(s)\int_{T-s}^T x_2(u)duds \\ &\le b_{12}(M+x_1{}^*)M\sigma_1 < +\infty, \end{aligned}$$

and so $V(T)=V_1(T)+V_2(T)$ is bounded. On the other hand, following from Lemma 2.2 that under assumption (A_1), $\int_0^{+\infty} x_2(s)ds < +\infty$, consequently, $\int_T^t x_2(s)ds < +\infty$. Therefore we have

$$\int_T^t (x_1(s)-x_1{}^*)^2ds < +\infty, \;\; \int_T^t (u(s)-u^*)^2ds < +\infty.$$

From Lemma 2.1 $x_1{}'(t)$ and $u'(t)$ are bounded. Therefore, $(x_1(t)-x_1{}^*)^2$ and $(u(t)-u^*)^2$ are uniformly continuous on $[0,+\infty)$. Then by Barbalat's Lemma, we have

$$\lim_{t\to+\infty}(x_1(t)-x_1{}^*)^2=0, \;\; \lim_{t\to+\infty}(u(t)-u^*)^2=0,$$

that is,

$$\lim_{t\to+\infty}x_1(t)=x_1{}^*, \;\; \lim_{t\to+\infty}u(t)=u^*.$$

The proof of the theorem is complete.

Theorem 2.2 *Let* $(x_1(t),x_2(t),u(t))^T$ *be any solution of system* (1.4) *with initial condition (1.5). Assume that inequalities*

$$\frac{b_{11}e_1+d_1f_1}{b_{21}e_1+d_2f_1} > \frac{a_1}{a_2} > \frac{b_{12}e_1+d_1f_2}{b_{22}e_1+d_2f_2} \tag{A_2}$$

hold, then system (1.4) has a unique positive equilibrium $(x_1^\diamond, x_2^\diamond, u^\diamond)$*, where*

$$\begin{aligned} x_1^\diamond &= \frac{e_1\big(a_1(b_{22}e_1+d_2f_2)-a_2(b_{12}e_1+d_1f_2)\big)}{(b_{11}e_1+d_1f_1)(b_{22}e_1+d_2f_2)-(b_{12}e_1+d_1f_2)(b_{21}e_1+d_2f_1)}, \\ x_2^\diamond &= \frac{e_1\big(a_2(b_{11}e_1+d_1f_1)-a_1(b_{21}e_1+d_2f_1)\big)}{(b_{11}e_1+d_1f_1)(b_{22}e_1+d_2f_2)-(b_{12}e_1+d_1f_2)(b_{21}e_1+d_2f_1)}, \\ u^\diamond &= \frac{f_1}{e_1}x_1^\diamond+\frac{f_2}{e_1}x_2^\diamond. \end{aligned}$$

Further assume that

$$\frac{f_1d_2b_{12}+f_2d_1b_{21}}{2f_2d_1b_{22}} < \frac{2f_1d_2b_{11}}{f_1d_2b_{12}+f_2d_1b_{21}} \qquad (A_3)$$

holds, then the unique positive equilibrium $(x_1^\diamond, x_2^\diamond, u^\diamond)$ *is globally asymptotically stable, that is*

$$\lim_{t\to+\infty} x_i(t) = x_i^\diamond, \ \lim_{t\to+\infty} u(t) = u^\diamond, \ i = 1,2.$$

Proof. Define a Lyapunov function as follows:

$$\overline{V}_1(t) = \sum_{i=1}^{2} \eta_i\Big(x_i - x_i^\diamond - x_i^\diamond \ln\frac{x_i}{x_i^\diamond}\Big) + \eta_3(u-u^\diamond)^2,$$

where $\eta_1 = \dfrac{2\eta_3 f_1}{d_1}, \eta_2 = \dfrac{2\eta_3 f_2}{d_2}, \eta_3 = 1$. Calculating the derivative of $\overline{V}_1$ along the solution $(x_1(t),x_2(t),u(t))^T$ of the system (1.4), we have

$$\begin{aligned}
\overline{V}_1'(t) &= \eta_1(x_1(t)-x_1^\diamond)\Big[-b_{11}(x_1(t)-x_1^\diamond) - b_{12}\Big(\int_0^{+\infty} K_1(s)(x_2(t-s)-x_2^\diamond)ds\Big)\\
&\qquad -d_1(u(t)-u^\diamond)\Big]\\
&\quad +\eta_2(x_2(t)-x_2^\diamond)\Big[-b_{21}\Big(\int_0^{+\infty} K_2(s)(x_1(t-s)-x_1^\diamond)ds\Big) - b_{22}(x_2(t)-x_2^\diamond)\\
&\qquad -d_2(u(t)-u^\diamond)\Big]\\
&\quad +2\eta_3(u(t)-u^\diamond)\Big(-e_1(u(t)-u^\diamond) + f_1(x_1(t)-x_1^\diamond) + f_2(x_2(t)-x_2^\diamond)\Big)\\
&= -\eta_1 b_{11}(x_1(t)-x_1^\diamond)^2 - \eta_2 b_{22}(x_2(t)-x_2^\diamond)^2 - 2\eta_3 e_1(u(t)-u^\diamond)^2\\
&\quad -\eta_1 b_{12}\int_0^{+\infty} K_1(s)(x_2(t-s)-x_2^\diamond)(x_1(t)-x_1^\diamond)ds\\
&\quad -\eta_2 b_{21}\int_0^{+\infty} K_2(s)(x_1(t-s)-x_1^\diamond)(x_2(t)-x_2^\diamond)ds.
\end{aligned}$$

It follows that

$$\begin{aligned}
\overline{V}_1'(t) &\leq -\eta_1 b_{11}(x_1(t)-x_1^\diamond)^2 - \eta_2 b_{22}(x_2(t)-x_2^\diamond)^2 - 2\eta_3 e_1(u(t)-u^\diamond)^2\\
&\quad +\eta_1 b_{12}\int_0^{+\infty} K_1(s)(x_2(t-s)-x_2^\diamond)(x_1(t)-x_1^\diamond)ds\\
&\quad +\eta_2 b_{21}\int_0^{+\infty} K_2(s)(x_1(t-s)-x_1^\diamond)(x_2(t)-x_2^\diamond)ds\\
&\leq -\eta_1 b_{11}(x_1(t)-x_1^\diamond)^2 - \eta_2 b_{22}(x_2(t)-x_2^\diamond)^2 - 2\eta_3 e_1(u(t)-u^\diamond)^2\\
&\quad +\eta_1 b_{12}\Big(\frac{\theta_1}{2}(x_1(t)-x_1^\diamond)^2 + \frac{1}{2\theta_1}\int_0^{+\infty} K_1(s)(x_2(t-s)-x_2^\diamond)^2 ds\Big)\\
&\quad +\eta_2 b_{21}\Big(\frac{\theta_2}{2}\int_0^{+\infty} K_2(s)(x_1(t-s)-x_1^\diamond)^2 ds + \frac{1}{2\theta_2}(x_2(t)-x_2^\diamond)^2\Big).
\end{aligned}$$

Hence, we have

$$\begin{aligned}\overline{V}_1'(t) &\leq -(\eta_1 b_{11}-\eta_1 b_{12}\frac{\theta_1}{2})(x_1(t)-x_1^\diamond)^2+\eta_1 b_{12}\frac{1}{2\theta_1}\int_0^{+\infty}K_1(s)(x_2(t-s)-x_2^\diamond)^2ds\\ &\quad -(\eta_2 b_{22}-\eta_2 b_{21}\frac{1}{2\theta_2})(x_2(t)-x_2^\diamond)^2+\eta_2 b_{21}\frac{\theta_2}{2}\int_0^{+\infty}K_2(s)(x_1(t-s)-x_1^\diamond)^2ds\\ &\quad -2\eta_3 e_1(u(t)-u^\diamond)^2.\end{aligned} \tag{2.8}$$

Let

$$\overline{V}_2(t)=\eta_1 b_{12}\frac{1}{2\theta_1}\int_0^{+\infty}K_1(s)\int_{t-s}^t(x_2(u)-x_2^\diamond)^2duds+\eta_2 b_{21}\frac{\theta_2}{2}\int_0^{+\infty}K_2(s)\int_{t-s}^t(x_1(u)-x_1^\diamond)^2duds.$$

Calculating the derivative $\overline{V}_2(t)$, we obtain

$$\begin{aligned}\overline{V}_2'(t) &= \eta_1 b_{12}\frac{1}{2\theta_1}(x_2(t)-x_2^\diamond)^2-\eta_1 b_{12}\frac{1}{2\theta_1}\int_0^{+\infty}K_1(s)(x_2(t-s)-x_2^\diamond)^2ds\\ &\quad +\eta_2 b_{21}\frac{\theta_2}{2}(x_1(t)-x_1^\diamond)^2-\eta_2 b_{21}\frac{\theta_2}{2}\int_0^{+\infty}K_2(s)(x_1(t-s)-x_1^\diamond)^2ds.\end{aligned} \tag{2.9}$$

Define

$$\overline{V}(t)=\overline{V}_1(t)+\overline{V}_2(t).$$

It follows from (2.8) and (2.9) that

$$\begin{aligned}\overline{V}'(t) &\leq -(\eta_1 b_{11}-\eta_1 b_{12}\frac{\theta_1}{2}-\eta_2 b_{21}\frac{\theta_2}{2})(x_1(t)-x_1^\diamond)^2\\ &\quad -(\eta_2 b_{22}-\eta_2 b_{21}\frac{1}{2\theta_2}-\eta_1 b_{12}\frac{1}{2\theta_1})(x_2(t)-x_2^\diamond)^2\\ &\quad -2\eta_3 e_1(u(t)-u^\diamond)^2.\end{aligned} \tag{2.10}$$

Denote $\tau_1=\eta_1 b_{11}-\eta_1 b_{12}\frac{\theta_1}{2}-\eta_2 b_{21}\frac{\theta_2}{2}, \tau_2=\eta_2 b_{22}-\eta_2 b_{21}\frac{1}{2\theta_2}-\eta_1 b_{12}\frac{1}{2\theta_1}$. Taking

$$\theta_1=\theta_2=\frac{1}{2}\Big(\frac{f_1 d_2 b_{12}+f_2 d_1 b_{21}}{2f_2 d_1 b_{22}}+\frac{2f_1 d_2 b_{11}}{f_1 d_2 b_{12}+f_2 d_1 b_{21}}\Big),$$

we have

$$\begin{aligned}\tau_1 &= \frac{4f_1 f_2 d_1 d_2 b_{11} b_{22}-(f_1 d_2 b_{12}+f_2 d_1 b_{21})^2}{4f_2 b_{22} d_1 d_2 d_1},\\ \tau_2 &= \frac{2f_2 b_{22}\Big(4f_1 f_2 d_1 d_2 b_{11} b_{22}-(f_1 d_2 b_{12}+f_2 d_1 b_{21})^2\Big)}{d_2\Big(4f_1 f_2 d_1 d_2 b_{11} b_{22}+(f_1 d_2 b_{12}+f_2 d_1 b_{21})^2\Big)}.\end{aligned} \tag{2.11}$$

It follows from (A_3) that $\tau_i>0, i=1,2$. Then

$$\overline{V}'(t)\leq -\tau_1(x_1(t)-x_1^\diamond)^2-\tau_2(x_2(t)-x_2^\diamond)^2-2e_1(u(t)-u^\diamond)^2,$$

$\overline{V}(t)$ is nonincreasing, $x_i^{'}(t), i=1,2$ and $u^{'}(t)$ are bounded. Then $|x_i(t)-x_i^{\diamond}|, i=1,2$ and $|u(t)-u^{\diamond}|$ are uniformly continuous on $[0,+\infty)$. From Lemma 2.1 there exists a $M>0$ such that $0<x_i(t), u(t)<M, i=1,2$, Integrating both sides of (2.11) on the interval $[T,t)$, then we have

$$\overline{V}(t)+\tau_1\int_T^t(x_1(s)-x_1^{\diamond})^2ds+\tau_2\int_T^t(x_2(s)-x_2^{\diamond})^2ds+2e_1\int_T^t(u(s)-u^{\diamond})^2ds<\overline{V}(T).$$

Obviously $\overline{V}(T)$ is bounded. Therefore

$$\int_T^t(x_i(s)-x_i^{\diamond})^2ds<\frac{\overline{V}(T)}{\tau_i}<+\infty, i=1,2, \int_T^t(u(s)-u^{\diamond})^2ds<\frac{\overline{V}(T)}{2e_1}<+\infty.$$

By Barbalat's Lemma, we have

$$\lim_{t\to+\infty}(x_i(t)-x_i^{\diamond})^2=0, i=1,2, \ \lim_{t\to+\infty}(u(t)-u^{\diamond})^2=0,$$

that is

$$\lim_{t\to+\infty}x_i(t)=x_i{}^{\diamond}, \ \lim_{t\to+\infty}u(t)=u^{\diamond}, \ i=1,2.$$

This completes the proof of Theorem 2.2.

3. Influence of feedback control

Condition (A_1) is relate to the coefficients e_1, d_i and $f_i, i=1,2$. To obtain some more detail insight into this set of condition, we will discuss (A_1) in four cases:

(1) If inequalities $\frac{b_{11}}{b_{21}}>\frac{d_1}{d_2}, \frac{b_{12}}{b_{22}}>\frac{d_1}{d_2}$ hold, then there exists four subcases:

$$\frac{a_1}{a_2}>\frac{b_{11}}{b_{21}}>\frac{b_{11}e_1+d_1f_1}{b_{21}e_1+d_2f_1}, \ \frac{a_1}{a_2}>\frac{b_{12}}{b_{22}}>\frac{b_{12}e_1+d_1f_2}{b_{22}e_1+d_2f_2}; \qquad (H_1)$$

$$\frac{a_1}{a_2}>\frac{b_{11}}{b_{21}}>\frac{b_{11}e_1+d_1f_1}{b_{21}e_1+d_2f_1}, \ \frac{b_{12}}{b_{22}}>\frac{a_1}{a_2}>\frac{b_{12}e_1+d_1f_2}{b_{22}e_1+d_2f_2}; \qquad (H_2)$$

$$\frac{b_{11}}{b_{21}}>\frac{a_1}{a_2}>\frac{b_{11}e_1+d_1f_1}{b_{21}e_1+d_2f_1}, \ \frac{a_1}{a_2}>\frac{b_{12}}{b_{22}}>\frac{b_{12}e_1+d_1f_2}{b_{22}e_1+d_2f_2}; \qquad (H_3)$$

$$\frac{b_{11}}{b_{21}}>\frac{a_1}{a_2}>\frac{b_{11}e_1+d_1f_1}{b_{21}e_1+d_2f_1}, \ \frac{b_{12}}{b_{22}}>\frac{a_1}{a_2}>\frac{b_{12}e_1+d_1f_2}{b_{22}e_1+d_2f_2}. \qquad (H_4)$$

(2) If inequality $\frac{b_{11}}{b_{21}}>\frac{d_1}{d_2}>\frac{b_{12}}{b_{22}}$ holds, then there exists two subcases:

$$\frac{a_1}{a_2}>\frac{b_{11}}{b_{21}}>\frac{b_{11}e_1+d_1f_1}{b_{21}e_1+d_2f_1}, \ \frac{a_1}{a_2}>\frac{b_{12}e_1+d_1f_2}{b_{22}e_1+d_2f_2}>\frac{b_{12}}{b_{22}}; \qquad (H_5)$$

$$\frac{b_{11}}{b_{21}} > \frac{a_1}{a_2} > \frac{b_{11}e_1 + d_1 f_1}{b_{21}e_1 + d_2 f_1}, \ \frac{a_1}{a_2} > \frac{b_{12}e_1 + d_1 f_2}{b_{22}e_1 + d_2 f_2} > \frac{b_{12}}{b_{22}}. \tag{H_6}$$

(3) If inequality $\frac{b_{11}}{b_{21}} < \frac{d_1}{d_2} < \frac{b_{12}}{b_{22}}$ holds, then there exists two subcases:

$$\frac{a_1}{a_2} > \frac{b_{11}e_1 + d_1 f_1}{b_{21}e_1 + d_2 f_1} > \frac{b_{11}}{b_{21}}, \ \frac{b_{12}}{b_{22}} > \frac{a_1}{a_2} > \frac{b_{12}e_1 + d_1 f_2}{b_{22}e_1 + d_2 f_2}; \tag{H_7}$$

$$\frac{a_1}{a_2} > \frac{b_{11}e_1 + d_1 f_1}{b_{21}e_1 + d_2 f_1} > \frac{b_{11}}{b_{21}}, \ \frac{a_1}{a_2} > \frac{b_{12}}{b_{22}} > \frac{b_{12}e_1 + d_1 f_2}{b_{22}e_1 + d_2 f_2}. \tag{H_8}$$

(4) If inequalities $\frac{b_{11}}{b_{21}} < \frac{d_1}{d_2}, \frac{b_{12}}{b_{22}} < \frac{d_1}{d_2}$ hold, then there exists one subcase:

$$\frac{a_1}{a_2} > \frac{b_{11}e_1 + d_1 f_1}{b_{21}e_1 + d_2 f_1} > \frac{b_{11}}{b_{21}}, \ \frac{a_1}{a_2} > \frac{b_{12}e_1 + d_1 f_2}{b_{22}e_1 + d_2 f_2} > \frac{b_{12}}{b_{22}}. \tag{H_9}$$

Above analysis shows that there are altogether nine possible subcases such that condition (A_1) holds.

Remark 3.1. Note that subcases $(H_1), (H_5), (H_8)$ and (H_9) imply (D_4), thus species x_2 is extinct in system (1.1). It follows from Theorem 2.1 that species x_2 in system (1.4) is extinct too, that is, suitable feedback control variable can make extinct species still keep the property of extinction.

Remark 3.2. Note that subcases (H_2) and (H_7) imply (D_2), which means that system (1.1) is bistable, i. e. , the dynamic behavior of the system (1.1) is determined by the initial conditions. It follows from Theorem 2.1 that by choosing suitable feedback control variable, then species x_2 will be driven to extinct in system (1.4).

Remark 3.3. Note that subcases $(H_3), (H_6)$ imply (D_1). Then system (1.1) is globally stable. In this case, by choosing some suitable feedback control variable, species x_2 will be driven to extinct in system (1.4).

Remark 3.4. Note that subcases (H_4) implies (D_3), then species x_1 is extinct in system (1.1). It follows from Theorem 2.1 that by choosing suitable feedback control species x_2 will extinct in system (1.4), while species x_1 will convergence to a steady state.

Remark 3.5. If condition (A_1) is replaced by

$$\frac{a_1}{a_2} < \frac{b_{11}e_1 + d_1 f_1}{b_{21}e_1 + d_2 f_1}, \ \frac{a_1}{a_2} < \frac{b_{12}e_1 + d_1 f_2}{b_{22}e_1 + d_2 f_2}. \tag{A_4}$$

Then some similar result as that of Theorem 2.1 can be estabilished, where the roles of x_1, x_2 are interchanged.

By simple calculating, condition (A_3) is equivalent to

$$(f_1d_2b_{12} - f_2d_1b_{21})^2 < 4f_1f_2d_1d_2(b_{11}b_{22} - b_{12}b_{21})$$

then condition (A_3) implies inequality

$$\frac{b_{11}}{b_{21}} > \frac{b_{12}}{b_{22}} \tag{B_1}$$

holds. Then condition (A_2) implying (B_1) contains ten subcases:

(1) If inequalities $\frac{b_{11}}{b_{21}} > \frac{d_1}{d_2}, \frac{b_{12}}{b_{22}} > \frac{d_1}{d_2}$ hold, then there exists three subcases:

$$\frac{b_{11}}{b_{21}} > \frac{b_{12}}{b_{22}} > \frac{b_{11}e_1 + d_1f_1}{b_{21}e_1 + d_2f_1} > \frac{a_1}{a_2} > \frac{b_{12}e_1 + d_1f_2}{b_{22}e_1 + d_2f_2}; \tag{I_1}$$

$$\frac{b_{11}}{b_{21}} > \frac{b_{11}e_1 + d_1f_1}{b_{21}e_1 + d_2f_1} > \frac{b_{12}}{b_{22}} > \frac{a_1}{a_2} > \frac{b_{12}e_1 + d_1f_2}{b_{22}e_1 + d_2f_2}; \tag{I_2}$$

$$\frac{b_{11}}{b_{21}} > \frac{b_{11}e_1 + d_1f_1}{b_{21}e_1 + d_2f_1} > \frac{a_1}{a_2} > \frac{b_{12}}{b_{22}} > \frac{b_{12}e_1 + d_1f_2}{b_{22}e_1 + d_2f_2}. \tag{I_3}$$

(2) If inequality $\frac{b_{11}}{b_{21}} > \frac{d_1}{d_2} > \frac{b_{12}}{b_{22}}$ holds, then there exists one subcase:

$$\frac{b_{11}}{b_{21}} > \frac{b_{11}e_1 + d_1f_1}{b_{21}e_1 + d_2f_1} > \frac{a_1}{a_2} > \frac{b_{12}e_1 + d_1f_2}{b_{22}e_1 + d_2f_2} > \frac{b_{12}}{b_{22}}. \tag{I_4}$$

(3) If inequality $\frac{b_{11}}{b_{21}} < \frac{d_1}{d_2} < \frac{b_{12}}{b_{22}}$ holds, then there exists three subcases:

$$\frac{b_{11}e_1 + d_1f_1}{b_{21}e_1 + d_2f_1} > \frac{b_{11}}{b_{21}} > \frac{b_{12}}{b_{22}} > \frac{a_1}{a_2} > \frac{b_{12}e_1 + d_1f_2}{b_{22}e_1 + d_2f_2}; \tag{I_5}$$

$$\frac{b_{11}e_1 + d_1f_1}{b_{21}e_1 + d_2f_1} > \frac{b_{11}}{b_{21}} > \frac{a_1}{a_2} > \frac{b_{12}}{b_{22}} > \frac{b_{12}e_1 + d_1f_2}{b_{22}e_1 + d_2f_2}; \tag{I_6}$$

$$\frac{b_{11}e_1 + d_1f_1}{b_{21}e_1 + d_2f_1} > \frac{a_1}{a_2} > \frac{b_{11}}{b_{21}} > \frac{b_{12}}{b_{22}} > \frac{b_{12}e_1 + d_1f_2}{b_{22}e_1 + d_2f_2}. \tag{I_7}$$

(4) If inequalities $\frac{b_{11}}{b_{21}} < \frac{d_1}{d_2}, \frac{b_{12}}{b_{22}} < \frac{d_1}{d_2}$ hold, then there exists three subcases:

$$\frac{b_{11}e_1 + d_1f_1}{b_{21}e_1 + d_2f_1} > \frac{b_{11}}{b_{21}} > \frac{a_1}{a_2} > \frac{b_{12}e_1 + d_1f_2}{b_{22}e_1 + d_2f_2} > \frac{b_{12}}{b_{22}}; \tag{I_8}$$

$$\frac{b_{11}e_1 + d_1f_1}{b_{21}e_1 + d_2f_1} > \frac{a_1}{a_2} > \frac{b_{11}}{b_{21}} > \frac{b_{12}e_1 + d_1f_2}{b_{22}e_1 + d_2f_2} > \frac{b_{12}}{b_{22}}; \tag{I_9}$$

$$\frac{b_{11}e_1+d_1f_1}{b_{21}e_1+d_2f_1} > \frac{a_1}{a_2} > \frac{b_{12}e_1+d_1f_2}{b_{22}e_1+d_2f_2} > \frac{b_{11}}{b_{21}} > \frac{b_{12}}{b_{22}}. \tag{I_{10}}$$

Any of the ten subcases and condition (A_3) hold, then the system (1.4) is globally stable.

Remark 3.6. Note that subcases $(I_1), (I_2), (I_5)$ imply (D_3), then species x_1 is extinct in system (1.1). And $(I_7), (I_9), (I_{10})$ imply (D_4), thus species x_2 is extinct in system (1.1). It follows from Theorem 2.2 that under certain conditions, suitable feedback control variable can make extinct species in system (1.1) becomes globally stable in system (1.4) .

Remark 3.7. Note that subcases $(I_3), (I_4), (I_6)$ and (I_8) imply (D_1), Then system (1.1) is globally stable. Theorem 2.2 shows that under certain conditions, suitable feedback control variable can make the stable species still keeps the property of stability.

4. Examples

In this section we shall give several examples to illustrate the feasibility of main results in the previous section.

Example 4.1. *Consider the following equations*

$$\begin{aligned} x_1'(t) &= x_1(t)\Big(1.3 - x_1(t) - 1.5\int_0^{+\infty} e^{-s}x_2(t-s)ds - u(t)\Big), \\ x_2'(t) &= x_2(t)\Big(1 - \int_0^{+\infty} e^{-s}x_1(t-s)ds - 1.2x_2(t) - 2u(t)\Big), \\ u'(t) &= -u(t) + 1.2x_1(t) + 4x_2(t). \end{aligned} \tag{4.1}$$

By simple calculation, one has $\frac{a_1}{a_2} = 1.3 > \frac{b_{11}}{b_{21}} = 1 > \frac{b_{11}e_1+d_1f_1}{b_{21}e_1+d_2f_1} \approx 0.6471$, $\frac{a_1}{a_2} = 1.3 > \frac{b_{12}}{b_{22}} = 1.25 > \frac{b_{12}e_1+d_1f_2}{b_{22}e_1+d_2f_2} \approx 0.5978$, *that is, subcase* (H_1) *in Theorem 2.1 holds. It follows from Theorem* 2.1 *that the boundary equilibrium* $(x_1{}^*, 0, u^*) = (0.5909, 0, 0.7091)$ *of system* (4.1) *is globally asymptotically stable. Figure 1 shows the dynamics behavior of system* (4.1).

Example 4.2. *Consider the following equations*

$$\begin{aligned} x_1'(t) &= x_1(t)\Big(1.5 - 3x_1(t) - 1.5\int_0^{+\infty} e^{-s}x_2(t-s)ds - u(t)\Big), \\ x_2'(t) &= x_2(t)\Big(1.2 - 3\int_0^{+\infty} e^{-s}x_1(t-s)ds - x_2(t) - 2u(t)\Big), \\ u'(t) &= -0.8u(t) + 1.2x_1(t) + 3x_2(t). \end{aligned} \tag{4.2}$$

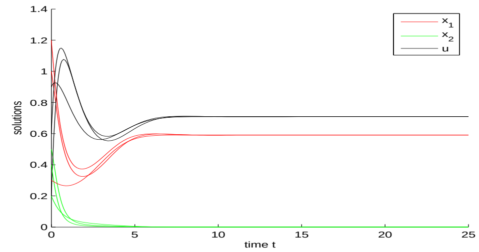

FIGURE 1. Dynamic behaviors of the solution $(x_1(t), x_2(t), u(t))^T$ of system (4.1), with the initial conditions $(x_1(\theta), x_2(\theta), u(\theta))^T = (0.3, 0.2, 0.9)^T$,$(1.2, 0.4, 0.6)^T$ and $(1.0, 0.5, 0.2)^T$ for $\theta \in (-\infty, 0]$, respectively.

By simple calculation, one has $\frac{a_1}{a_2} = 1.25 > \frac{b_{11}}{b_{21}} = 1 > \frac{b_{11}e_1 + d_1 f_1}{b_{21}e_1 + d_2 f_1} = 0.75$, $\frac{b_{12}}{b_{22}} = 1.5 > \frac{a_1}{a_2} = 1.25 > \frac{b_{12}e_1 + d_1 f_2}{b_{22}e_1 + d_2 f_2} \approx 0.6176$, *that is, subcase* (H_2) *in Theorem* 2.1 *holds. It follows from Theorem* 2.1 *that the boundary equilibrium* $({x_1}^*, 0, u^*) = (\frac{1}{3}, 0, \frac{1}{2})$ *of system* (4.2) *is globally asymptotically stable. Figure 2 shows the dynamics behavior of system* (4.2).

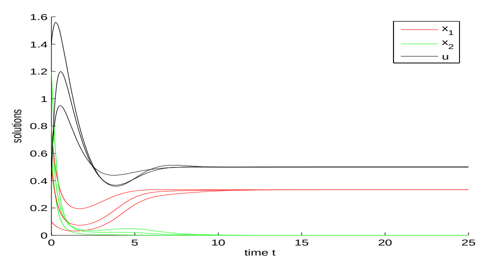

FIGURE 2. Dynamic behaviors of the solution $(x_1(t), x_2(t), u(t))^T$ of system (4.2), with the initial conditions $(x_1(\theta), x_2(\theta), u(\theta))^T = (0.8, 0.6, 0.5)^T$, $(0.1, 0.8, 1.4)^T$ and $(0.5, 1.2, 0.4)^T$ for $\theta \in (-\infty, 0]$, respectively.

Example 4.3. *Consider the following equations*

$$\begin{aligned} x_1'(t) &= x_1(t)\Big(1.5 - 3x_1(t) - 3\int_0^{+\infty} e^{-s}x_2(t-s)ds - u(t)\Big), \\ x_2'(t) &= x_2(t)\Big(1 - 1.5\int_0^{+\infty} e^{-s}x_1(t-s)ds - 3x_2(t) - 1.5u(t)\Big), \\ u'(t) &= -1.5u(t) + 3x_1(t) + 2x_2(t). \end{aligned} \tag{4.3}$$

By simple calculation, one has $\frac{b_{11}}{b_{21}} = 2 > \frac{a_1}{a_2} = 1.5 > \frac{b_{11}e_1 + d_1 f_1}{b_{21}e_1 + d_2 f_1} \approx 1.1111$, $\frac{a_1}{a_2} = 1.5 > \frac{b_{12}}{b_{22}} = 1 > \frac{b_{12}e_1 + d_1 f_2}{b_{22}e_1 + d_2 f_2} = 0.8667$, *that is, subcase* (H_3) *in Theorem 2.1 holds. It follows from Theorem* 2.1 *that the boundary equilibrium* $(x_1{}^*, 0, u^*) = (0.3, 0, 0.6)$ *of system* (4.3) *is globally asymptotically stable. Figure 3 shows the dynamics behavior of system* (4.3).

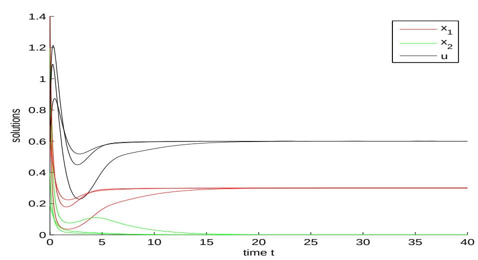

FIGURE 3. Dynamic behaviors of the solution $(x_1(t), x_2(t), u(t))^T$ of system (4.3), with the initial conditions $(x_1(\theta), x_2(\theta), u(\theta))^T = (0.9, 1.2, 0.7)^T$, $(0.7, 0.2, 0.6)^T$ and $(1.4, 0.4, 0.8)^T$ for $\theta \in (-\infty, 0]$, respectively.

Example 4.4. *Consider the following equations*

$$\begin{aligned} x_1'(t) &= x_1(t)\Big(1.4 - 3x_1(t) - 2\int_0^{+\infty} e^{-s}x_2(t-s)ds - u(t)\Big), \\ x_2'(t) &= x_2(t)\Big(1 - \int_0^{+\infty} e^{-s}x_1(t-s)ds - x_2(t) - 2u(t)\Big), \\ u'(t) &= -u(t) + 2x_1(t) + 4x_2(t). \end{aligned} \tag{4.4}$$

By calculation, one has $\frac{b_{11}}{b_{21}} = 3 > \frac{a_1}{a_2} = 1.4 > \frac{b_{11}e_1 + d_1 f_1}{b_{21}e_1 + d_2 f_1} = 1$, $\frac{b_{12}}{b_{22}} = 2 > \frac{a_1}{a_2} = 1.4 > \frac{b_{12}e_1 + d_1 f_2}{b_{22}e_1 + d_2 f_2} \approx 0.6667$, *that is, subcase* (H_4) *in Theorem 2.1 holds. It follows from Theorem* 2.1 *that the boundary equilibrium* $(x_1{}^*, 0, u^*) = (0.28, 0, 0.56)$ *of system* (4.4) *is globally* *asymptotically stable. Figure 4 shows the dynamics behavior of system* (4.4).

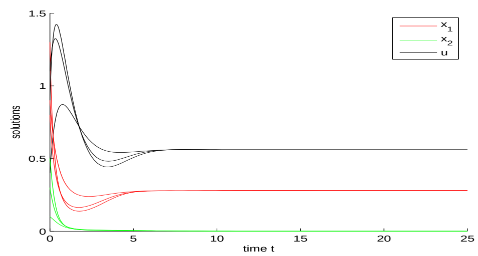

FIGURE 4. Dynamic behaviors of the solution $(x_1(t), x_2(t), u(t))^T$ of system (4.4), with the initial conditions $(x_1(\theta), x_2(\theta), u(\theta))^T = (0.9, 0.1, 0.4)^T$, $(1.3, 0.5, 0.9)^T$ and $(0.8, 0.3, 1.1)^T$ for $\theta \in (-\infty, 0]$, respectively.

Example 4.5. *consider the following equations*

$$\begin{aligned} x_1'(t) &= x_1(t)\Big(1.4 - 3x_1(t) - 2\int_0^{+\infty} e^{-s} x_2(t-s)ds - 5u(t)\Big), \\ x_2'(t) &= x_2(t)\Big(1 - \int_0^{+\infty} e^{-s} x_1(t-s)ds - x_2(t) - 4.5u(t)\Big), \\ u'(t) &= -u(t) + 1x_1(t) + 1.5x_2(t). \end{aligned} \tag{4.5}$$

By calculation, one has $\frac{b_{11}}{b_{21}} = 3 > \frac{b_{12}}{b_{22}} = 2 > \frac{b_{11}e_1 + d_1 f_1}{b_{21}e_1 + d_2 f_1} \approx 1.4545 > \frac{a_1}{a_2} = 1.4 > \frac{b_{12}e_1 + d_1 f_2}{b_{22}e_1 + d_2 f_2} \approx 1.2258$, *that is, subcase* (I_1) *in Theorem 2.2 holds.* $\frac{f_1 d_2 b_{12} + f_2 d_1 b_{21}}{2 f_2 d_1 b_{22}} = 1.1 < \frac{2 f_1 d_2 b_{11}}{f_1 d_2 b_{12} + f_2 d_1 b_{21}} \approx 1.6364$, *that is condition* (A_3) *holds. It follows from Theorem* 2.2 *that the positive equilibrium* $(x_1^\diamond, x_2^\diamond, u^\diamond) = (0.1385, 0.0308, 0.1847)$ *of system* (4.5) *is globally asymptotically stable. Figure 5 shows the dynamics behavior of system* (4.5).

5. Discussion

Depending on the relationship of the coefficients, the traditional Lotka-Volterra competitive system admits four kinds of dynamic behaviors: coexistence, extinction and bistable. Gopalsamy and Weng [10] proposed and studied the dynamic behaviors of system (1.2), sufficient

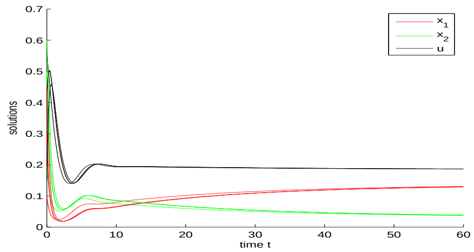

FIGURE 5. Dynamic behaviors of the solution $(x_1(t), x_2(t), u(t))^T$ of system (4.5), with the initial conditions $(x_1(\theta), x_2(\theta), u(\theta))^T = (0.3, 0.5, 0.4)^T$, $(0.1, 0.2, 0.55)^T$ and $(0.45, 0.6, 0.15)^T$ for $\theta \in (-\infty, 0]$, respectively.

conditions which guarantee the existence of a unique globally attractive positive equilibrium are obtained. Li, Han and Chen [11] proposed and studied the dynamic behaviors of system (1.3), they showed that feedback control variables can only change the position of the equilibrium but not change the stability property of the system if the Lotka-Volterra system is globally stable; They also found that for the extinct case, by choosing suitable control variables, extinct species can become globally stable, or still keep the property of extinction.

In this paper, we propose and study a Lotka-Volterra system with one feedback control. At first sight, one might conjecture the system admits some similar dynamic behaviors as that of system (1.2) and (1.3). However, our study shows that the system admits more complicate dynamic behaviors.

Concern with the extinction of the system, Our first finding is that feedback control variable can change the stability property of the positive equilibrium of the system, by choosing suitable feedback control variable, the species could no longer coexistence, one of the species will be driven to extinction despite the original Lotka-Volterra system is globally stable, such a finding is different to that of [10, 11]. Our second finding is similar to that of [11], that is, for the extinct case, by choosing suitable control variables, extinct species can be still keep the property of extinction. Our third finding is that for the bistable case (the dynamic behavior of the solution is determined by the initial condition), by choosing the suitable feedback control variable, despite the choice of initial condition, one of the species will be driven to extinction. This is the first time that such a phenomenon been discussed. Our forth finding is that by choosing the suitable feedback control variable, the extinct species in system (1.1) could become permanent while the p ermanent species in system (1.l) could become extinct.

Concern with the stability property of the system, we show that by choosing the suitable feedback control variable, the extinct species could become globally stable, or the global stability property is still contains if the original Lotka-Volerra system is globally stable.

To sum up, for the system with one feedback control variable, we may draw the conclusion: Simple system, complicate dynamic behaviors. The feedback control variable is one of the most important factor to determine the dynamic behaviors of the system. Such an finding may be used to protect the endangered species.

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