J. Nonlinear Funct. Anal. 2016 (2016), Article ID 46

MATHRES

Journal of Nonlinear Functional Analysis

Available online at http://jnfa.mathres.org

STABILITY ANALYSIS OF A TWO SPECIES AMENSALISM MODEL WITH HOLLING II FUNCTIONAL RESPONSE AND A COVER FOR THE FIRST SPECIES

RUNXIN WU[1,*], LIANG ZHAO[2], QIAOXIA LIN[3]

[1]College of Mathematics and Physics, Fujian University of Technology, Fuzhou 350014, Fujian, China

[2]School of Information and Statistics, Guangxi University of Finance and Economics,

Nannin, 530003, Guangxi, China

[3]College of Mathematics and Computer Sciences, Fuzhou University, Fuzhou 350116, Fujian, China

Abstract. In this paper, a two species amensalism model with Holling II functional response and a cover for the first species is investigated. Our study shows that if the cover is enough large to accumulate the first species, then two species could coexist in a stable state; while if the cover is limited, then the first species maybe driven to extinction. Numerical simulations are carried out to illustrate the feasibility of our findings.

Keywords. Amensalism model; Boundary equilibrium; Lyapunov function; Stability.

1. Introduction

During the past decade, many scholars [1-10] investigated the dynamic behaviors of the amensalism model, here, amensalism means that an interaction between two species, where a species inflicts harm to the other species without any costs or benefits received by the other. However, only recently did scholars ([7], [10]) paid attention to the influence of refuge on the ammensalism model.

*Corresponding author

E-mail address: runxinwu@163.com (R. Wu)

Received September 8, 2016

Sita Rambabu, Narayan and Bathul [7] considered a two species amensalism model with a partial cover for the first species to protect it from the second species, the model is as follows

$$\begin{aligned}
\frac{dx}{dt} &= a_1x(t) - b_1x^2(t) - c_1(1-k)x(t)y(t), \\
\frac{dy}{dt} &= a_2y(t) - b_2y^2(t),
\end{aligned}$$

where $a_i, b_i, i = 1,2$ and c_1 are all positive constants, k is a cover provided for the species x, and $0 < k < 1$. The series solution of above system was approximated by the Homotopy analysis method (HAM). Recently, Xie, Chen and He [10] argued that one should give positive answer to show how the cover (represented by k) influence the dynamic behaviors of the system (1.1). After gave a thoroughly analysis of the global dynamic behaviors of system (1.1), they showed that if the cover is enough large, then two species could coexist in a stable state.

It bring to our attention that in system (1.1), the authors made the assumption that the relationship between two species is linearizing. It is well known that in the predator-prey system, more appropriate model should incorporating some kind of functional response [12]-[31]. And among those functional response, the most common and fundamental one is the Holling II functional response, which was proposed by Holling in 1965; see [11] and the references therein. Stimulated by the works of [10], [17] and [18], we propose the following two species amensalism model with Holling II functional response and a cover for the first species

$$\begin{aligned}
\frac{dx}{dt} &= a_1x(t) - b_1x^2(t) - \frac{c_1(1-k)x(t)y(t)}{1+d_1y(t)}, \\
\frac{dy}{dt} &= a_2y(t) - b_2y^2(t).
\end{aligned} \tag{1.1}$$

where $a_i, b_i, i = 1,2$ c_1 and d_1 are all positive constants, k is a cover provided for the species x, and $0 < k < 1$.

The aim of this paper is to give the detail analysis of the dynamic behaviors of system (1.1) and to find out how the cover influence the dynamic behaviors of the system (1.1). We arrange the paper as follows: In the next section, we will investigate the existence and local stability property of the equilibria of system (1.1). In Section 3, we will investigate the global stability property of the system; In Section 4, several examples together with their numeric simulations are presented to show the feasibility of our main results. We end this paper by a briefly discussion.

2. The existence and stability of the equilibria

The equilibria of system (1.1) is determined by the equations

$$\begin{aligned} &a_1x - b_1x^2 - \frac{c_1(1-k)xy}{1+d_1y} = 0, \\ &a_2y - b_2y^2 = 0. \end{aligned} \tag{2.1}$$

Hence, system (1.1) admits the trivial equilibrium $E_0(0,0)$, boundary equilibria $E_1\big(\frac{a_1}{b_1},0\big)$ and $E_2\big(0,\frac{a_2}{b_2}\big)$. Also, if $k > 1 - \dfrac{a_1a_2d_1 + a_1b_2}{a_2c_1}$, then system (1.1) admits a unique positive equilibrium $E_3\big(x^*,y^*\big)$, where

$$x^* = \frac{a_1a_2d_1 + a_2c_1k + a_1b_2 - a_2c_1}{b_1(a_2d_1 + b_2)},\ y^* = \frac{a_2}{b_2}. \tag{2.2}$$

Concerned with the local stability property of the above four equilibria, we have

Theorem 2.1. *$E_0(0,0)$ and $E_1(\frac{a_1}{b_1},0)$ are unstable; If $k < 1 - \dfrac{a_1a_2d_1 + a_1b_2}{a_2c_1}$, then $E_2(0,\frac{a_2}{b_2})$ is stable and if $k > 1 - \dfrac{a_1a_2d_1 + a_1b_2}{a_2c_1}$, then $E_2(0,\frac{a_2}{b_2})$ is unstable, in this case, $E_3\Big(x^*,y^*\Big)$ is stable.*

Proof. The Jacobian matrix of the system (1.1) is calculated as

$$J(x,y) = \begin{pmatrix} B_1 & B_2 \\ 0 & -2b_2y + a_2 \end{pmatrix},$$

where

$$B_1 = a_1 - 2b_1x - \frac{c_1(1-k)y}{1+d_1y},$$

$$B_2 = -\frac{c_1(1-k)x}{(1+d_1y)^2}.$$

The Jacobian of the system about the equilibrium point $E_1(0,0)$ is given by

$$\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}.$$

The eigenvalues of the matrix are a_1 and a_2, $E_0(0,0)$ is a unstable.

For $E_1(\frac{a_1}{b_1},0)$, its Jacobian matrix is given by

$$\begin{pmatrix} -a_1 & -\dfrac{c_1(1-k)a_1}{b_1} \\ 0 & a_2 \end{pmatrix}.$$

The eigenvalues of the matrix are $-a_1$ and a_2, $E_1(\frac{a_1}{b_1},0)$ is unstable.

For $E_2(0,\frac{a_2}{b_2})$, its Jacobian matrix is given by

$$\begin{pmatrix} \dfrac{a_2a_1d_1+a_2c_1k+a_1b_2-a_2c_1}{a_2d_1+b_2} & 0 \\ 0 & -a_2 \end{pmatrix}.$$

The eigenvalues of the matrix are $\dfrac{a_2a_1d_1+a_2c_1k+a_1b_2-a_2c_1}{a_2d_1+b_2}$ and $-a_2$. Hence, if $a_2a_1d_1+a_2c_1k+a_1b_2 > a_2c_1$ hold (i.e., $k > 1-\dfrac{a_1a_2d_1+a_1b_2}{a_2c_1}$), $E_2(0,\frac{a_2}{b_2})$ is unstable, and if $a_2a_1d_1+a_2c_1k+a_1b_2 < a_2c_1$ hold (i.e., $k < 1-\dfrac{a_1a_2d_1+a_1b_2}{a_2c_1}$), $E_2(0,\frac{a_2}{b_2})$ is stable.

The Jacobian matrix about the equilibrium E_3 is given by

$$\begin{pmatrix} -\dfrac{a_2a_1d_1+a_2c_1k+a_1b_2-a_2c_1}{a_2d_1+b_2} & * \\ 0 & -a_2 \end{pmatrix},$$

if $k > 1-\dfrac{a_1a_2d_1+a_1b_2}{a_2c_1}$ hold, then $-\dfrac{a_2a_1d_1+a_2c_1k+a_1b_2-a_2c_1}{a_2d_1+b_2} < 0$, hence, above matrix has two negative eigenvalues and E_3 is stable. This ends the proof of Theorem 2.1.

Remark 2.2. When $d_1 = 0$, then Theorem 2.1 degenerate to Theorem 2.1 in [10], thus, we generalize Theorem 2.1 of [10] to the system with Holling II functional response. Theorem 2.1 also shows that if the positive equilibrium is exist, then it is locally stable.

Remark 2.3. It seems strange since the dynamic behaviors of the traditional Lotka-Volerra predator-prey model and predator-prey model with Holling II functional response has substantial difference, depending on the relationship of the parameters, the Lotka-Volterra predator-prey model could have a globally attractive positive equilibrium or the globally attractive boundary equilibrium; while for the predator-prey system with the Holling II functional response, if the positive equilibrium is unstable, the system could admits a unique globally stable limit cycles. Our Theorem 2.1 shows that the dynamic behaviours of system (1.1) is similar to the Lotka-Volterra one, maybe the reason lies in that the second equation in (1.1) is independent of x, this leads to the fact that the density of species y finally attractive to $\frac{a_2}{b_2}$.

3. stability of the equilibria

This section we will further investigate the global stability of the boundary equilibrium E_2 and the positive equilibrium E_3.

Now let's state several lemmas which will be useful in the proving of main results.

Lemma 3.1. [29] *If $a > 0, b > 0$ and $\dot{x} \geq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have*

$$\liminf_{t \to +\infty} x(t) \geq \frac{b}{a}.$$

If $a > 0, b > 0$ and $\dot{x} \leq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have

$$\limsup_{t \to +\infty} x(t) \leq \frac{b}{a}.$$

Lemma 3.2. [10] *System $\dfrac{dy}{dt} = a_2 y(t) - b_2 y^2(t)$ has a unique globally attractive positive equilibrium $y^* = \frac{a_2}{b_2}$.*

Theorem 3.3. *If $k < 1 - \dfrac{a_1 a_2 d_1 + a_1 b_2}{a_2 c_1}$ hold, then $E_2(0, \frac{a_2}{b_2})$ is globally stable.*

Proof. $k < 1 - \dfrac{a_1 a_2 d_1 + a_1 b_2}{a_2 c_1}$ is equivalent to $a_2 a_1 d_1 + a_2 c_1 k + a_1 b_2 < a_2 c_1$, that is,

$$a_1(b_2 + a_2 d_1) - c_1(1 - k)a_2 < 0. \tag{3.1}$$

Hence, we have

$$a_1(1 + \frac{a_2}{b_2} d_1) - c_1(1 - k)\frac{a_2}{b_2} < 0. \tag{3.2}$$

And so, one could choose $\varepsilon > 0$ small enough such that

$$a_1(1 + (\frac{a_2}{b_2} + \varepsilon)d_1) - c_1(1 - k)(\frac{a_2}{b_2} - \varepsilon) < 0, \tag{3.3}$$

which is equivalent to

$$a_1 - \frac{c_1(1-k)(\frac{a_2}{b_2}-\varepsilon)}{1+(\frac{a_2}{b_2}+\varepsilon)d_1} < 0. \tag{3.4}$$

For this ε, it follows from Lemma 3.2 that there exists a $T > 0$, such that every positive solution $y(t)$ of

$$\frac{dy}{dt} = a_2 y(t) - b_2 y^2(t)$$

satisfies

$$\frac{a_2}{b_2} - \varepsilon < y(t) < \frac{a_2}{b_2} + \varepsilon. \tag{3.5}$$

Now we consider the Lyapunov function

$$V(x,y) = x + y - y^* - y^* \ln\frac{y}{y^*}, \tag{3.6}$$

where $y^* = \frac{a_2}{b_2}$. Calculating the derivative of V along the solution of the system (1.1), by using equalities (3.4), (3.5) and the fact $a_2 = b_2 y^*$, for $t > T$, we have

$$\begin{aligned}
\dot{V} &= a_1 x - b_1 x^2 - \frac{c_1(1-k)xy}{1+d_1 y} + (a_2 y - b_2 y^2)(1 - \frac{y^*}{y}) \\
&= -b_1 x^2 + \Big(a_1 - \frac{c_1(1-k)y}{1+d_1 y}\Big)x + (b_2 y^* - b_2 y)(y - y^*) \\
&\le -b_1 x^2 + \Big(a_1 - \frac{c_1(1-k)(\frac{a_2}{b_2}-\varepsilon)}{1+(\frac{a_2}{b_2}+\varepsilon)d_1}\Big)x - b_2(y-y^*)^2.
\end{aligned} \tag{3.7}$$

Obviously, $\frac{dV}{dt} < 0$ strictly for all $x, y > 0$ except the positive equilibrium $E_2(0, \frac{a_2}{b_2})$, where $\frac{dV}{dt} = 0$. Thus, $V(x,y)$ satisfies Lyapunov's asymptotic stability theorem, and the boundary equilibrium$E_2(0, \frac{a_2}{b_2})$ of system (1.1) is globally stable. This ends the proof of Theorem 3.1.

Remark 3.4. Theorem 3.3 shows that the conditions which ensure the locally stable of the boundary equilibrium E_2 is coincidence with the globally stable ones.

Theorem 3.5. *If $k > 1 - \dfrac{a_1 a_2 d_1 + a_1 b_2}{a_2 c_1}$ hold, then $E_3\big(x^*, y^*\big)$ is globally stable.*

Proof. $k > 1 - \dfrac{a_1 a_2 d_1 + a_1 b_2}{a_2 c_1}$ is equivalent to

$$a_1(b_2 + a_2 d_1) - c_1(1-k)a_2 > 0, \tag{3.8}$$

or

$$a_1(1 + \frac{a_2}{b_2} d_1) - c_1(1-k)\frac{a_2}{b_2} > 0. \tag{3.9}$$

And so, one could choose $\varepsilon > 0$ small enough such that

$$a_1\Big(1+(\frac{a_2}{b_2}-\varepsilon)d_1\Big)-c_1(1-k)(\frac{a_2}{b_2}-\varepsilon)>0 \tag{3.10}$$

and

$$a_1\Big(1+(\frac{a_2}{b_2}+\varepsilon)d_1\Big)-c_1(1-k)(\frac{a_2}{b_2}+\varepsilon)>0 \tag{3.11}$$

hold, which is equivalent to

$$a_1-\frac{c_1(1-k)(\frac{a_2}{b_2}-\varepsilon)}{1+(\frac{a_2}{b_2}-\varepsilon)d_1}>0 \tag{3.12}$$

and

$$a_1-\frac{c_1(1-k)(\frac{a_2}{b_2}+\varepsilon)}{1+(\frac{a_2}{b_2}+\varepsilon)d_1}>0 \tag{3.13}$$

hold. For this ε, it follows from Lemma 3.1 in [10] that there exists a $T > 0$, such that every positive solution $y(t)$ of

$$\frac{dy}{dt}=a_2y(t)-b_2y^2(t)$$

satisfies

$$\lim_{t\to+\infty}y(t)=\frac{a_2}{b_2}=y^*, \tag{3.14}$$

and so

$$\frac{a_2}{b_2}-\varepsilon<y(t)<\frac{a_2}{b_2}+\varepsilon.$$

Hence, to end the proof of Theorem 3.5, it is enough to show that under the assumption of Theorem 3.5, $\lim\limits_{t\to+\infty} x(t)=x^*$. For $t \geq T$, it follows from (3.14), the monotonicity of the function $f(y)=\dfrac{c_1(1-k)y}{1+d_1y}$ and the first equation of system (1.1) that

$$\begin{aligned}\frac{dx}{dt} &\leq a_1x(t)-b_1x^2(t)-\frac{c_1(1-k)(\frac{a_2}{b_2}-\varepsilon)}{1+d_1(\frac{a_2}{b_2}-\varepsilon)}x(t)\\ &= x(t)\Big(a_1-\frac{c_1(1-k)(\frac{a_2}{b_2}-\varepsilon)}{1+d_1(\frac{a_2}{b_2}-\varepsilon)}-b_1x(t)\Big).\end{aligned} \tag{3.15}$$

Applying Lemma 3.1 to (3.15) leads to

$$\limsup_{t\to+\infty}x(t)\leq\frac{a_1-\dfrac{c_1(1-k)(\frac{a_2}{b_2}-\varepsilon)}{1+d_1(\frac{a_2}{b_2}-\varepsilon)}}{b_1}. \tag{3.16}$$

Setting $\varepsilon \to 0$ in (3.16) leads to

$$\limsup_{t\to+\infty} x(t) \le \frac{a_1 - \dfrac{c_1(1-k)\frac{a_2}{b_2}}{1+d_1\frac{a_2}{b_2}}}{b_1} = \frac{a_1a_2d_1 + a_2c_1k + a_1b_2 - a_2c_1}{b_1(a_2d_1+b_2)} = x^*. \tag{3.17}$$

For $t \ge T$, again it follows from (3.14), the monotonicity of the function $f(y) = \dfrac{c_1(1-k)y}{1+d_1y}$ and the first equation of system (1.1) that

$$\begin{aligned} \frac{dx}{dt} &\ge a_1x(t) - b_1x^2(t) - \frac{c_1(1-k)(\frac{a_2}{b_2}+\varepsilon)}{1+d_1(\frac{a_2}{b_2}+\varepsilon)}x(t) \\ &= x(t)\Big(a_1 - \frac{c_1(1-k)(\frac{a_2}{b_2}+\varepsilon)}{1+d_1(\frac{a_2}{b_2}+\varepsilon)} - b_1x(t)\Big). \end{aligned} \tag{3.18}$$

Applying Lemma 3.1 to (3.18) leads to

$$\liminf_{t\to+\infty} x(t) \ge \frac{a_1 - \dfrac{c_1(1-k)(\frac{a_2}{b_2}+\varepsilon)}{1+d_1(\frac{a_2}{b_2}+\varepsilon)}}{b_1}. \tag{3.19}$$

Setting $\varepsilon \to 0$ in (3.19) leads to

$$\liminf_{t\to+\infty} x(t) \ge \frac{a_1 - \dfrac{c_1(1-k)\frac{a_2}{b_2}}{1+d_1\frac{a_2}{b_2}}}{b_1} = \frac{a_1a_2d_1 + a_2c_1k + a_1b_2 - a_2c_1}{b_1(a_2d_1+b_2)} = x^*. \tag{3.20}$$

(3.17) together with (3.20) leads to

$$\lim_{t\to+\infty} x(t) = x^*.$$

This ends the proof of Theorem 3.5.

Remark 3.6. Theorem 3.5 shows that if the positive equilibrium E_3 is exist, then it is globally stable.

Remark 3.7. If

$$\frac{a_1b_2}{aa_2c_1} < 1 < \frac{a_1a_2d_1 + a_1b_2}{a_2c_1}, \tag{3.21}$$

then for all $k \in (0,1)$, system (1.1) admits a unique positive equilibrium, one could easily see that if d_1 is enough large, ($d_1 > \dfrac{a_2c_1 - a_1b_2}{a_1a_2}$), then (3.21) holds. With the increasing of d_1, from the expression $\dfrac{y}{1+d_1y}$ one could see that the influence of the second species to the first species decreasing, and if it exceed the threshold $\frac{a_2c_1 - a_1b_2}{a_1a_2}$, the first species could survival well and the cover has no influence on the coexistence of the two species.

However, from Theorem 3.3 in [10], we know that for the Lotka-Volterra type amensalism model, without the cover, condition $\frac{a_1b_2}{a_2c_1} < 1$ is enough to ensure the first species will be driven to extinction. From this point, the Holling II functional response increasing the stability property of the system.

Remark 3.8. Noting that $x^*(k) = \frac{a_1a_2d_1 + a_2c_1k + a_1b_2 - a_2c_1}{b_1(a_2d_1 + b_2)}$ and $\frac{dx^*(k)}{dk} = \frac{a_2c_1}{b_1(a_2d_1 + b_2)} > 0$, hence, x^* is the strictly increasing function of the k, which means that the cover can increase the densities of the first species.

4. Numeric simulations

Now let us consider the following examples.

Example 4.1. Consider the following system

$$\begin{aligned} \frac{dx}{dt} &= x(t) - \tfrac{1}{4}x^2(t) - \frac{(1-k)x(t)}{1+d_1y(t)}y(t), \\ \frac{dy}{dt} &= \tfrac{1}{2}y(t) - y^2(t). \end{aligned} \tag{4.1}$$

In this system, corresponding to system (1.1), we take $a_1 = b_2 = c_1 = 1, a_2 = \frac{1}{2}, b_1 = \frac{1}{4}$. $k \in (0,1)$ and d_1 are two positive constants to be determined. Noting that $\frac{a_1b_2}{a_2c_1} = 2 > 1$, hence, for any $k \in (0,1)$ and any $d_1 \in R^+$, condition of Theorem 3.2 holds, and so, it follows from Theorem 3.5 that system (4.1) admits a unique positive equilibrium (x^*, y^*) which is globally attractive. Specially, if we take $k = \frac{1}{2}, d_1 = 1$, the unique positive equilibrium $(\frac{10}{3}, \frac{1}{2})$ is globally attractive. Figure 1 supports this assertion.

Example 4.2. Consider the following system

$$\begin{aligned} \frac{dx}{dt} &= x(t) - \tfrac{1}{4}x^2(t) - \frac{(1-k)x(t)}{1+d_1y(t)}y(t), \\ \frac{dy}{dt} &= 2y(t) - y^2(t). \end{aligned} \tag{4.2}$$

In this system, corresponding to system (1.1), we take $a_1 = b_2 = c_1 = 1, a_2 = 2, b_1 = \frac{1}{4}$. $k \in (0,1)$ and d_1 are positive constants to be determined. Obviously, if $d_1 > \dfrac{a_2c_1 - a_1b_2}{a_1a_2} = \dfrac{1}{2}$, then from Remark 3.7, for any $k \in (0,1)$, condition of Theorem 3.5 holds, and so, it follows from Theorem 3.5 that system (4.2) admits a unique positive equilibrium (x^*, y^*) which is globally attractive. Specially, if we take $k = \frac{1}{4}, d_1 = 1$, the unique positive equilibrium $(2,2)$ is globally attractive. Figure 2 supports this assertion.

Example 4.3. Consider the following system

$$\begin{aligned} \frac{dx}{dt} &= x(t) - \tfrac{1}{4}x^2(t) - \frac{(1-k)x(t)}{1+d_1y(t)}y(t), \\ \frac{dy}{dt} &= 2y(t) - y^2(t). \end{aligned} \tag{4.3}$$

In this system, corresponding to system (1.1), we take $a_1 = b_2 = c_1 = 1, a_2 = 2, b_1 = \frac{1}{4}$. $k \in (0,1)$ and d_1 are two positive constants to be determined. Obviously, if $d_1 < \dfrac{a_2c_1 - a_1b_2}{a_1a_2} = \dfrac{1}{2}$, then if $k > 1 - \dfrac{a_1a_2d_1 + a_1b_2}{a_2c_1} = \dfrac{1}{4}$, condition of Theorem 3.2 holds, and so, it follows from Theorem 3.5 that system (4.3) admits a unique positive equilibrium (x^*, y^*), which is globally attractive. Specially, if we take $k = \frac{1}{2}, d_1 = \frac{1}{4}$, the unique positive equilibrium $(\frac{4}{3}, 2)$ is globally attractive. Figure 3 supports this assertion. Also, if $k < 1 - \dfrac{a_1a_2d_1 + a_1b_2}{a_2c_1} = \dfrac{1}{4}$, from Theorem 3.1 that the boundary equilibrium $(0, \frac{a_2}{b_2})$ is globally attractive. Take $k = \frac{1}{8}, d_1 = \frac{1}{4}$, the boundary equilibrium $(0,2)$ is globally attractive. Figure 4 supports this assertion.

5. Discussion

During the last decade, many scholars investigated the dynamic behaviors of the predator-prey system incorporating a prey refuge, see [11]-[29]. Kar [17] studied a Holling II prey-predator system incorporating a prey refuge. His study shows that "a refuge can be important for the biological control of a pest, however, increasing the amount of refuge can increase prey densities and lead to population outbreaks." Chen, Chen and Xie [18] studied the dynamic behaviors of the Leslie-Gower predator-prey system incorporating a prey refuge, their study shows that the unique positive equilibrium of the system is globally stable, therefore, for the system they considered, prey refuge has no influence on the persistent property of the system. That

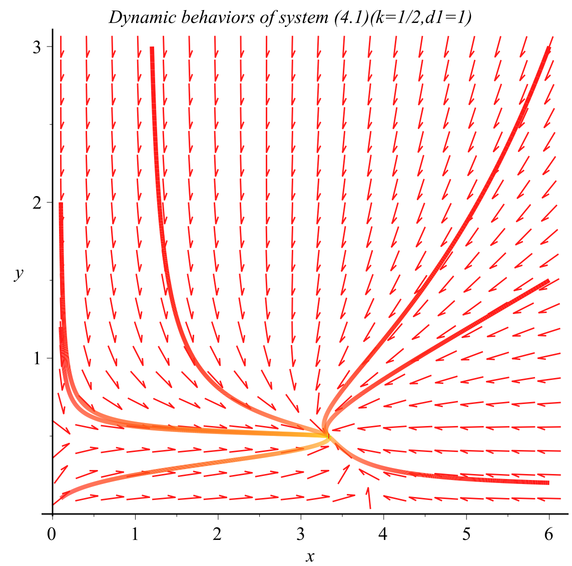

FIGURE 1. Numeric simulations of system (4.1) with $k = \frac{1}{2}, d_1 = 1$, the initial conditions $(x(0), y(0)) = (0.1, 1.2), (0.1, 2), (0.1, 0.1), (6, 0.2)$ $(6, 3)$ and $(6, 1.5)$, respectively.

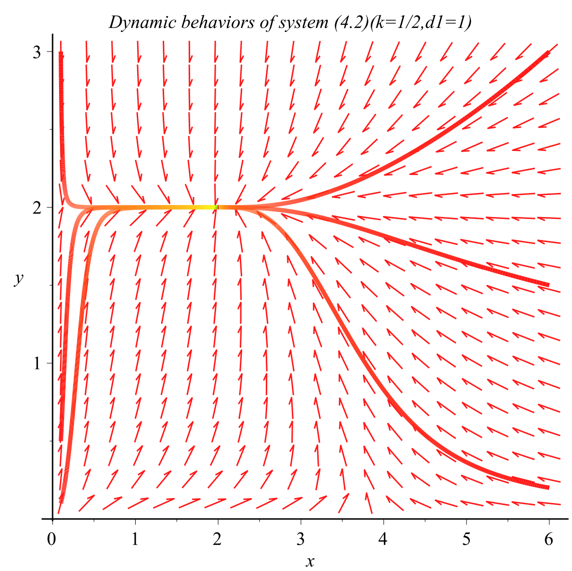

FIGURE 2. Numeric simulations of system (4.2) with $k = \frac{1}{4}, d_1 = 1$, the initial conditions $(x(0), y(0)) = (0.1, 0.1), (0.1, 3), (0.1, 0.5), (6, 0.2)$ $(6, 3)$ and $(6, 1.5)$, respectively.

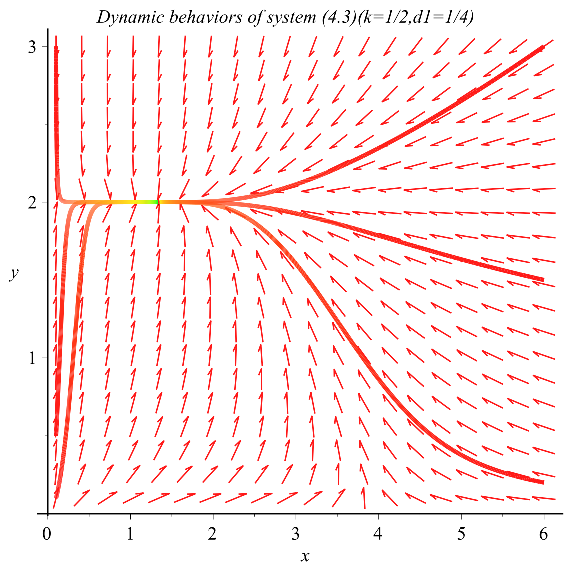

FIGURE 3. Numeric simulations of system (4.3) with $k = \frac{1}{2}$, $d_1 = \frac{1}{4}$, the initial conditions $(x(0), y(0)) = (0.1, 0.1), (0.1, 3), (0.1, 0.5), (6, 0.2)$ $(6, 3)$ and $(6, 1.5)$, respectively.

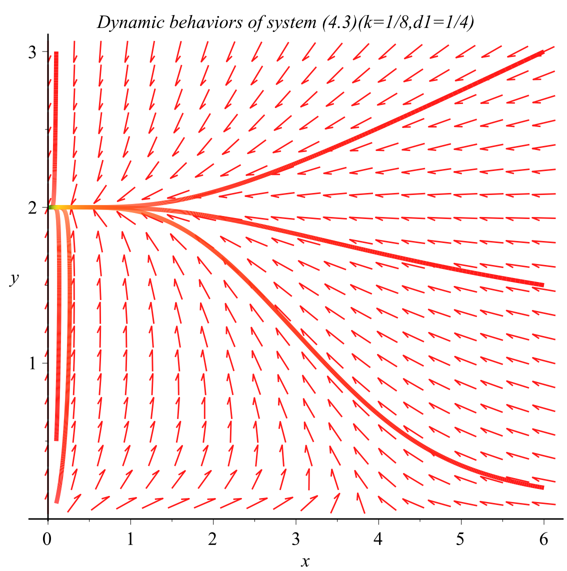

FIGURE 4. Numeric simulations of system (4.3) with $k = \frac{1}{8}$, $d_1 = \frac{1}{4}$, the initial conditions $(x(0), y(0)) = (0.1, 0.1), (0.1, 3), (0.1, 0.5), (6, 0.2)$ $(6, 3)$ and $(6, 1.5)$, respectively.

is, depending on the different predator-prey system, the influence of prey refuge maybe very different.

It bring to our attention that to this day, still few work ([7,10]) on the amensalism model with refuge, and so, based on recent work of Xie, Chen and He [10], we further consider the amensalism model with Holling II functional response and a cover for the first species. Our study shows that despite the introducing of the Holling II functional response, the dynamic behaviors of system (1.1) is similar to the dynamic behaviors of the Lotka-Volterra type amensalism model with a cover for the first species. That is, depending on the parameters k and d_1, the boundary equilibrium or the unique positive equilibrium is globally attractive (for more detail discussion, one could refer to Remark 3.3). Numeric simulations also support our findings. We mentioned here that a more suitable population model should include some past state of the species, and this leads to the mathematical modeling with delay, we leave this for future investigation.

Acknowledgments

The authors are grateful to anonymous referees for their excellent suggestions, which greatly improve the presentation of the paper. This research was supported by the Natural Science Foundation of Fujian Province (2015J01012, 2015J01019, 2015J05006) and the Scientific Research Foundation of Fuzhou University (XRC-1438).

References

[1] R. Y. Han, Y. L. Xue, L. Y. Yang, On the existence of positive periodic solution of a Lotka-Volterra amensalism model, J. Rongyang Univ. Ed. 33 (2015), 22-26.

[2] F. D. Chen, W. X. He, R. Y. Han, On discrete amensalism model of Lotka-Volterra, J. Beihua Univ. Natural Sci. Ed. 16 (2015), 141-144.

[3] F. D. Chen, M. S. Zhang, R. Y. Han, Existence of positive periodic solution of a discrete Lotka-Volterra amensalism model, J. Shengyang Univ. Nature Sci. Ed. 27 (2015), 251-254.

[4] G. C. Sun, Qualitative analysis on two populations ammenslism model, J. Jiamusi Unive. Nature Sci. Ed. 21 (2003), 283-286.

[5] Z. F. Zhu, Q. L. Chen, Mathematic analysis on commensalism Lotka-Volterra model of populations, J. Jixi Univ. Nature Sci. Ed. 8 (2008), 100-101.

[6] Z. Zhang, Stability and bifurcation analysis for a amensalism system with delays, Math. Numer. Sinica, 30 (2008), 213-224.

[7] B. Sita Rambabu, K. L. Narayan, S. Bathul, A mathematical study of two species amensalism model with a cover for the first species by homotopy analysis method, Adv. Appl. Sci. Res. 3 (2012), 1821-1826.

[8] K. V. L. N. Acharyulu, P.P. Mohan, N. Ch. Pattabhi Ramacharyulu, An ecological mathematical model of an immigrated ammensal and a migrated enemy model with mortality rate for ammensal species-an analytical investigation, Int. J. Pure Appl. Sci. Tech. 4 (2011), 71-84.

[9] K. V. L. N. Acharyulu, N. Ch. Pattabhi Ramacharyulu, On the carrying capacity of enemy species, inhibition coefficient of ammensal species and dominance reversal time in an ecological ammensalism –a special case study with numerical approach, Int. J. Adv. Sci. Tech. 43 (2012), 49-57.

[10] X. D. Xie, F. D. Chen, M. X. He, Dynamic behaviors of two species amensalism model with a cover for the firstspecies, J. Math. Comput. Sci. 16 (2016), 395-401.

[11] C.S. Holling, The functional response of predators to prey density and its role in mimicry and population regulations, Mem. Entomol. Soc. Can. 45 (1965), 3-60.

[12] Q. Yue, Dynamics of a modified Leslie-Gower predator-prey model with Holling-type II schemes and a prey refuge, SpringerPlus, 5 (2016), 1-12.

[13] Y. Wu, F. Chen, W. Chen & Y. Lin, Dynamic behaviors of a nonautonomous discrete predator-prey system incorporating a prey refuge and Holling type ii functional response, Discrete Dyn. Nature Soc. 2012 (2012), 1951-1965.

[14] F. Chen, Y. Wu, Z. Ma, Stability property for the predator-free equilibrium point of predator-prey systems with a class of functional response and prey eefuges, Discrete Dyn. Nature Soc. 2012 (2012), 477-502.

[15] R. Yang, C. Zhang, Dynamics in a diffusive predator-prey system with a constant prey refuge and delay, Nonlinear Anal. Real World Appl. 31 (2016), 1-22.

[16] E. Gonzlez-Olivares, R. Ramos-Jiliberto, Dynamic consequences of prey refuges in a simple model system: more prey, fewer predators and enhanced stability, Ecological Model. 166 (2003), 135-146.

[17] T. K. Kar, Stability analysis of a prey-predator model incorporating a prey refuge, Commun. Nonlinear Sci. Numer. Simulation, 10 (2005), 681-691.

[18] F. D. Chen, L. J. Chen, X. D. Xie, On a Leslie-Gower predator-prey model incorporating a prey refuge, Nonlinear Anal. Real World Appl. 10 (2009), 2905-2908.

[19] L. J. Chen, F. D. Chen and L. J. Chen, Qualitative analysis of a predator-prey model with Holling type II functional response incorporating a constant prey refuge, Nonlinear Anal. Real World Appl. 11 (2010), 246-252.

[20] L. L. Ji, C. Q. Wu, Qualitative analysis of a predator-prey model with constant-rate prey harvesting incorporating a constant prey refuge, Nonlinear Anal. Real World Appl. 11 (2010), 2285-2295.

[21] Y. D. Tao, X. Wang, X. Y. Song, Effect of prey refuge on a harvested predator-prey model with generalized functional response, Commun. Nonlinear Sc. Numer. Simulation, 16 (2011), 1052-1059.

[22] Y. J. Huang, F. D. Chen, Z. Li, Stability analysis of a prey-predator model with Holling type III response function incorporating a prey refuge, Appl. Math. Comput. 182 (2006), 672-683.

[23] V. Krivan, On the Gause predator-prey model with a refuge: A fresh look at the history, J. Theoretical Biolo. 274 (2011), 67-73.

[24] Z. H. Ma, W. D. Li, S. F. Wang, The effect of prey refuge in a simple predator-prey model, Ecological Model. 222 (2011), 3453-3454.

[25] L. Chen, F. Chen, Y Wang, Influence of predator mutual interference and prey refuge on Lotka-Volterra predator-prey dynamics, Commun. Nonlinear Sci. Numer. Simulations, 18 (2013), 3174-3180.

[26] Z. Ma, et al., Dynamic behaviors of a Lotka-Volterra predator-prey model incorporating a prey refuge and predator mutual interference, Appl. Math. Comput. 219 (2013), 7945-7953.

[27] L. J. Chen, F. D. Chen, Dynamical analysis of a predator-prey model with square root functional response, J. Nonlinear Funct. Anal. 2015 (2015) Article ID 8.

[28] F. Chen, Z. Ma, H. Zhang, Global asymptotical stability of the positive equilibrium of the Lotka-Volterra prey-predator model incorporating a constant number of prey refuges, Nonlinear Anal. Real World Appl. 13 (2012), 2790-2793.

[29] F. D. Chen, Z. Li, Y. J. Huang, Note on the permanence of a competitive system with infinite delay and feedback controls, Nonlinear Anal. Real World Appl. 8 (2007), 680-687.

[30] L. J. Chen, F. D. Chen, Globaly analysis of a harvested predator-prey model incorporating a constant prey erfuge, Int. J. Biomath. 3 (2011), 205-223.

[31] L. J. Chen, F. D. Chen, Global Stability and bifurcation of a ratio-dependent predator-prey model with prey refuge, Acta Mathe. Sinica 57 (2014), 301-310.