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PERIODICITY IN PREDATOR-PREY SYSTEMS WITH MULTIPLE DELAYS AND FEEDBACK CONTROLS

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Abstract. Two classes of non-autonomous two species Lotka-Volterra predator-prey systems with multiple time delays and feedback controls are discussed. Some sufficient conditions on the existence of positive periodic solutions are established by using of the continuation theorem.

Keywords. Predator-prey system; Multiple time delays; Continuation theorem; Positive periodic solution; Feedback control.

2010 Mathematics Subject Classification. 34D20, 34D23, 34K20, 34K25.

1. Introduction

In this paper, we consider the following two classes of non-autonomous two species Lotka-Volterra predator-prey systems with multiple time delays and feedback controls

$$
\begin{aligned}
\dot{x}_1(t) = \;& x_1(t)\big[r_1(t) - a_{11}(t)x_1(t-\tau_{11}(t)) - a_{12}(t)\int_{-\tau_{12}}^{0} k_{12}(s)x_2(t+s)ds \\
& -b_1(t)u_1(t) - c_1(t)u_1(t-\sigma_1)\big], \\
\dot{x}_2(t) = \;& x_2(t)\big[-r_2(t) + a_{21}(t)\int_{-\tau_{21}}^{0} k_{21}(s)x_1(t+s)ds - a_{22}(t)x_2(t-\tau_{22}(t)) \\
& -b_2(t)u_2(t) - c_2(t)u_2(t-\sigma_2)\big], \\
\dot{u}_i(t) = \;& -\Lambda_i(t)u_i(t) + d_i(t)x_i(t) + e_i(t)x_i(t-\varepsilon_i), \quad i=1,2,
\end{aligned}
\tag{1.1}
$$

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and

$$
\begin{aligned}
\dot{x}_1(t) = &\ x_1(t)\big[r_1(t) - a_{11}(t)\int_{-\tau_{11}}^{0} k_{11}(s)x_1(t+s)ds - a_{12}(t)x_2(t-\tau_{12}(t))\\
&-b_1(t)u_1(t) - c_1(t)u_1(t-\sigma_1)\big],\\
\dot{x}_2(t) = &\ x_2(t)\big[-r_2(t) + a_{21}(t)x_1(t-\tau_{21}(t)) - a_{22}(t)\int_{-\tau_{22}}^{0} k_{22}(s)x_2(t+s)ds\\
&-b_2(t)u_2(t) - c_2(t)u_2(t-\sigma_2)\big],\\
\dot{u}_i(t) = &\ -\Lambda_i(t)u_i(t) + d_i(t)x_i(t) + e_i(t)x_i(t-\varepsilon_i), \quad i=1,2.
\end{aligned} \tag{1.2}
$$

Traditional two-species non-autonomous Lotka-Volterra predator-prey systems with multiple pure delays and without feedback controls are take the form

$$
\begin{aligned}
\dot{x}_1(t) = &\ x_1(t)\big[r_1(t) - a_{11}(t)x_1(t-\tau_{11}(t)) - a_{12}(t)\int_{-\tau_{12}}^{0} k_{12}(s)x_2(t+s)ds\big],\\
\dot{x}_2(t) = &\ x_2(t)\big[-r_2(t) + a_{21}(t)\int_{-\tau_{21}}^{0} k_{21}(s)x_1(t+s)ds - a_{22}(t)x_2(t-\tau_{22}(t))\big],
\end{aligned} \tag{1.3}
$$

and

$$
\begin{aligned}
\dot{x}_1(t) = &\ x_1(t)\big[r_1(t) - a_{11}(t)\int_{-\tau_{11}}^{0} k_{11}(s)x_1(t+s)ds - a_{12}(t)x_2(t-\tau_{12}(t))\big],\\
\dot{x}_2(t) = &\ x_2(t)\big[-r_2(t) + a_{21}(t)x_1(t-\tau_{21}(t)) - a_{22}(t)\int_{-\tau_{22}}^{0} k_{22}(s)x_2(t+s)ds\big].
\end{aligned} \tag{1.4}
$$

In the theory of mathematical biology and mathematical ecology, as we well know, systems like (1.1)-(1.4) are very important mathematical models which describe multiple-time two-species interactional population dynamics in a non-autonomous environment.

However, the environments of most natural populations undergo temporal variation and the variation of the environment plays an important role in many population dynamical systems. The effects of a periodically varying environment are important for evolutionary theory as the selective forces on systems in a fluctuating environment differ from those in a stable environment. Hence, the assumption of periodicity of the parameters in a way incorporates the periodicity of the environment [1]. For these reasons, it is realistic to assume that the parameters in the models are periodic functions of period ω. Therefore, to consider periodic environmental factor, it is reasonable to study Lotka-Volterra systems with periodic coefficients. In particularly, the Lotka-Volterra predator-prey system is the most important means to explain the ecological phenomenon and one of the most hot themes in both mathematical ecology and mathematical biology. There has been a great deal of literature on the study of the periodic solutions for various type continuous-time nonlinear population dynamical systems. Many important and good results were obtained recently, for example, see [1]-[24] and the references cited therein.

On the other hand, recently, the study of the existence of positive periodic solutions for the population dynamical systems with feedback control and time delays have been studied extensively [14]-[24] . However, about the non-autonomous periodic Lotka-Volterra predator-prey systems with multiple time delays and feedback controls have not been fully investigated. In the present paper, our aim is to establish some sufficient conditions on the the existence of positive periodic solutions of the system (1.1) and system (1.2).

2. Preliminaries

In system (1.1) and system (1.2), we have that $x_1(t)$ is the prey population density and $x_2(t)$ is the predator population density at time t, respectively. $b_i(t), c_i(t), \Lambda_i(t), d_i(t), e_i(t)$ $(i=1,2)$ represent the feedback control coefficients at time t, respectively. $\sigma_i > 0, \varepsilon_i > 0 (i=1,2)$ and $\tau_{i,j} \geq 0$ $(i,j=1,2)$, are constants where $\tau_{i,j} \geq 0$ $(i,j=1,2)$ may be $+\infty$. Throughout this paper, for system (1.1) and system (1.2) we introduce the following hypotheses.

(H_1) $\tau_{ij}(t)(i,j=1,2)$ and $r_i(t)(i=1,2)$ are continuous ω-periodic functions with $\tau'_{ij}(t) < 1$ and $\int_0^\omega r_i(t)dt > 0$. $a_{ij}(t)(i,j=1,2)$ are continuous positive ω-periodic functions. $k_{ij}(s)$ $(i,j=1,2)$ are nonnegative integrable functions on $[-\tau_{ij}, 0]$ $(i,j=1,2)$ satisfying $\int_{-\tau_{ij}}^0 k_{ij}(s)ds = 1$. σ_i and ε_i (i=1,2) are positive constants.

In this paper, for system (1.1) and system (1.2), we consider the solution with the following initial condition

$$\begin{aligned} x_i(t) &= \phi_i(t), \quad \text{for all} \quad t \in [-\tau, 0],\ i=1,2, \\ u_i(t) &= \psi_i(t), \quad \text{for all} \quad t \in [-\tau, 0],\ i=1,2, \end{aligned} \tag{2.1}$$

where $\phi_i(t)$ $(i=1,2)$ are nonnegative continuous functions defined on $[-\tau, 0)$ satisfying $\phi_i(0) > 0$ $(i=1,2)$ and $\tau = \max_{t\in[0,\omega]}\{\tau_{ij}(t), \tau_{ij}, \sigma_i, \varepsilon_i(i,j=1,2)\}$.

Throughout this paper, for any ω-periodic continuous function $f(t)$ we denote

$$f^L = \min_{t\in[0,\omega]} f(t), \quad f^M = \max_{t\in[0,\omega]} f(t), \quad \bar{f} = \frac{1}{\omega}\int_0^\omega f(t)dt. \tag{2.2}$$

The following lemmas paly an important role in this paper.

Lemma 2.1. [21] *Suppose* $\tau \in C^1(R,R)$ *with* $\tau(t+\omega) \equiv \tau(t)$ *and* $\tau'(t) < 1$,$\forall t \in [0,\omega]$. *Then the function* $t-\tau(t)$ *has a unique inverse function* $\mu(t)$ *satisfying* $\mu \in C(R,R)$, $\mu(u+\omega) = \mu(u)+\omega$,$\forall u \in R$.

Lemma 2.2. [25] *Let* L *be a Fredholm operator of index zero and let* N *be* L*-compact on* $\overline{\Omega}$. *If*

(a) *for each* $\lambda \in (0,1)$ *and* $x \in \partial\Omega \cap DomL$, $Lx \neq \lambda Nx$;

(b) *for each* $x \in \partial\Omega \cap KerL$, $QNx \neq 0$;

(c) $deg\{JQN, \Omega \cap KerL, 0\} \neq 0$,

then the operator equation $Lx = Nx$ *has at least one solution lying in* $DomL \cap \overline{\Omega}$.

3. Main results

In order to obtain the existence of positive periodic solutions of system (1.1) and system (1.2), firstly, we introduce the following Lemma.

Lemma 3.1. *Suppose that* $(x_1^*(t), x_2^*(t), u_1^*(t), u_2^*(t))$ *is an* ω*-periodic solution of (1.1) and (1.2) with initial conditions (2.1), then* $(x_1^*(t), x_2^*(t), u_1^*(t), u_2^*(t))$ *satisfies the following system (3.1) and system (3.2).*

$$\begin{aligned}
\dot{x}_1(t) = \quad & x_1(t)\big[r_1(t) - a_{11}(t)x_1(t-\tau_{11}(t)) - a_{12}(t)\int_{-\tau_{12}}^{0} k_{12}(s)x_2(t+s)ds \\
& -b_1(t)u_1(t) - c_1(t)u_1(t-\sigma_1)\big], \\
\dot{x}_2(t) = \quad & x_2(t)\big[-r_2(t) + a_{21}(t)\int_{-\tau_{21}}^{0} k_{21}(s)x_1(t+s)ds - a_{22}(t)x_2(t-\tau_{22}(t)) \\
& -b_2(t)u_2(t) - c_2(t)u_2(t-\sigma_2)\big],
\end{aligned} \tag{3.1}$$

and

$$\begin{aligned}
\dot{x}_1(t) = \quad & x_1(t)\big[r_1(t) - a_{11}(t)\int_{-\tau_{11}}^{0} k_{11}(s)x_1(t+s)ds - a_{12}(t)x_2(t-\tau_{12}(t)) \\
& -b_1(t)u_1(t) - c_1(t)u_1(t-\sigma_1)\big], \\
\dot{x}_2(t) = \quad & x_2(t)\big[-r_2(t) + a_{21}(t)x_1(t-\tau_{21}(t)) - a_{22}(t)\int_{-\tau_{22}}^{0} k_{22}(s)x_2(t+s)ds \\
& -b_2(t)u_2(t) - c_2(t)u_2(t-\sigma_2)\big],
\end{aligned} \tag{3.2}$$

where

$$u_i(t) = \int_t^{t+\omega} [d_i(\mu)x_i(\mu) + e_i(\mu)x_i(\mu-\varepsilon_i)]G_i(t,\mu)d\mu, \qquad i = 1,2,$$

and

$$G_i(t,\mu) = \frac{\exp\{\int_t^{\mu} \Lambda_i(\theta)d\theta\}}{\exp\{\int_0^{\omega} \Lambda_i(\theta)d\theta\} - 1}, \qquad i = 1,2.$$

The converse is also true.

Proof. Lemma 3.1 can be proved by using the similar method given by Yin and Li in the proof of Lemma 2 in Ref. 20, and hence here we omit it.

It is easy to see that system (3.1) and (3.2) are equivalent to the following system (3.3) and system (3.4)

$$\begin{aligned}
\dot{x}_1(t) = \; & x_1(t)\big[r_1(t) - a_{11}(t)x_1(t-\tau_{11}(t)) - a_{12}(t)\int_{-\tau_{12}}^{0} k_{12}(s)x_2(t+s)ds \\
& -b_1(t)u_1(t) - c_1(t)u_1(t-\sigma_1)\big], \\
\dot{x}_2(t) = \; & x_2(t)\big[-r_2(t) + a_{21}(t)\int_{-\tau_{21}}^{0} k_{21}(s)x_1(t+s)ds - a_{22}(t)x_2(t-\tau_{22}(t)) \\
& -b_2(t)u_2(t) - c_2(t)u_2(t-\sigma_2)\big],
\end{aligned} \tag{3.3}$$

and

$$\begin{aligned}
\dot{x}_1(t) = \; & x_1(t)\big[r_1(t) - a_{11}(t)\int_{-\tau_{11}}^{0} k_{11}(s)x_1(t+s)ds - a_{12}(t)x_2(t-\tau_{12}(t)) \\
& -b_1(t)u_1(t) - c_1(t)u_1(t-\sigma_1)\big], \\
\dot{x}_2(t) = \; & x_2(t)\big[-r_2(t) + a_{21}(t)x_1(t-\tau_{21}(t)) - a_{22}(t)\int_{-\tau_{22}}^{0} k_{22}(s)x_2(t+s)ds \\
& -b_2(t)u_2(t) - c_2(t)u_2(t-\sigma_2)\big],
\end{aligned} \tag{3.4}$$

where

$$u_i(t) = \int_t^{t+\omega} K(x_i)G_i(t,\mu)d\mu, \qquad G_i(t,\mu) = \frac{\exp\{\int_t^{\mu} \Lambda_i(\theta)d\theta\}}{\exp\{\int_0^{\omega} \Lambda_i(\theta)d\theta\} - 1},$$

and

$$K(x_i) = d_i(\mu)x_i(\mu) + e_i(\mu)x_i(\mu - \varepsilon_i), \quad i = 1,2,\cdots,n.$$

It is clear that in order to prove that system (1.1) and (1.2) with initial conditions (2.1) have at least one ω-periodic solution, we only need to prove that system (3.3) and system (3.4) have at least one ω-periodic solution.

Now, for convenience of statements, we denote $\bar{R}_i = \frac{1}{\omega}\int_0^{\omega} |r_i(t)|dt, \qquad i = 1,2.$

(H_2) $\quad \bar{r}_1 - \dfrac{\Gamma + \bar{H}_1}{a_{21}^L}\bar{r}_2 > 0,$

(H_3) $\bar{r}_1 - \frac{a_{11}^M + \bar{H}_1}{a_{21}^L}\bar{r}_2 > 0$, where

$$\Gamma = \max\{a_{11}^M, [\frac{a_{11}(\varphi_1(t))}{1-\tau_{11}'(\varphi_1(t))}]^M\},$$

and

$$\begin{aligned} H_1(t) = b_1(t)\int_t^{t+\omega}(d_1(\mu)+e_1(\mu))G_1(t,\mu)d\mu \\ + c_1(t)\int_{t-\sigma_1}^{t+\omega-\sigma_1}(d_1(\mu)+e_1(\mu))G_1(t-\sigma_1,\mu)d\mu. \end{aligned}$$

The following theorem is about the existence of positive periodic solutions of system (1.1).

Theorem 3.2. *Suppose that* (H_1) *and* (H_2) *hold, then system (1.1) has at least one positive* $\omega-$*periodic solution.*

Proof. Let

$$x_1(t) = \exp\{y_1(t)\} \quad \text{and} \quad x_2(t) = \exp\{y_2(t)\}.$$

From (3.3), we have the following system

$$\begin{aligned} \dot{y}_1(t) = {} & r_1(t) - a_{11}(t)\exp\{y_1(t-\tau_{11}(t))\} - a_{12}(t)\int_{-\tau_{12}}^0 k_{12}(s)\exp\{y_2(t+s)\}ds \\ & -b_1(t)U_1(t) - c_1(t)U_1(t-\sigma_1), \\ \dot{y}_2(t) = {} & -r_2(t) + a_{21}(t)\int_{-\tau_{21}}^0 k_{21}(s)\exp\{y_1(t+s)\}ds - a_{22}(t)\exp\{y_2(t-\tau_{22}(t))\} \\ & -b_2(t)U_2(t) - c_2(t)U_2(t-\sigma_1), \\ U_i(t) = {} & \int_t^{t+\omega} K(e^{y_i})G_i(t,\mu)d\mu, \qquad i=1,2, \end{aligned} \tag{3.5}$$

where

$$K(e^{y_i}) = d_i(\mu)\exp\{y_i(\mu)\} + e_i(\mu)\exp\{y_i(\mu-\varepsilon_i)\}.$$

Now, we introduce the normed vector spaces X and Z as follows. Let $C(R,R^2)$ denote the space of all continuous functions $y(t) = (y_1(t), y_2(t)) : R \to R^2$. We take

$$X = Z = \{y(t) \in C(R,R^2) : y(t) \text{ an } \omega\text{-periodic function}\},$$

with norm

$$\| y \| = \max_{t\in[0,\omega]} |y_1(t)| + \max_{t\in[0,\omega]} |y_2(t)|.$$

It is obvious that X and Z are the Banach spaces. We define a linear operator $L : \mathrm{Dom}\, L \subset X \to Z$ and a continuous operator $N : X \to Z$ by $Ly(t) = \dot{y}(t)$, and $Ny(t) = (Ny_1(t), Ny_2(t))$, where

$$\begin{aligned} Ny_1(t) = \;& r_1(t) - a_{11}(t)\exp\{y_1(t-\tau_{11}(t))\} - a_{12}(t)\int_{-\tau_{12}}^{0} k_{12}(s)\exp\{y_2(t+s)\}ds \\ & -b_1(t)U_1(t) - c_1(t)U_1(t-\sigma_1), \\ Ny_2(t) = \;& -r_2(t) + a_{21}(t)\int_{-\tau_{21}}^{0} k_{21}(s)\exp\{y_1(t+s)\}ds - a_{22}(t)\exp\{y_2(t-\tau_{22}(t))\} \\ & -b_2(t)U_2(t) - c_2(t)U_2(t-\sigma_1). \end{aligned} \tag{3.6}$$

Further, we define continuous projectors $P : X \to X$ and $Q : Z \to Z$ by

$$Py(t) = \frac{1}{\omega}\int_0^{\omega} y(t)dt, \quad Qv(t) = \frac{1}{\omega}\int_0^{\omega} v(t)dt.$$

We easily see $\mathrm{Im}\, L = \{v \in Z : \int_0^{\omega} v(t)dt = 0\}$ and $\mathrm{Ker}\, L = R^2$. It is obvious that $\mathrm{Im}\, L$ is closed in Z and $\dim\mathrm{Ker}\, L = 2$. Since for any $v \in Z$ there are unique $v_1 \in R^n$ and $v_2 \in \mathrm{Im}\, L$ with

$$v_1 = \frac{1}{\omega}\int_0^{\omega} v(t)dt, \quad v_2(t) = v(t) - v_1$$

such that $v(t) = v_1 + v_2(t)$, we have $\mathrm{codim}\mathrm{Im}\, L = 2$. Therefore, L is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L) $K_p : \mathrm{Im}\, L \to \mathrm{Ker}\, P \cap \mathrm{Dom}\, L$ is given in the following form

$$K_p v(t) = \int_0^t v(s)ds - \frac{1}{\omega}\int_0^{\omega}\int_0^t v(s)dsdt.$$

For convenience, we denote $F(t) = (F_1(t), F_2(t))$ as follows

$$\begin{aligned} F_1(t) = \;& r_1(t) - a_{11}(t)\exp\{y_1(t-\tau_{11}(t))\} - a_{12}(t)\int_{-\tau_{12}}^{0} k_{12}(s)\exp\{y_2(t+s)\}ds \\ & -b_1(t)U_1(t) - c_1(t)U_1(t-\sigma_1), \\ F_2(t) = \;& -r_2(t) + a_{21}(t)\int_{-\tau_{21}}^{0} k_{21}(s)\exp\{y_1(t+s)\}ds - a_{22}(t)\exp\{y_2(t-\tau_{22}(t))\} \\ & -b_2(t)U_2(t) - c_2(t)U_2(t-\sigma_1). \end{aligned} \tag{3.7}$$

Thus, we have

$$QNy(t) = \frac{1}{\omega}\int_0^{\omega} F(t)dt \tag{3.8}$$

and

$$
\begin{aligned}
K_p(I-Q)Ny(t) = & \ K_pINy(t) - K_pQNy(t) \\
= & \int_0^t F(s)ds - \frac{1}{\omega}\int_0^{\omega}\int_0^t F(s)dsdt \\
& +\Big(\frac{1}{2}-\frac{t}{\omega}\Big)\int_0^{\omega} F(s)ds.
\end{aligned} \tag{3.9}
$$

From (3.8) and (3.9), we can see that QN and $K_p(I-Q)N$ are continuous operators. Furthermore, it can be verified that $\overline{K_p(I-Q)N(\overline{\Omega})}$ is compact for any open bounded set $\Omega \subset X$ by using Arzela-Ascoli theorem and $QN(\overline{\Omega})$ is bounded. Therefore, N is L-compact on $\overline{\Omega}$ for any open bounded subset $\Omega \subset X$.

Now, we can search for an appropriate open bounded subset Ω for the application of the continuation theorem (Lemma 2.2) to system (3.5).

Corresponding to the operator equation $Ly(t) = \lambda Ny(t)$ with parameter $\lambda \in (0,1)$, we have

$$
\dot{y}_i(t) = \lambda F_i(t), \quad i = 1,2, \tag{3.10}
$$

where $F_i(t)$ $(i = 1,2)$ are given in Eqs.(3.7).

Assume that $y(t) = (y_1(t), y_2(t)) \in X$ is a solution of system (3.10) for some parameter $\lambda \in (0,1)$. By integrating system (3.10) over the interval $[0,\omega]$, we obtain

$$
\begin{aligned}
& \int_0^{\omega} \Big[r_1(t) - a_{11}(t)\exp\{y_1(t-\tau_{11}(t))\} - a_{12}(t)\int_{-\tau_{12}}^0 k_{12}(s)\exp\{y_2(t+s)\}ds \\
& -b_1(t)U_1(t) - c_1(t)U_1(t-\sigma_1)\Big]dt = 0, \\
& \int_0^{\omega} \Big[-r_2(t) + a_{21}(t)\int_{-\tau_{21}}^0 k_{21}(s)\exp\{y_1(t+s)\}ds - a_{22}(t)\exp\{y_2(t-\tau_{22}(t))\} \\
& -b_2(t)U_2(t) - c_2(t)U_2(t-\sigma_1)\Big]dt = 0.
\end{aligned} \tag{3.11}
$$

By (3.11), we have

$$
\begin{aligned}
& \int_0^{\omega} \Big[a_{11}(t)\exp\{y_1(t-\tau_{11}(t))\} + a_{12}(t)\int_{-\tau_{12}}^0 k_{12}(s)\exp\{y_2(t+s)\}ds \\
& +b_1(t)U_1(t) + c_1(t)U_1(t-\sigma_1)\Big]dt = \bar{r}_1\omega, \\
& \int_0^{\omega} \Big[a_{21}(t)\int_{-\tau_{21}}^0 k_{21}(s)\exp\{y_1(t+s)\}ds - a_{22}(t)\exp\{y_2(t-\tau_{22}(t))\} \\
& -b_2(t)U_2(t) - c_2(t)U_2(t-\sigma_1)\Big]dt = \bar{r}_2\omega.
\end{aligned} \tag{3.12}
$$

It follows from (3.10) and (3.12) that

$$
\begin{aligned}
&\textstyle\int_0^\omega |\dot{y}_1(t)|dt \\
&= \lambda \int_0^\omega \Big| r_1(t) - a_{11}(t)\exp\{y_1(t-\tau_{11}(t))\} - a_{12}(t)\int_{-\tau_{12}}^0 k_{12}(s)\exp\{y_2(t+s)\}ds \\
&-b_1(t)U_1(t) - c_1(t)U_1(t-\sigma_1)\Big| dt \\
&\le \int_0^\omega |r_1(t)|dt + \int_0^\omega \big[a_{11}(t)\exp\{y_1(t-\tau_{11}(t))\} + a_{12}(t)\int_{-\tau_{12}}^0 k_{12}(s)\exp\{y_2(t+s)\}ds \\
&+b_1(t)U_1(t) + c_1(t)U_1(t-\sigma_1)\big] dt \\
&\le (\bar{r_1} + \bar{R}_1)\omega,
\end{aligned}
$$

and

$$
\begin{aligned}
&\textstyle\int_0^\omega |\dot{y}_2(t)|dt \\
&= \lambda \int_0^\omega \Big| -r_2(t) + a_{21}(t)\int_{-\tau_{21}}^0 k_{21}(s)\exp\{y_1(t+s)\}ds - a_{22}(t)\exp\{y_2(t-\tau_{22}(t))\} \\
&-b_2(t)U_2(t) - c_2(t)U_2(t-\sigma_2)\Big| dt \\
&\le \int_0^\omega |r_2(t)|dt + \int_0^\omega \big[a_{21}(t)\int_{-\tau_{21}}^0 k_{21}(s)\exp\{y_1(t+s)\}ds - a_{22}(t)\exp\{y_2(t-\tau_{22}(t))\} \\
&-b_2(t)U_2(t) - c_2(t)U_2(t-\sigma_2)\big] dt \\
&\le (\bar{r_2} + \bar{R}_2)\omega,
\end{aligned}
$$

that is,

$$
\int_0^\omega |\dot{y}_i(t)|dt \le (\bar{r}_i + \bar{R}_i)\omega := C_i, \qquad i = 1,2. \tag{3.13}
$$

For each $i, j = 1,2$, we have

$$
\begin{aligned}
&\int_0^\omega a_{ij}(t)\int_{-\tau_{ij}}^0 k_{ij}(s)\exp\{y_i(t+s)\}dsdt \\
=\;& \int_{-\tau_{ij}}^0 \int_0^\omega a_{ij}(t)k_{ij}(s)\exp\{y_i(t+s)\}dtds \\
=\;& \int_{-\tau_{ij}}^0 \int_s^{s+\omega} a_{ij}(v-s)k_{ij}(s)\exp\{y_i(v)\}dvds \\
=\;& \int_{-\tau_{ij}}^0 \int_0^\omega a_{ij}(v-s)k_{ij}(s)\exp\{y_i(v)\}dvds \\
=\;& \int_0^\omega \int_{-\tau_{ij}}^0 a_{ij}(v-s)k_{ij}(s)\exp\{y_i(v)\}dsdv \\
=\;& \int_0^\omega \Big(\int_{-\tau_{ij}}^0 a_{ij}(t-s)k_{ij}(s)ds\Big)\exp\{y_i(t)\}dt.
\end{aligned} \tag{3.14}
$$

Let $z_i(t) = t - \tau_{ii}(t)$ $(i = 1,2)$. From Lemma 2.1 and H_1, we get that function $z_i(t)$ has a unique ω periodic inverse function $\varphi_i(t)$. For every $i = 1,2$, we have

$$\int_0^{\omega} a_{ii}(t)\exp\{y_i(t-\tau_{ii}(t))\}dt = \int_{-\tau_{ii}(0)}^{\omega-\tau_{ii}(\omega)} \frac{a_{ii}(\varphi_i(t))}{1-\tau_{ii}'(\varphi_i(t))}\exp\{y_i(t)\}dt.$$

It is clear that

$$\frac{a_{ii}(\varphi_i(t))}{1-\tau_{ii}'(\varphi_i(t))} =: \Gamma_i(t), \quad i = 1,2,$$

are ω periodic functions. Then for every $i = 1,2$, we have

$$\int_0^{\omega} a_{ii}(t)\exp\{y_i(t-\tau_{ii}(t))\}dt = \int_0^{\omega} \Gamma_i(t)\exp\{y_i(t)\}dt. \tag{3.15}$$

From the continuity of $y(t) = (y_1(t), y_2(t))$, there exist constants $\xi_i, \eta_i \in [0,\omega]$ $(i = 1,2)$ such that

$$y_i(\xi_i) = \max_{t\in[0,\omega]} y_i(t), \quad y_i(\eta_i) = \min_{t\in[0,\omega]} y_i(t), \quad i = 1,2. \tag{3.16}$$

From (3.12) and (3.14-3.16), we further obtain

$$y_1(\eta_1) \le \ln\big(\frac{\bar{r}_1}{\bar{\Gamma}_1}\big) =: B_1,\ y_2(\eta_2) \le \ln\big(\frac{\bar{r}_1}{\bar{A}_1}\big) =: B_2,\ y_1(\xi_1) \ge \ln\big(\frac{\bar{r}_2}{\bar{A}_2}\big) =: B_3, \tag{3.17}$$

where

$$A_1(t) = \int_0^{\omega}\int_{-\tau_{12}}^{0} k_{12}(s)a_{12}(t-s)dsdt, \quad A_2(t) = \int_0^{\omega}\int_{-\tau_{21}}^{0} k_{21}(s)a_{21}(t-s)dsdt.$$

On the other hand, from the second equation of (3.12) and (3.14-3.16), we have

$$a_{21}^L\int_0^{\omega}\exp\{y_1(t)\}dt \le \bar{r}_2\omega + (\bar{a}_{22}+\bar{H}_2)\omega\exp\{y_2(\xi_2)\}, \tag{3.18}$$

where

$$H_i(t) = b_i(t)\int_t^{t+\omega}(d_i(\mu)+e_i(\mu))G_i(t,\mu)d\mu + c_i(t)\int_{t-\sigma_i}^{t+\omega-\sigma_i}(d_i(\mu)+e_i(\mu))G_i(t-\sigma_i,\mu)d\mu,$$

$i = 1,2$. Further from the first equation of (3.12) and (3.14-3.16,3.18), we can obtain

$$\bar{r}_1\omega \le (\bar{A}_1+\bar{H}_1)\omega\exp\{y_2(\xi_2)\} + \frac{\Gamma}{a_{21}^L}\bar{r}_2\omega + \frac{\Gamma}{a_{21}^L}(\bar{a}_{22}+\bar{H}_2)\omega\exp\{y_2(\xi_2)\},$$

where

$$\Gamma = \max\{a_{11}^M, \Gamma_1^M\}.$$

In view of the condition of Theorem 3.2, we have

$$0 < (\bar{r}_1 - \frac{\Gamma+\bar{H}_1}{a_{21}^L}\bar{r}_2)\Delta \le (\bar{r}_1 - \frac{\Gamma}{a_{21}^L}\bar{r}_2)\Delta \le \exp\{y_2(\xi_2)\}, \tag{3.19}$$

where

$$\Delta = \frac{1}{\bar{A}_1 + \bar{H}_1 + \frac{\Gamma}{a_{21}^L}(\bar{a}_{22} + \bar{H}_2)}.$$

From (3.19), we have

$$y_2(\xi_2) \geq \ln A_3 =: B_4, \tag{3.20}$$

where

$$A_3 = a_{21}^L(\bar{r}_1 - \frac{\Gamma + \bar{H}_1}{a_{21}^L}\bar{r}_2)\Delta.$$

From (3.13), (3.17) and (3.20), we have

$$y_i(t) \leq y_i(\eta_i) + \int_0^\omega |\dot{y}_i(t)|dt \leq B_i + C_i =: M_i \quad i = 1,2, \tag{3.21}$$

and

$$y_i(t) \geq y_i(\xi_i) - \int_0^\omega |\dot{y}_i(t)|dt \geq B_{i+2} - C_i =: N_i \quad i = 1,2. \tag{3.22}$$

Therefore, from (3.21), (3.22), we have

$$\max_{t\in[0,\ \omega]} |y_i(t)| \leq \max\left\{|M_i|, |N_i|\right\} =: H_i, \qquad i = 1,2.$$

It is easy to see that the constants $H_i\,(i = 1,2)$ are independent of parameter $\lambda \in (0,1)$. For any $y = (y_1, y_2) \in R^2$, from (3.6), we obtain $QNu = (QNy_1, QNy_2)$, where

$$\begin{aligned} QNu_1 = &\quad \bar{r}_1 - (\bar{a}_{11} + \bar{H}_1)\exp\{y_1\} - \bar{a}_{12}\exp\{y_2\}, \\ QNu_2 = &\quad -\bar{r}_2 + \bar{a}_{21}\exp\{y_1\} - (\bar{a}_{22} + \bar{H}_2)\exp\{y_2\}. \end{aligned}$$

We consider the following system of algebraic equations

$$\begin{aligned} &\bar{r}_1 - (\bar{a}_{11} + \bar{H}_1)\upsilon_1 - \bar{a}_{12}\upsilon_2 = 0, \\ &-\bar{r}_2 + \bar{a}_{21}\upsilon_1 - (\bar{a}_{22} + \bar{H}_2)\upsilon_2 = 0. \end{aligned}$$

By direct calculation, we have

$$\upsilon_1 = \frac{(\bar{a}_{22} + \bar{H}_2)\bar{r}_1 + \bar{a}_{12}\bar{r}_2}{(\bar{a}_{11} + \bar{H}_1)(\bar{a}_{22} + \bar{H}_2) + \bar{a}_{12}\bar{a}_{21}},$$

and

$$\upsilon_2 = \frac{\bar{a}_{21}\bar{r}_1 - \bar{r}_2(\bar{a}_{11} + \bar{H}_1)}{(\bar{a}_{11} + \bar{H}_1)(\bar{a}_{22} + \bar{H}_2) + \bar{a}_{12}\bar{a}_{21}} > \frac{a_{21}^L(\bar{r}_1 - \frac{\Gamma + \bar{H}_1}{a_{21}^L}\bar{r}_2)}{(\bar{a}_{11} + \bar{H}_1)(\bar{a}_{22} + \bar{H}_2) + \bar{a}_{12}\bar{a}_{21}}.$$

From the assumption of Theorem 3.2, the system of algebraic equations has a unique positive solution $\upsilon = (\upsilon_1, \upsilon_2)$. Hence, the equation $QNy = 0$ has a unique solution $y^* = (y_1^*, y_2^*) = (\ln\upsilon_1, \ln\upsilon_2) \in R^2$.

Choosing constant $H > 0$ large enough such that $|y_1^*| + |y_2^*| < H$ and $H > H_1 + H_2$, we define a bounded open set $\Omega \subset X$ as follows

$$\Omega = \{y \in X : \|y\| < H\}.$$

It is clear that Ω satisfies conditions (a) and (b) of Lemma 2.2. On the other hand, by directly calculating we can obtain

$$\deg\{JQN, \Omega \cap \operatorname{Ker} L, (0,0)\}$$

$$= \operatorname{sgn} \begin{vmatrix} -(\bar{a}_{11} + \bar{H}_1)K_1 & -\bar{a}_{12}K_2 \\ \bar{a}_{21}K_1 & -(\bar{a}_{22} + \bar{H}_2)K_2 \end{vmatrix},$$

where $K_i = \exp\{y_i\}(i = 1,2)$. Since

$$\begin{vmatrix} -(\bar{a}_{11} + \bar{H}_1)K_1 & -\bar{a}_{12}K_2 \\ \bar{a}_{21}K_1 & -(\bar{a}_{22} + \bar{H}_2)K_2 \end{vmatrix} \neq 0,$$

we have $\deg\{JQN, \Omega \cap \operatorname{Ker} L, (0,0)\} \neq 0$. This shows that Ω satisfies condition (c) of Lemma 2.2. Therefore, system (3.5) has a ω-periodic solution $y^*(t) = (y_1^*(t), y_2^*(t)) \in \bar{\Omega}$. Hence, system (1.1) has a positive ω-periodic solution $x^*(t) = (x_1^*(t), x_2^*(t))$.

From the proof of Theorem 3.2, on the existence of positive periodic solutions of system (1.2), we have the following result.

Theorem 3.3. *Suppose that* $(\mathbf{H_1})$ *and* (H_3) *hold. Then system (1.2) has at least one positive* $\omega-$ *periodic solution.*

Proof. Theorem 3.3 can be proved by using the similar method with Theorem 3.2, and hence here we omit it.

Remark 3.4. From Theorem 3.2 and Theorem 3.3, we find that as long as system (1.1) has a positive periodic solution. Then system (1.2) has a positive periodic solution.

4. Applications

In this section, to show the generality of our results, we apply Theorem 3.2 and Theorem 3.3 to some special cases of system (1.1) and system (1.2). Firstly, we consider the following predator-prey systems with pure delays.

$$
\begin{aligned}
\dot{x}_1(t) = & \; x_1(t)\big[r_1(t) - a_{11}(t)x_1(t-\tau_{11}(t)) - a_{12}(t)x_2(t-\tau_{12}(t)) \\
& -b_1(t)u_1(t) - c_1(t)u_1(t-\sigma_1)\big], \\
\dot{x}_2(t) = & \; x_2(t)\big[-r_2(t) + a_{21}(t)x_1(t-\tau_{21}(t)) - a_{22}(t)x_2(t-\tau_{22}(t)) \\
& -b_2(t)u_2(t) - c_2(t)u_2(t-\sigma_2)\big], \\
\dot{u}_i(t) = & \; -\Lambda_i(t)u_i(t) + d_i(t)x_i(t) + e_i(t)x_i(t-\varepsilon_i), \quad i=1,2,
\end{aligned}
\tag{4.1}
$$

and

$$
\begin{aligned}
\dot{x}_1(t) = & \; x_1(t)\big[r_1(t) - a_{11}(t)\int_{-\tau_{11}}^{0} k_{11}(s)x_1(t+s)ds - a_{12}(t)\int_{-\tau_{12}}^{0} k_{12}(s)x_2(t+s)ds \\
& -b_1(t)u_1(t) - c_1(t)u_1(t-\sigma_1)\big], \\
\dot{x}_2(t) = & \; x_2(t)\big[-r_2(t) + a_{21}(t)\int_{-\tau_{21}}^{0} k_{21}(s)x_1(t+s)ds - a_{22}(t)\int_{-\tau_{22}}^{0} k_{22}(s)x_2(t+s)ds \\
& -b_2(t)u_2(t) - c_2(t)u_2(t-\sigma_2)\big], \\
\dot{u}_i(t) = & \; -\Lambda_i(t)u_i(t) + d_i(t)x_i(t) + e_i(t)x_i(t-\varepsilon_i), \quad i=1,2.
\end{aligned}
\tag{4.2}
$$

As a direct consequence of Theorems 3.2 and Theorems 3.3, we have the following result.

Corollary 4.1. *Suppose that the assumptions of Theorem 3.2 hold, then system (4.1) has at least one positive $\omega-$ periodic solution.*

Corollary 4.2. *Suppose that the assumptions of Theorem 3.3 hold, then system (4.2) has at least one positive $\omega-$ periodic solution.*

Next, we consider the following predator-prey systems with predator density-independence and pure delays

$$
\begin{aligned}
\dot{x}_1(t) = & \; x_1(t)\big[r_1(t) - a_{11}(t)x_1(t-\tau_{11}(t)) - a_{12}(t)\int_{-\tau_{12}}^{0} k_{12}(s)x_2(t+s)ds \\
& -b_1(t)u_1(t) - c_1(t)u_1(t-\sigma_1)\big] \\
\dot{x}_2(t) = & \; x_2(t)\big[-r_2(t) + a_{21}(t)\int_{-\tau_{21}}^{0} k_{21}(s)x_1(t+s)ds \\
& -b_2(t)u_2(t) - c_2(t)u_2(t-\sigma_2)\big] \\
\dot{u}_i(t) = & \; -\Lambda_i(t)u_i(t) + d_i(t)x_i(t) + e_i(t)x_i(t-\varepsilon_i), \quad i=1,2,
\end{aligned}
\tag{4.3}
$$

and

$$
\begin{aligned}
\dot{x}_1(t) = \;& x_1(t)\big[r_1(t) - a_{11}(t)\int_{-\tau_{11}}^{0} k_{11}(s)x_1(t+s)ds - a_{12}(t)x_2(t-\tau_{12}(t)) \\
& -b_1(t)u_1(t) - c_1(t)u_1(t-\sigma_1)\big] \\
\dot{x}_2(t) = \;& x_2(t)\big[-r_2(t) + a_{21}(t)x_1(t-\tau_{21}(t)) \\
& -b_2(t)u_2(t) - c_2(t)u_2(t-\sigma_2)\big] \\
\dot{u}_i(t) = \;& -\Lambda_i(t)u_i(t) + d_i(t)x_i(t) + e_i(t)x_i(t-\varepsilon_i), \quad i = 1,2.
\end{aligned}
\tag{4.4}
$$

From Theorems 3.2 and Theorems 3.3, on the existence of positive periodic solutions of system (4.3) and (4.4), we have the following result.

Corollary 4.3. *Suppose that the assumptions of Theorem 3.2 hold. Then system (4.3) has at least one positive $\omega-$ periodic solution.*

Corollary 4.4. *Suppose that the assumptions of Theorem 3.3 hold. Then system (4.4) has at least one positive $\omega-$ periodic solution.*

Remark 4.5. From the Theorem 3.2, Theorem 3.3 and Corollaries 4.1-4.4, we find that as long as system (1.1) has a positive periodic solution. Then system (1.2) and systems (4.1-4.4), have a positive periodic solution.

Finally, we consider the following predator-prey system with pure delays.

$$
\begin{aligned}
\dot{x}_1(t) = \;& x_1(t)\big[r_1(t) - a_{11}(t)x_1(t-\tau_{11}(t)) - a_{12}(t)\int_{-\tau_{12}}^{0} k_{12}(s)x_2(t+s)ds \\
& -b_1(t)u_1(t) - c_1(t)u_1(t-\sigma_1)\big] \\
\dot{x}_2(t) = \;& x_2(t)\big[r_2(t) + a_{21}(t)\int_{-\tau_{21}}^{0} k_{21}(s)x_1(t+s)ds - a_{22}(t)x_2(t-\tau_{22}(t)) \\
& -b_2(t)u_2(t) - c_2(t)u_2(t-\sigma_2)\big] \\
\dot{u}_i(t) = \;& -\Lambda_i(t)u_i(t) + d_i(t)x_i(t) + e_i(t)x_i(t-\varepsilon_i), \quad i = 1,2,
\end{aligned}
\tag{4.5}
$$

and

$$
\begin{aligned}
\dot{x}_1(t) = & \ x_1(t)\big[r_1(t) - a_{11}(t)\int_{-\tau_{11}}^{0} k_{11}(s)x_1(t+s)ds - a_{12}(t)x_2(t-\tau_{12}(t)) \\
& -b_1(t)u_1(t) - c_1(t)u_1(t-\sigma_1)\big] \\
\dot{x}_2(t) = & \ x_2(t)\big[r_2(t) + a_{21}(t)x_1(t-\tau_{21}(t)) - a_{22}(t)\int_{-\tau_{22}}^{0} k_{22}(s)x_2(t+s)ds \\
& -b_2(t)u_2(t) - c_2(t)u_2(t-\sigma_2)\big] \\
\dot{u}_i(t) = & \ -\Lambda_i(t)u_i(t) + d_i(t)x_i(t) + e_i(t)x_i(t-\varepsilon_i), \quad i = 1,2.
\end{aligned} \tag{4.6}
$$

Now, for convenience of statements we denote

(H_4) $\quad \bar{r}_1 - \dfrac{a_{12}^M}{\Gamma_0 + \bar{H}_2}\bar{r}_2 > 0,$

(H_5) $\quad \bar{r}_1 - \dfrac{a_{21}^M}{a_{22}^L + \bar{H}_2}\bar{r}_2 > 0,$

where

$$\Gamma_0 = \min\{a_{22}^L, [\frac{a_{22}(\varphi_1(t))}{1-\tau_{22}'(\varphi_1(t))}]^L\},$$

and

$$H_2(t) = b_2(t)\int_t^{t+\omega}(d_2(\mu)+e_2(\mu))G_2(t,\mu)d\mu + c_2(t)\int_{t-\sigma_2}^{t+\omega-\sigma_2}(d_2(\mu)+e_2(\mu))G_2(t-\sigma_2,\mu)d\mu.$$

From the proof of Theorem 3.2, on the existence of positive periodic solutions of system (4.5) and (4.6), we have the following result.

Theorem 4.6. *Suppose that (H_1) and (H_4) hold. Then system (4.5) has at least one positive $\omega-$ periodic solution.*

Theorem 4.7. *Suppose that (H_1) and (H_5) hold. Then system (4.6) has at least one positive $\omega-$ periodic solution.*

Theorem 4.6 and Theorem 4.7 can be proved by using the similar method with Theorem 3.2. Hence, we here omit it.

Remark 4.8. From the Theorem 4.6 and Theorem 4.7, we find that as long as system (4.5) has a positive periodic solution. Then system (4.6) has a positive periodic solution.

5. Conclusions

In this paper, two classes of general non-autonomous two species Lotka-Volterra predator-prey systems with multiple time delays and feedback controls are proposed and analyzed to study the effect of time delays and feedback controls on the existence of positive periodic solutions of the system. By means of the continuation theorem, we easily obtained the sufficient conditions for the existence of positive periodic solutions of the system. From the conditions of Theorems 3.2, 3.3, 4.6, 4.7 and Corollaries 4.1, 4.2, 4.3, 4.4, we can see that the time delays and feedback controls have effect on the existence of positive periodic solutions, and conditions $(\mathbf{H_2})$-$(\mathbf{H_5})$ are very crucial to find the criteria for the existence of positive periodic solutions. Further, from the conditions $(\mathbf{H_2})$-$(\mathbf{H_5})$, we see that the intrinsic growth rate of the prey is more significant than the intrinsic growth rate of the predator and the intrinsic growth rate of the prey will determine the existence of positive periodic solutions of the system. The results indicate that if the intrinsic growth rate of the prey is large enough, then it can maintain the existence of positive periodic solutions of the system.

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