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GLOBAL DYNAMICS OF A SYRPHID FLY-APHID MODEL WITH STAGE STRUCTURE FOR PREDATOR POPULATION

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Abstract. A syrphid fly-aphid model with stage structure for predator population is considered, in which only the immature predator-natural enemy syrphid fly can prey on pest-aphid. By applying Lyapunov functions and LaSalles invariance principle, we show that the global asymptotic stability of the interior equilibrium for the considered model. We also obtain results on the existence and stability of periodic solutions. Numerical simulations are also performed to illustrate the validity of our results.

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Keywords. Stage structure; Global stability; Periodic solution; Lyapunov function; LaSalles invariance principle.

1. Introduction

Since W. G. Aiello and H. I. Freedman [1] built and studied a time delay model of single species growth with stage structure, model with stage structure consisting of immature and mature stages has been paid many attentions by numerous scholars due to it embodies the specific stages in different habits of its whole life history of biological population (particularly mammalian population) in the natural world: see[2-10] and references therein. The authors in [2-4] studied the global properties and the existence and stability of periodic solutions of a predator-prey model with nonlinear functional response and stage structure for the predator. W. Wang, *et al.* [5] and R. Xu, *et al.* [6] showed the combined effects of stage structure for predator and time delay on the global dynamics of predator-prey model. The literatures [7-10] proposed and investigated the dynamics of predator-prey models with harvesting and stage structure. Although

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these established models with stage structure is different, but all assume that only mature predator is capable of preying on prey, which is inconsistent with observed fact. F.Y. Wei, *et al.* [11] focus on the stabilities of the equilibria to a predatorCprey model with stage structure incorporating prey refuge. Although these established models with stage structure is different, but all assume that only mature predator is capable of preying on prey, which is inconsistent with observed fact. The authors in [12] pointed out, in the real world, many predator population, especially natural enemy insect larvae are capable of preying on preys, and the predation ability of some larvae is even more than adults, such as the ladybug larvae. Some species such as syrphid flies just larvae prey on aphids and the adults don't hunt. Recently, L. H. Huo, *et al.* [13] proposed the following syrphid fly-aphid model with stage structure for predator population,

$$\begin{aligned} \dot{x} &= rx\Big(1-\frac{x}{K}\Big)-\frac{axy_1}{1+bx}, \\ \dot{y}_1 &= \frac{maxy_1}{1+bx}+ey_2-d_1y_1-dy_1, \\ \dot{y}_2 &= kdy_1-d_2y_2, \end{aligned} \tag{1.1}$$

where $x(t)$, $y_1(t)$ and $y_2(t)$ denotes the density of prey population-pest aphid, immature and mature predator population-natural enemy syrphid fly at time t, respectively; r,K,a,b,e,d,d_1,d_2, k, m are all positive constants. Here only the immature predators can hunt preys and the term $\frac{ax}{1+bx}$ denotes the Holling type II functional response of the immature predators. r represents the intrinsic growth rate and K the carrying capacity of the prey; m is the conversion factor denoting the number of newly born immature predators for each captured prey; d_1,d_2 are the death rate of the immature and mature predators, respectively. d denotes the transformation rate at which immature predators become mature predators and k is the success rates of transformation; e is the reproduction rate of mature predators.

Model (1.1) admits two boundary equilibria $E_1(0,0,0)$ and $E_2(K,0,0)$ and an interior equilibrium point $E^*(x^*,y_1^*,y_2^*)$, where

$$x^*=\frac{A}{ma-bA},\ y_1^*=\frac{r}{a}(1+bx^*)(1-\frac{x^*}{K}),\ y_2^*=\frac{kd}{d_2}y_1^*,\ A=d_1+d-\frac{ekd}{d_2}.$$

The authors in [13] derived the following conclusions using the qualitative theory of ordinary differential equations and uniform persistence theory.

Theorem A. *$E_1(0,0,0)$ is always unstable; $E_2(K,0,0)$ is unstable for $0<A<\frac{maK}{1+bK}$ and locally asymptotically stable for $A>\frac{maK}{1+bK}$; $E^*(x^*,y_1^*,y_2^*)$ is locally asymptotically stable for $\max\left\{\frac{ma(bK-1)}{b(1+bK)},0\right\}<A<\frac{maK}{1+bK}$.*

Theorem B. *Model (1.1) is permanent for $0<A<\frac{maK}{1+bK}$.*

Above research results provide a theoretical basis for integrated pest management, but the existence of periodic solutions and the global stability properties of the interior equilibrium are not addressed in [13]. The main purpose of biological control is to control quantity of pest population by utilizing predation of natural enemy on pest and to minimize the use of pesticides, therefore, the extinction of natural enemy-predator population is important in the context of pest management. In this paper, the aim is to derive sufficient conditions of coexistence of both predator and prey population and also extinction of predator population.

The rest of the paper is organized as follows. Analysis on global stability property of the interior equilibrium and the existence and stability of periodic solutions for the considered model are given in Section 2. Some numeric simulations which illustrate the feasibility of our finding are given in Section 3. This paper ends by a brief conclusion.

2. Dynamical analysis of model (1.1)

2.1. Global stability for E_2 and E^*

Theorem 2.1. *$E_2(K,0,0)$ is globally asymptotically stable for $A > \frac{maK}{1+bK}$.*

Proof. From $A > \frac{maK}{1+bK}$, that is, $d_1 + d - \frac{ekd}{d_2} > \frac{maK}{1+bK}$, we can derive that

$$\frac{(d+d_1)(1+Kb)-maK}{madkK} > \frac{e(1+Kb)}{mad_2K}$$

and we can choose a constant u such that

$$\frac{(d+d_1)(1+Kb)-maK}{madkK} > u > \frac{e(1+Kb)}{mad_2K}. \tag{2.1}$$

Let us consider the following Lyapunov function

$$V = \frac{1}{a}\left(\frac{x}{K} - 1 - \ln\frac{x}{K}\right) + \frac{1+bK}{maK}y_1 + uy_2.$$

It follows from (2.1) that

$$\dot{V} = -\frac{r}{a}\left(1-\frac{x}{K}\right)^2 - \Big(\frac{(d+d_1)(1+Kb)-maK}{maK} - udk\Big)y_1 - \left(ud_2 - \frac{e(1+Kb)}{maK}\right)y_2 \le 0.$$

It is easy to see that $\dot{V} = 0$ holds if and only if $x = K, y_1 = 0, y_2 = 0$, from Lasalle's invariance principle, we derive that E_2 is globally asymptotically stable. This completes the proof.

It follows from Theorem B that there exists an $\underline{x} > 0$ such that $\liminf_{t\to\infty} x(t) \ge \underline{x}$. By constructing a Lyapunov function (similarly to that of [4]), we obtain the following global stability property of the interior equilibrium.

Theorem 2.2. *Assume that* $\underline{x} > \frac{K}{2}$. *Then* $E^*(x^*, y_1^*, y_2^*)$ *is globally asymptotically stable for* $0 < A < \frac{maK}{1+bK}$.

Proof. It follows from $\underline{x} > \frac{K}{2}$ that there exists a $T_0 > 0$ such that $x(t) > \frac{K}{2}$ for all $t \geq T_0$ and then $x^* > \frac{K}{2}$. From the proof of Theorem 3.3 in [13], we derive that $E^*(x^*, y_1^*, y_2^*)$ is locally asymptotically stable under the conditions of Theorem 2.2. Let

$$f(x) = rx(1 - \frac{x}{K}),\ g(x) = \frac{ax}{1+bx}.$$

Considering the Lyapunov function

$$V = \int_{x^*}^{x} \frac{g(s) - g(x^*)}{g(s)} \mathrm{d}s + \frac{1}{m} \int_{y_1^*}^{y_1} \frac{s - y_1^*}{s} \mathrm{d}s + \frac{e}{md_2} \int_{y_2^*}^{y_2} \frac{s - y_2^*}{s} \mathrm{d}s.$$

Then, we have

$$\begin{aligned}
\dot{V} &= \frac{g(x) - g(x^*)}{g(x)} \dot{x} + \frac{y_1 - y_1^*}{my_1} \dot{y}_1 + \frac{e(y_2 - y_2^*)}{md_2 y_2} \dot{y}_2 \\
&= \frac{g(x) - g(x^*)}{g(x)} (f(x) - g(x)y_1) + \frac{y_1 - y_1^*}{my_1} (mg(x) - (d_1 + d)y_1 + ey_2) \\
&\quad + \frac{e(y_2 - y_2^*)}{md_2 y_2} (kdy_1 - d_2 y_2) \\
&= \frac{f(x)}{g(x)} (g(x) - g(x^*)) + g(x^*)y_1 - g(x)y_1^* - \frac{d_1 + d}{m} y_1 + \frac{d_1 + d}{m} y_1^* - \frac{ey_1^* y_2}{my_1} \\
&\quad + \frac{ekd}{md_2} y_1 - \frac{ekdy_2^* y_1}{md_2 y_2} + \frac{e}{m} y_2^*.
\end{aligned}$$

In view of

$$y_1^* = \frac{f(x^*)}{g(x^*)},\ kdy_1^* = d_2 y_2^*,\ g(x^*) = \frac{ax^*}{1+bx^*} = \frac{d_1 + d}{m} - \frac{ekd}{md_2},$$

it is easy to compute that

$$g(x^*)y_1 - \frac{d_1 + d}{m} y_1 + \frac{ekd}{md_2} y_1 = 0$$

and

$$-g(x)y_1^* + \frac{d_1 + d}{m} y_1^* = -g(x)y_1^* + g(x^*)y_1^* + \frac{ekd}{md_2} y_1^* = -y_1^*(g(x) - g(x^*)) + \frac{e}{m} y_2^*.$$

It follows that

$$\begin{aligned}
\dot{V} &= \frac{f(x)}{g(x)} (g(x) - g(x^*)) - y_1^*(g(x) - g(x^*)) - \frac{ey_1^* y_2}{my_1} - \frac{ekdy_2^* y_1}{md_2 y_2} + \frac{2e}{m} y_2^* \\
&= \frac{(f(x) - f(x^*))(g(x) - g(x^*))}{g(x)} - \frac{f(x^*)}{g(x)g(x^*)} (g(x) - g(x^*))^2 \\
&\quad - \frac{ey_2^*}{m} (\frac{y_1^* y_2}{y_2^* y_1} + \frac{kdy_1}{d_2 y_2} - 2).
\end{aligned}$$

If $x(t) > \frac{K}{2}$ for all $t \geq T_0$, since $f(x)$ is strictly decreasing on $[\frac{K}{2}, \infty)$ and $g(x)$ is strictly increasing on $[0, \infty)$, it follows that

$$\frac{(f(x) - f(x^*))(g(x) - g(x^*))}{g(x)} \leq 0, \tag{2.2}$$

where the equality holds if and only if $x = x^*$. It follows from basic inequality that

$$\frac{y_1^* y_2}{y_2^* y_1} + \frac{kdy_1}{d_2 y_2} = \frac{d_2 y_2}{kdy_1} + \frac{kdy_1}{d_2 y_2} \geq 2, \tag{2.3}$$

where the equality holds if and only if

$$\frac{d_2 y_2}{kdy_1} = \frac{kdy_1}{d_2 y_2} = 1,$$

that is, $kdy_1 = d_2 y_2$. Note that $kdy_1^* = d_2 y_2^*$, which implies equality of (2.3) holds if and only if $y_1 = y_1^*,\ y_2 = y_2^*$. From (2.2) and (2.3), we derive that $\dot{V} \leq 0$, and $\dot{V} = 0$ holds if and only if $x = x^*,\ y_1 = y_1^*,\ y_2 = y_2^*$. Then, from Lasalle's invariance principle, we derive that E^* is globally asymptotically stable. This completes the proof.

2.2. Existence of Periodic Solution

We first introduce the following definition and lemma in order to study the existence of periodic solution.

Definition 2.3. The autonomous differential system

$$\dot{x}(t) = \Psi(x), \quad \Psi : \Omega' \subset R^n \to R^n, \tag{2.4}$$

is said to be competitive in Ω' if there is a diagonal matrix $H = \mathrm{diag}(\varepsilon_1, \varepsilon_2, ..., \varepsilon_n)$, $\varepsilon_i \in \{-1, 1\}, i = 1, ..., n$ such that $HJ_{(2.4)}H$ has non-positive off-diagonal elements for all $x \in \Omega'$.

Lemma 2.4. (See[14]) *Assume that (1) system(2.4) is permanent; (2) system(2.4) is competitive and irreducible in Ω'; (3) $\Omega' \subset R^3$ is open and p convex set; (4) there exists a unique positive equilibrium x^* in Ω' and det$(\Psi'(x^*)) < 0$. Then the following one of conclusions holds*

(I) x^* is stable. Also if x^* is stable but not globally stable, then system(2.4) admits one periodic orbit which is orbitally unstable.

(II) there exists nontrivial periodic solution which is orbitally stable in Ω'. Furthermore, if Ψ is analytic in Ω', x^* is unstable, then system(2.4) admits at least one periodic orbit(does not exceed finite orbits) and admits at least one which is asymptotically orbitally stable.

The Jacobian matrix of model (1.1) is given by

$$J(E)=\begin{pmatrix} r-\frac{2r}{K}x-\frac{ay_1}{(1+bx)^2} & -\frac{ax}{1+bx} & 0 \\ \frac{may_1}{(1+bx)^2} & -d_1-d+\frac{max}{1+bx} & e \\ 0 & kd & -d_2 \end{pmatrix}.$$

Let

$$d_2+\frac{ekd}{d_2}-r+\frac{2r}{K}x^*+\frac{ay_1^*}{(1+bx^*)^2}>0, \tag{2.5}$$

$$\begin{aligned}\Big[(d_2+\frac{ekd}{d_2}-r+\frac{2r}{K}x^*+\frac{ay_1^*}{(1+bx^*)^2})(d_2+\frac{ekd}{d_2})+\frac{ay_1^*}{(1+bx^*)^2}\Big]&\\ \cdot\Big(r-\frac{2rx^*}{K}-\frac{ay_1^*}{(1+bx^*)^2}\Big)<\frac{aekdAy_1^*}{d_2(1+bx^*)^2},&\end{aligned} \tag{2.6}$$

$$\begin{aligned}\Big[(d_2+\frac{ekd}{d_2}-r+\frac{2r}{K}x^*+\frac{ay_1^*}{(1+bx^*)^2})(d_2+\frac{ekd}{d_2})+\frac{ay_1^*}{(1+bx^*)^2}\Big]&\\ \cdot\Big(r-\frac{2rx^*}{K}-\frac{ay_1^*}{(1+bx^*)^2}\Big)>\frac{aekdAy_1^*}{d_2(1+bx^*)^2}.&\end{aligned} \tag{2.7}$$

Then, under the assumption $0<A<\frac{maK}{1+bK}$, applying the classical Routh-Hurwitz theorem, we derive that E^* is locally asymptotically stable if (2.5) and (2.6) hold and unstable if (2.7) holds. Moreover, the condition of Theorem A, that is, $A>\max\left\{\frac{ma(bK-1)}{b(1+bK)},0\right\}$ enough to ensure the inequalities (2.5) and (2.6) hold.

By Lemma 2.4, we obtain the following existence theorem of periodic solution, which is similar to the obtained result in [4].

Theorem 2.5. *Assume that* $0<A<\frac{maK}{1+bK}$ *and* E^* *is not globally asymptotically stable.*
(I) If (2.5) and (2.6) hold, then model (1.1) admits at least one periodic orbit which is orbitally unstable.
(II) If (2.7) holds, then model (1.1) admits at least one periodic orbit(does not exceed finite orbits) and admits at least one which is asymptotically orbitally stable.

Proof. Let $\mathrm{diag}(-1,1,-1)J(E)\mathrm{diag}(-1,1,-1)\triangleq B$, where

$$B=\begin{pmatrix} r-\frac{2r}{K}x-\frac{ay_1}{(1+bx)^2} & \frac{ax}{1+bx} & 0 \\ -\frac{may_1}{(1+bx)^2} & -d_1-d+\frac{max}{1+bx} & -e \\ 0 & -kd & -d_2 \end{pmatrix},$$

and $\det J(E^*)=-\frac{md_2a^2x^*y_1^*}{(1+bx^*)^3}<0$. Notice that all the off-diagonal elements of B are non-positive for all $(x,y_1,y_2)\in\Omega_0$, where $\Omega_0=\{(x,y_1,y_2)\in R^3: x\le 0, y_1\ge 0, y_2\le 0\}$, and so model (1.1) is competitive in Ω_0. Then, it follows from the results in [13] that model (1.1) is competitive in $\Omega = \{(x, y_1, y_2) \in R^3 | x > 0, y_1 > 0, y_2 > 0\}$ with respect to the partial order defined by the orthant Ω_0. Since model (1.1) is permanent, it is easy to see that model (1.1) is point dissipative and irreducible. Then

(I) If (2.5) and (2.6) hold, then E^* is locally but not globally asymptotically stable, from Lemma 2.4 (I), we derive that model (1.1) admits at least one periodic orbit which is orbitally unstable.

(II) If (2.7) holds, then E^* is unstable, from Lemma 2.4 (II), we derive that model (1.1) admits at least one periodic orbit(does not exceed finite orbits) and admits at least one which is asymptotically orbitally stable. This completes the proof.

3. Numerical Simulations

Example 3.1.

$$\begin{aligned} \dot{x} &= x\left(1 - \frac{x}{5}\right) - \frac{xy_1}{1+x}, \\ \dot{y}_1 &= \frac{mxy_1}{1+x} + 0.5y_2 - 0.5y_1 - 0.5y_1, \\ \dot{y}_2 &= 0.25y_1 - 0.5y_2, \end{aligned} \tag{3.1}$$

where $K = 5,\ r = a = b = 1,\ e = k = d = d_1 = d_2 = 0.5.$

For these values of parameters, we verify dynamical properties of model (1.1) by choosing parameter m appropriately.

(1) Let $m = 0.85$, simple computations show that $A = 0.75,\ \dfrac{maK}{1+bK} \approx 0.7083$, then the condition of Theorem 2.1 is satisfied and $E_2(5,0,0)$ is globally asymptotically stable (see Figure 1.).

(2) Let $m = 1$, then $A = 0.75,\ \dfrac{maK}{1+bK} \approx 0.8333,\ x^* = 3 > \dfrac{K}{2}$, from Theorem 2.2, we derive that $E^*(3, 1.6, 0.8)$ is globally asymptotically stable. Figure 2. shows above dynamics.

(3) Let $m = 1.2$, then $A = 0.75,\ \dfrac{maK}{1+bK} = 1$, that is, $0 < A < \dfrac{maK}{1+bK}$ and a set of computations show (2.7) is satisfied, then $E^*(1.6667, 1.7778, 0.8889)$ is unstable, from Theorem 2.5 (II), we derive that model (3.1) admits at least one periodic orbit(does not exceed finite orbits) and admits at least one which is asymptotically orbitally stable. Figure 4. shows above dynamics.

Furthermore, numerical simulation indicates that for $m = 1.1$ and then $x^* = 2.1429 < \dfrac{K}{2}$, the interior equilibrium $E^*(2.1429, 1.7959, 0.8980)$ of model (3.1) is also globally asymptotically stable (see Figure 3.).

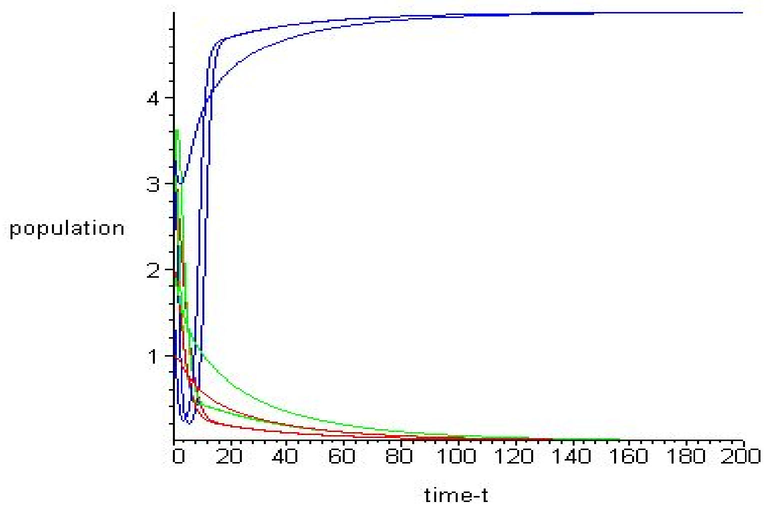

FIGURE 1. Dynamic behaviors of model (3.1) for $m = 0.85$ and $E_2(5,0,0)$ is globally asymptotically stable, the initial conditions $(x(0), y_1(0), y_2(0)) = (3.5,2,1)$, $(2,4,2)$ and $(4.5,3,4)$, respectively, where $x(t)-$ blue, $y_1(t)-$ green, $y_2(t)-$ red.

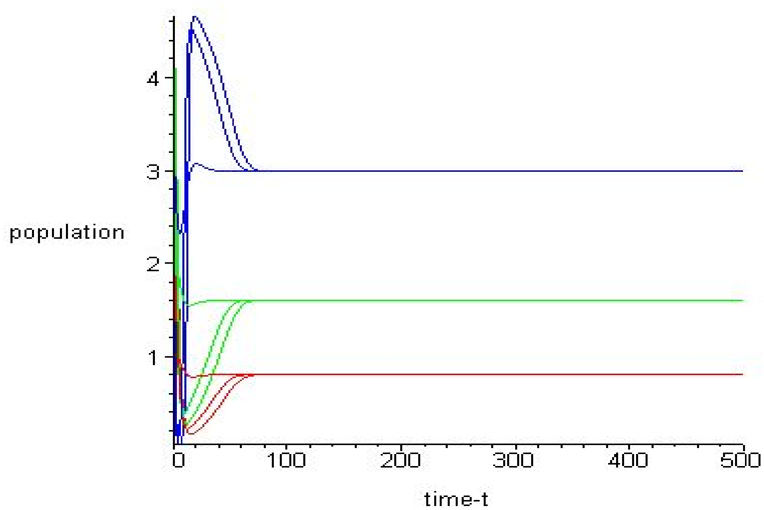

FIGURE 2. Dynamic behaviors of model (3.1) for $m = 1$ and $E^*(3,1.6,0.8)$ is globally asymptotically stable, the initial conditions $(x(0), y_1(0), y_2(0)) = (3.5,2,1)$, $(2,4,2)$ and $(4.5,3,4)$, respectively, where $x(t)-$ blue, $y_1(t)-$ green, $y_2(t)-$ red.

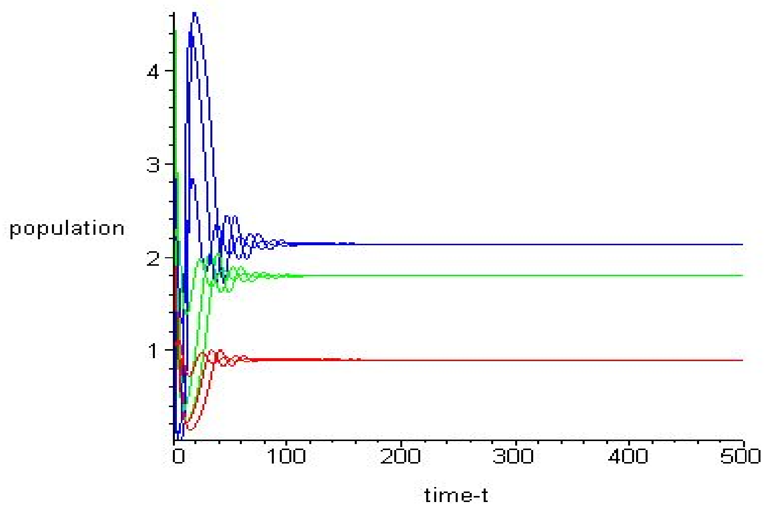

FIGURE 3. Dynamic behaviors of model (3.1) for $m = 1.1$ and $E^*(2.1429, 1.7959, 0.8980)$ is globally asymptotically stable, the initial conditions $(x(0), y_1(0), y_2(0)) = (3.5, 2, 1)$, $(2, 4, 2)$ and $(4.5, 3, 4)$, respectively, where $x(t)-$ blue, $y_1(t)-$ green, $y_2(t)-$ red.

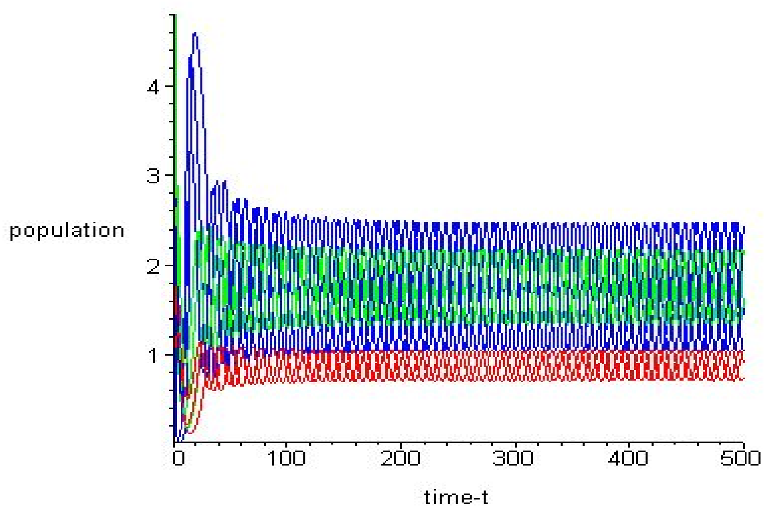

FIGURE 4. Dynamic behaviors of model (3.1) for $m = 1.2$ and there exists one periodic solution which is asymptotically orbitally stable, the initial conditions $(x(0), y_1(0), y_2(0)) = (3.5, 2, 1)$, $(2, 4, 2)$ and $(4.5, 3, 4)$, respectively, where $x(t)-$ blue, $y_1(t)-$ green, $y_2(t)-$ red.

Example 3.2.

$$\begin{aligned} \dot{x} &= x\Big(1-\frac{x}{5}\Big)-\frac{xy_1}{1+x}, \\ \dot{y}_1 &= \frac{1.2xy_1}{1+x}+0.5y_2-0.5y_1-0.5y_1, \\ \dot{y}_2 &= 0.235y_1-0.5y_2, \end{aligned} \tag{3.2}$$

where $K=5,\ r=a=b=1,\ e=d=d_1=d_2=0.5,\ k=0.47,\ m=1.2.$

Differently to [4], we find above parameter values under which the assumption condition of Theorem 2.5 and (2.5), (2.6) are satisfied, that is, $E^*(1.7586, 1.7883, 0.8405)$ is locally but not globally asymptotically stable, Figure 5. indicates that model (3.2) admits one nontrivial periodic solution.

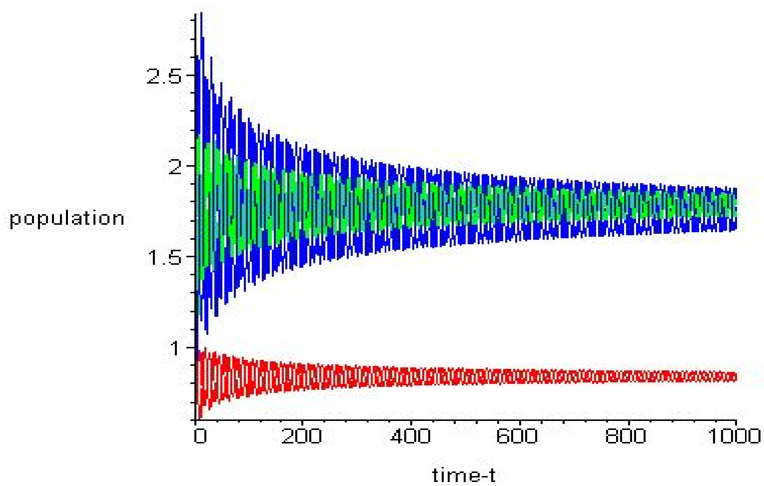

FIGURE 5. Model (3.2) admits one nontrivial periodic solution surrounding E^* and $E^*(1.7586, 1.7883, 0.8405)$ is locally asymptotically stable with the initial conditions $(x(0), y_1(0), y_2(0)) = (2, 1.2, 0.9)$ and $(1.4, 2, 1.5)$, respectively, where $x(t)-$ blue, $y_1(t)-$ green, $y_2(t)-$ red.

4. Conclusions

In this paper, we revisit a syrphid fly-aphid model proposed by L. H. Huo, *et al.* [13], where predator species takes two stage structure: immature and mature, and only the immature predator species could capture prey species. Some sufficient conditions for global stability and existence of nontrivial periodic solution have been obtained. Our results indicate that the considered model exists a threshold value $A = \frac{maK}{1+bK}$. When A is less than $\frac{maK}{1+bK}$, there exists a unique interior equilibrium E^* which is globally asymptotically stable or there exists at least one non-trivial periodic solution surrounding E^*, that is, all natural enemy and pest populations tend to a constant population level or periodic oscillation. When A is more than $\frac{maK}{1+bK}$, there exists a boundary equilibrium $E_2(K,0,0)$ which is globally asymptotically stable, that is, the natural enemy population goes to extinction and the pest population breaks out. Therefore, we can use this threshold value $A = \frac{maK}{1+bK}$ to control the pest population, moreover,control all natural enemy and pest populations to a constant population level and to prevent the outbreak of the pest population, which provide the prediction and decision basis for biological control and guide integrated pest management.

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