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DYNAMIC BEHAVIORS OF A LOTKA-VOLTERRA PREDATOR-PREY MODEL INCORPORATING A CONSTANT NUMBER OF PREY REFUGE AND FEEDBACK CONTROLS

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Abstract. A Lotka-Volterra predator-prey model incorporating a constant number of prey refuge and feedback controls is studied in this paper. By constructing suitable Lyapunov functions, sufficient conditions which ensure the global stability of the positive equilibrium and boundary equilibriums are obtained, respectively. Numerical simulations are also presented to illustrate the feasibility of the main results.

Keywords. Global stability; Feedback control; Equilibrium; Lyapunov function.

2010 Mathematics Subject Classification. 34D23, 92B05.

1. Introduction

Prey refuge is widely believed to be an important factor in predator-prey communities. A prey refuge can be broadly defined to include any strategy that decreases predation risk, e.g., spatial or temporal refuges, prey aggregations, or reduced prey activity; see [1]-[3] and the references therein. The existence of refuges has important effects on the coexistence of predator and prey, and many studies confirmed that preventing prey extinction and damping predator-prey oscillations are the most significant roles of prey refuges; see [4]-[17] and the references therein.

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The effects of a constant proportion or a constant number of refuges with the predator-prey model were studied in [1]-[14]. Recently, Ma [10] and Chen, Ma and Zhang [16] investigated the dynamic behaviors of the traditional Lotka-Volterra predator-prey systems incorporating a constant number of prey refuge, i.e.,

$$\begin{aligned} x'(t) &= rx(1-\frac{x}{k}) - c(x-R)y, \\ y'(t) &= e(x-R)y - dy, \end{aligned} \tag{1.1}$$

where $x(t)$ and $y(t)$ denote the densities of the prey and predator populations at time t, respectively; r,k,c,e,d and R are all positive constants; and a refuge protecting a constant number (R) of the prey. This leaves $(x-R)$ of the prey available to the predator. Obviously, the above system admits two equilibria $(k,0)$ and $(\widetilde{x},\widetilde{y})$, where

$$\widetilde{x} = \frac{d}{e} + R, \quad \widetilde{y} = \frac{er\widetilde{x}}{cd}(1-\frac{\widetilde{x}}{k}).$$

Concerned with the dynamic behaviors of system (1.1), the authors of [15] and [16] obtained the following result.

Theorem A. *Assume that the inequality*

$$(H_1) \;\; 0 < R < k - \frac{d}{e}$$

holds. Then $\left(\widetilde{x},\widetilde{y}\right)$ *is globally asymptotically stable.*

Concerned with the stability property of the boundary equilibrium of system (1.1), by applying the analysis technique of [17], we find the following result.

Theorem B. *Assume that the inequality*

$$(H_2) \;\; R > k - \frac{d}{e}$$

holds. Then $\left(k,0\right)$ *is globally asymptotically stable.*

Remark 1.1. Assume that $k < \frac{d}{e}$ holds. In this case, condition (H_2) always holds and the predator species will be driven to extinction. The reason may lies in that with the limited carrying capacity of the prey species, the amount of prey species is also limited, and the predator species will be driven to extinction due to the less of food resource.

Remark 1.2. The above results indicate that the prey and the predator species could be coexist in a stable state if the refuge is less then $k - \frac{d}{e}$, while if the refuge is large enough. The prey

species could be accumulated in the refuge, and the predator species will be driven to extinction due to the lack of food.

On the other hand, as was pointed out by Gopalsamy and Weng [18], in some cases an equilibrium is not the desirable one, and this requires altering the system structurally so as to make the population stabilise at lower value. One of the methods of accomplishing this is to introduce the feedback control variables and this can be implemented by means of biological controls. The dynamic behaviors of the single species and multispecies competition model with feedback controls have been investigated by many authors; see [19]-[31] and the references therein. However, seldom did scholars investigated the dynamic behaviors of the predator-prey system with feedback controls ([28], [29]). To the best of the authors' knowledge, to this day, still no scholars investigate the stability property of predator-prey model incorporate both a constant number of prey refuge and feedback controls, this motivates us to propose and investigate the following model:

$$\begin{aligned} x'(t) &= rx(1-\frac{x}{k}) - c(x-R)y - \alpha_1 u_1 x, \\ y'(t) &= e(x-R)y - dy - \alpha_2 u_2 y, \\ u_1'(t) &= -e_1 u_1 + f_1 x, \\ u_2'(t) &= -e_2 u_2 + f_2 y, \end{aligned} \tag{1.2}$$

where $x(t)$ and $y(t)$ denote the densities of the prey and predator populations at time t, respectively; $u_1(t)$ and $u_2(t)$ are feedback control variables, r describes the intrinsic growth rates of x; d describes the death rate of the predator; k measures the environmental carrying capacity of the prey species; $k, r, e, c, d, \alpha_i, e_i, f_i, i = 1,2$ and R are all positive constants, a constant number (R) of the prey are protected, and this leaves $(x-R)$ of the prey available to the predator. For practical biological meaning, $R < x(t), t \in [0,+\infty)$, we simply study system in $R^4_+ = \{(x,y,u_1,u_2) \in R^4 \mid x,y > 0, u_i > 0, i = 1,2\}$.

The rest of the paper is organized as follows. In Section 2, we state the existence of the equilibrium and investigate the stability property of model (1.2). In Section 3, numerical simulations are presented to illustrate the feasibility of our results. We end this paper by a brief discussion.

2. Global stability

Set

$$\begin{aligned}\bar{u}_1 &= \frac{\alpha_1 f_1}{e_1}, \bar{u}_2 = \frac{\alpha_2 f_2}{e_2},\\ \Delta &= \big(er\bar{u}_2 - ced - 2\bar{u}_2(d+Re)(\frac{r}{k}+\bar{u}_1)\big)^2\\ &\quad +4\Big[(\frac{r}{k}+\bar{u}_1)\bar{u}_2^2 + ce\bar{u}_2\Big]\Big[er(d+eR) - (\frac{r}{k}+\bar{u}_1)(d+eR)^2\Big].\end{aligned}$$

Lemma 2.1. *System (1.2) admits a boundary equilibrium $E_1(x^*, y^*, u_1^*, u_2^*)$, where $x^* = \dfrac{rk}{r+k\bar{u}_1}, y^* = 0, u_1^* = \dfrac{rkf_1}{e_1 r + e_1 k\bar{u}_1}, u_2^* = 0$.*

Lemma 2.2. *Assume that*

$$(H_3) \quad 0 < R < \frac{kr}{r+k\bar{u}_1} - \frac{d}{e}$$

holds. Then system (1.2) admits a unique positive equilibrium $E_2(x^{**}, y^{**}, u_1^{**}, u_2^{**})$, where

$$\begin{aligned}x^{**} &= \frac{d+y^{**}\bar{u}_2}{e} + R,\\ y^{**} &= \frac{er\bar{u}_2 - 2\bar{u}_2(d+Re)(\frac{r}{k}+\bar{u}_1) - ced + \sqrt{\Delta}}{2\bar{u}_2\big[\bar{u}_2(\frac{r}{k}+\bar{u}_1)+ce\big]},\\ u_1^{**} &= \frac{f_1}{e_1}x^{**},\ u_2^{**} = \frac{f_2}{e_2}y^{**}.\end{aligned}$$

Proof. It follows from (H_3) that $0 < R < \dfrac{kr}{r+k\bar{u}_1} - \dfrac{d}{e} < \dfrac{kr}{r+k\bar{u}_1}$. So, $\dfrac{r}{k} + \bar{u}_1 < \dfrac{r}{R}$. It follows that $eR(\dfrac{r}{k}+\bar{u}_1) < er$. Hence, we have

$$(d+eR)(\frac{r}{k}+\bar{u}_1) < er + d(\frac{r}{k}+\bar{u}_1). \tag{2.1}$$

We now consider the positive equilibrium of system (1.2), which is the solution of the following equations,

$$\begin{aligned}&rx(1-\frac{x}{k}) - c(x-R)y - \alpha_1 u_1 x = 0,\\ &e(x-R)y - dy - \alpha_2 u_2 y = 0,\\ &-e_1 u_1 + f_1 x = 0,\\ &-e_2 u_2 + f_2 y = 0.\end{aligned} \tag{2.2}$$

By the third and the forth equations of (2.2), one has

$$u_1 = \frac{f_1 x}{e_1}, u_2 = \frac{f_2 y}{e_2}.$$

This combine with the first and the second equations of (2.2) leads to

$$x = \frac{d + y\bar{u}_2}{e} + R, \tag{2.3}$$

$$x(r - \frac{r}{k}x) - c(x - R)y - \bar{u}_1 x^2 = 0. \tag{2.4}$$

Substituting (2.3) into (2.4), we find that

$$F(y) = Ay^2 + By + C = 0, \tag{2.5}$$

where

$$\begin{aligned} A &= -(\frac{r}{k} + \bar{u}_1)\bar{u}_2^2 - ce\bar{u}_2, \\ B &= er\bar{u}_2 - 2\bar{u}_2(d + eR)(\frac{r}{k} + \bar{u}_1) - ced, \\ C &= er(d + eR) - (\frac{r}{k} + \bar{u}_1)(d + eR)^2. \end{aligned}$$

The discriminant of equation (2.5) satisfies

$$\begin{aligned} \Delta &= \left(er\bar{u}_2 - ced - 2\bar{u}_2(d + Re)(\frac{r}{k} + \bar{u}_1)\right)^2 \\ &\quad + 4\left[(\frac{r}{k} + \bar{u}_1)\bar{u}_2^2 + ce\bar{u}_2\right]\left[er(d + eR) - (\frac{r}{k} + \bar{u}_1)(d + eR)^2\right] \\ &= (er\bar{u}_2 - ced)^2 + 4ced\bar{u}_2(d + Re)(\frac{r}{k} + \bar{u}_1) \\ &\quad + 4ce\bar{u}_2\left[er - (\frac{r}{k} + \bar{u}_1)(d + Re)\right](d + Re) \\ &\overset{\text{from (2.1)}}{>} (er\bar{u}_2 - ced)^2 + 4ced\bar{u}_2(d + Re)(\frac{r}{k} + \bar{u}_1) \\ &\quad + 4ce\bar{u}_2\left[er - er - d(\frac{r}{k} + \bar{u}_1)\right](d + Re) \\ &= (er\bar{u}_2 - ced)^2 > 0, \end{aligned}$$

which means that $F(y) = 0$ has two different solutions.

On the other hand, one could easily see that $F(0) > 0$. Therefore, $F(y) = 0$ admits a unique positive solution. Consequently, (2.2) has a unique positive equilibrium $E_2(x^{**}, y^{**}, u_1^{**}, u_2^{**})$, where

$$\begin{aligned} x^{**} &= \frac{d + y^{**}\bar{u}_2}{e} + R, \\ y^{**} &= \frac{er\bar{u}_2 - 2\bar{u}_2(d + Re)(\frac{r}{k} + \bar{u}_1) - ced + \sqrt{\Delta}}{2\bar{u}_2\left[\bar{u}_2(\frac{r}{k} + \bar{u}_1) + ce\right]}, \\ u_1^{**} &= \frac{f_1}{e_1}x^{**},\ u_2^{**} = \frac{f_2}{e_2}y^{**}. \end{aligned}$$

This completes the proof of the Lemma 2.2.

Theorem 2.3. *Assume that*

$$(H_4) \quad \frac{kr}{r+k\bar{u}_1} < \frac{d}{e},$$

holds. Then the boundary equilibrium E_1 of system (1.2) is globally stable.

Proof. It follows from the condition that $x^* < \frac{d}{e}$. Therefore, $ceR + cex^* < cd + ceR$. So,

$$\frac{c(R+x^*)}{d+eR} < \frac{c}{e}. \tag{2.6}$$

Define a Lyapunov function as follows:

$$V_1(t) = \eta_1\Big(x - x^* - x^*\ln\frac{x}{x^*}\Big) + \eta_2 y + \frac{\eta_3}{2}(u_1-u_1^*)^2 + \frac{\eta_4}{2}u_2^2,$$

where $\eta_i, i = 1,2,3,4$ are positive constants to be determined. Calculating the derivative of V_1 along any positive solution of system (1.2), we have

$$\begin{aligned}
\frac{dV_1}{dt}\Big|_{(1.2)} &= \eta_1(x - \frac{x^*x'}{x}) + \eta_2 y' + \eta_3(u_1-u_1^*)u_1' + \eta_4 u_2 u_2' \\
&= -\frac{r}{k}\eta_1(x-x^*)^2 - \alpha_1\eta_1(x-x^*)(u_1-u_1^*) - c\eta_1 xy + c\eta_1 x^* y + cR\eta_1 y \\
&\quad -\frac{\eta_1 cRyx^*}{x} - \eta_2 dy - \eta_2 eRy + \eta_2 exy - \alpha_2\eta_2 yu_2 - e_1\eta_3(u_1-u_1^*)^2 \\
&\quad +\eta_3 f_1(x-x^*)(u_1-u_1^*) - \eta_4 e_2 u_2^2 + \eta_4 f_2 u_2 y \\
&= -\frac{r}{k}\eta_1(x-x^*)^2 - \eta_3 e_1(u_1-u_1^*)^2 - \eta_4 e_2 u_2^2 - (\alpha_2\eta_2 - f_2\eta_4)yu_2 \\
&\quad -(\eta_1\alpha_1 - \eta_3 f_1)(x-x^*)(u_1-u_1^*) - (d\eta_2 + eR\eta_2 - cR\eta_1 - cx^*\eta_1)y \\
&\quad -(\eta_1 c - \eta_2 e)xy - \frac{\eta_1 cRyx^*}{x}.
\end{aligned}$$

Taking $\eta_1 = 1, \eta_3 = \frac{\alpha_1}{f_1}$, and from (2.6), we see that there exists a $\eta_2 > 0$ such that

$$\frac{c(R+x^*)}{d+eR} < \eta_2 < \frac{c}{e}$$

Consequently, we have $c\eta_1 - \eta_2 e > 0$, $(d+eR)\eta_2 - c(R+x^*)\eta_1 > 0$. Finally, let us choose $\eta_4 = \frac{\alpha_2\eta_2}{f_2} > 0$. Then $\alpha_2\eta_2 - \eta_4 f_2 = 0$. It follows that

$$\begin{aligned}
\frac{dV_1}{dt}\Big|_{(1.2)} &= -\frac{r}{k}(x-x^*)^2 - \frac{\alpha_1 e_1}{f_1}(u_1-u_1^*)^2 - \frac{e_2\alpha_2\eta_2}{f_2}u_2^2 \\
&\quad -\big((d+eR)\eta_2 - c(R+x^*)\big)y - (c-\eta_2 e)xy - \frac{cRyx^*}{x}.
\end{aligned}$$

Therefore, $\frac{dV_1}{dt} < 0$ strictly for all x, y, u_1, $u_2 > 0$ except the boundary equilibrium $(x^*, 0, u_1^*, 0)$, where $\frac{dV_1}{dt} = 0$. Thus, according to the LaSalle theorem, E_1 is globally stable.

Theorem 2.4. *Assume that (H_3) holds. Then the unique positive equilibrium E_2 of system (1.2) is globally stable.*

Proof. It follows from Lemma 2.2 that system (1.2) has a unique positive equilibrium. Define a Lyapunov function as follows:

$$V_2(t) = \eta_1\Big(x - x^{**} - x^{**}\ln\frac{x}{x^{**}}\Big) + \eta_2\Big(y - y^{**} - y^{**}\ln\frac{y}{y^{**}}\Big) + \frac{\eta_3}{2}\Big(u_1 - u_1^{**}\Big)^2 + \frac{\eta_4}{2}\Big(u_2 - u_2^{**}\Big)^2,$$

where $\eta_i > 0, i = 1,2,3,4$ are positive constants to be determined. Calculating the derivative of V_2 along the positive solution of system (1.2), we have

$$\begin{aligned}
\frac{dV_2}{dt}\Big|_{(1.2)} &= -\eta_1\frac{r}{k}(x-x^{**})^2 - \alpha_1\eta_1(x-x^{**})(u_1-u_1^{**}) \\
&\quad +\eta_1(x-x^{**})\frac{c(x^{**}-R)y^{**}x - c(x-R)yx^{**}}{xx^{**}} + e\eta_2(x-x^{**})(y-y^{**}) \\
&\quad -\alpha_2\eta_2(y-y^{**})(u_2-u_2^{**}) - e_1\eta_3(u_1-u_1^{**})^2 + f_1\eta_3(x-x^{**})(u_1-u_1^{**}) \\
&\quad -e_2\eta_4(u_2-u_2^{**})^2 + \eta_4 f_2(y-y^{**})(u_2-u_2^{**}) \\
&= -\eta_1\frac{r}{k}(x-x^{**})^2 - e_1\eta_3(u_1-u_1^{**})^2 - e_2\eta_4(u_2-u_2^{**})^2 \\
&\quad -\alpha_1\eta_1(x-x^{**})(u_1-u_1^{**}) - c\eta_1(x-x^{**})(y-y^{**}) - \frac{cRy}{xx^{**}}\eta_1(x-x^{**})^2 \\
&\quad +\frac{cR}{x^{**}}\eta_1(x-x^{**})(y-y^{**}) + e\eta_2(x-x^{**})(y-y^{**}) \\
&\quad -\big(\eta_2\alpha_2 - \eta_4 f_2\big)(y-y^{**})(u_2-u_2^{**}) + \eta_3 f_1(x-x^{**})(u_1-u_1^{**}) \\
&= -\big(\frac{r}{k} + \frac{cRy}{xx^{**}}\big)\eta_1(x-x^{**})^2 - e_1\eta_3(u_1-u_1^{**})^2 - e_2\eta_4(u_2-u_2^{**})^2 \\
&\quad -\big(\alpha_1\eta_1 - \eta_3 f_1\big)(x-x^{**})(u_1-u_1^{**}) - \big(\eta_2\alpha_2 - \eta_4 f_2\big)(y-y^{**})(u_2-u_2^{**}) \\
&\quad -\big(c\eta_1 - e\eta_2 - \frac{cR}{x^{**}}\eta_1\big)(x-x^{**})(y-y^{**}).
\end{aligned}$$

Taking $\eta_1 = 1$, $\eta_2 = \frac{c}{e_2}(1 - \frac{R}{x^{**}}) > 0$, $\eta_3 = \frac{\alpha_1}{f_1}$, $\eta_4 = \frac{\alpha_2 c}{f_2 e_2}(1 - \frac{R}{x^{**}}) > 0$, we have

$$\begin{aligned}
\frac{dV_2}{dt}\Big|_{(1.2)} &= -\Big(\frac{r}{k} + \frac{cRy}{xx^{**}}\Big)(x-x^{**})^2 - \frac{\alpha_1 e_1}{f_1}(u_1-u_1^{**})^2 \\
&\quad -\frac{c\alpha_2}{f_2}\Big(1 - \frac{R}{x^{**}}\Big)(u_2-u_2^{**})^2 < 0,
\end{aligned}$$

and so, $\frac{dV_2}{dt} < 0$ strictly for all $x, y, u_1, u_2 > 0$ except the positive equilibrium E_2 , where $\frac{dV_2}{dt} = 0$. According to the LaSalle theorem, E_2 is globally stable. This ends the proof of Theorem 2.4.

Remark 2.5. Assuming $0 < R < k - \frac{d}{e}$, we see from Theorem A that, for the system without feedback control (i.e., system (1.1)), the predator and prey species could be coexist in a stable state. However, with the introducing of feedback control variables, if

$$\bar{u}_1 = \frac{\alpha_1 f_1}{e_1} > \frac{er}{d} - \frac{r}{k},$$

then (H_4) follows, and predator species will be driven to extinction. The reason may lies in that $\bar{u}_1$ can be seen as the capture rate of prey species, with the increasing of capture rate, the predator species is not easy to obtain the enough food, and this finally leads to the extinction of predator species. If we choose the suitable feedback control $\bar{u}_1$ such that inequality

$$\bar{u}_1 < \frac{re}{d + eR} - \frac{r}{k},$$

then (H_3) follows. Thanks to Theorem 2.2, the prey species and predator species will be coexistence in a stable state.

Remark 2.6. Since $\bar{u}_1 = \frac{\alpha_1 f_1}{e_1} > 0$, it follows that

$$0 < \frac{kr}{r + k\bar{u}_1} - \frac{d}{e} < k - \frac{d}{e}.$$

Compare with the dynamic behaviors of systems (1.1) and (1.2), with the introducing of feedback control variables, the threshold of prey refuge for the predator species and prey species be coexistent become smaller. The reason may lies in that both the feedback control variables and prey refuge have negative effect on the prey and predator species. Hence, to ensure the prey species and the predator species coexistence in a stable state, one should restrict the refuge to a smaller range. On the other hand, using Remark 2.5 and (H_4), we see that $\frac{d}{e} > k > \frac{kr}{k\bar{u}_1 + r}$ holds. Then, one can obtain that if the system without feedback control variables is extinct (Remark 2.5), then the feedback control variables can only change the position of the boundary equilibrium and the extinct property is retained (Theorem 2.3).

3. Examples

The following examples show the feasibility of our main results.

Example 3.1. *In system (1.1), we choose* $r = 10, k = 10, R = 5, c = 2, e = 1, d = 1, \alpha_1 = 3, f_1 = 4, e_1 = 1, \alpha_2 = 2, f_2 = 3, e_2 = 4$. *Obviously,*

$$0 < R = 5 < k - \frac{d}{e} = 10 - \frac{1}{1} = 9.$$

Therefore, condition (H_1) *holds. From Theorem A, the prey species and the predator species will be coexistence in a stable state. However, for system (1.2), by simple calculation, one has*

$$\frac{kr}{r + k\bar{u}_1} = \frac{5}{6} < 1 = \frac{d}{e}.$$

So, condition (H_4) *holds. According to Theorem 2.1, the predator species will be driven to extinction. Figure 1 shows the dynamics of species of system (1.2).*

Example 3.2. *We only change* $\alpha_1 = 1, f_1 = 1, e_1 = 3$. *The rest of coefficients are the same as that of Example 3.1. From Example 3.1, we know that, in this case, for model (1.1) admits a positive equilibrium* $(\widetilde{x}, \widetilde{y})$, *which is globally asymptotically stable. Also, in this case, for model (1.2), by calculation, it follows that*

$$0 < R = 5 < \frac{kr}{r + k\bar{u}_1} - \frac{d}{e} = \frac{15}{2} - 1 = \frac{13}{2}.$$

Thus, condition (H_3) *holds. According to Theorem 2.2, the prey species and the predator species will be coexistent in a stable state. Figure 2 shows the dynamics of species of system (1.2).*

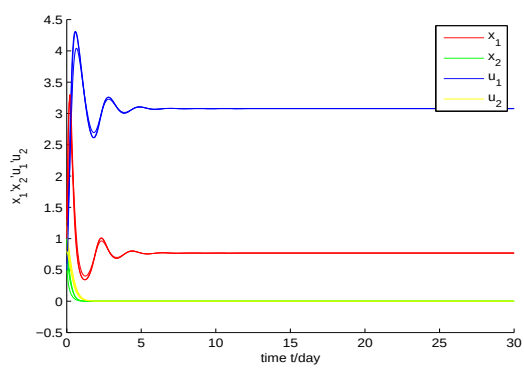

FIGURE 1. Dynamic behaviors of the solution $(x(t), y(t), u_1(t), u_2(t))$ of system (1.2) with the initial conditions $(x(0), y(0), u_1(0), u_2(0))$=$(1, 1, 0.5, 0.8)$, $(0.8, 0.6, 1, 1.2)$ and $(1.2, 1.3, 0.8, 1)$, respectively.

4. Discussion

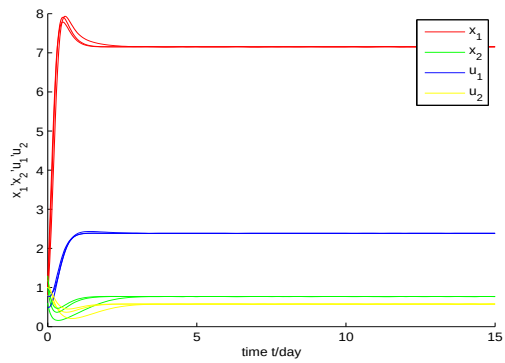

FIGURE 2. Dynamic behaviors of the solution $(x(t), y(t), u_1(t), u_2(t))$ of system (1.2) with the initial conditions $(x(0), y(0), u_1(0), u_2(0))$=$(1, 1, 0.5, 0.8)$, $(0.8, 0.6, 1, 1.2)$ and $(1.2, 1.3, 0.8, 1)$, respectively.

During the last decade, the dynamic behaviors of feedback control ecosystem become one of the most important topic in the study of population dynamics. Specially, Li, Han and Chen [24] proposed and studied an autonomous Lotka-Volterra competitive system with infinite delays and feedback controls. They showed that if the Lotka-Volterra competitive system without feedback control is globally stable, then the feedback controls only change the position of the unique positive equilibrium and retain the stable property, if the Lotka-Volterra competitive system is extinct, by choosing the suitable values of feedback control variables, the extinct species may become globally stable, or still keep the property of extinction. To the best of the authors knowledge, to this day, there are still no scholars study such kind of problem for the predator-prey system. This motivates us to propose and study the dynamic behaviors of system (1.2). Our main results (Theorem 2.3 and 2.4) indicate that, for the predator-prey system without feedback controls, if the system is globally stable, then depending on the choice of feedback control variables, predator species will be driven to extinction (Theorem 2.3) or the prey and predator specie could coexist in a stable state (Theorem 2.4). However, following from Remark 2.6, we see that if the system without feedback control variables is extinct (Remark 2.5), then the feedback control variables can only change the position of boundary equilibrium and the extinct property is retained (Theorem 2.3). Such a finding is very different to that of Li, Han and Chen [24]. We found that for the different population system, the role of feedback control variables are very different. And for different kind of population system, to ensure the system

be persistent, different feedback control strategy is needed. We mention here that, in system (1.2), we did not consider the influence of the delay, however, a more appropriate model should incorporate time delay to the system. We leave those for future study.

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