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ABSOLUTE STABILITY OF GENERAL NEUTRAL LURIE INDIRECT CONTROL SYSTEMS WITH UNBOUNDED COEFFICIENTS

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Abstract. The absolute stability problem of neutral Lurie indirect control systems with unbounded coefficients is investigated in this paper. By choosing a suitable Lyapunov-Krasovskii functional, several absolute stability criteria for neutral Lurie systems with single nonlinearity are obtained. Furthermore, the derived results are extended to multiple nonlinearities. The criteria proposed in this paper are particularly useful for neutral Lurie indirect control systems with unbounded coefficients. Our results are also applicable to such systems with bounded or constant coefficients. Numerical simulations demonstrate the effectiveness of the proposed approach.

Keywords. Nonlinear system; Neutral Lurie system; Absolute stability; Lyapunov stability theorem.

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1. Introduction

It is well known that time delay is often a significant source of instability. It is frequently encountered in engineering systems. There are mainly two types of time delay systems: the retarded type and the neutral type. Since time delay of neutral type systems not only exists in its own state but also in the derivative of the state, the retarded type system can be regarded as a special case of the neutral type system. Therefore, the stability analysis of neutral time delay systems is of much wider significance. In [1], by developing a discretized Lyapunov functional approach, the stability of linear delay-differential systems of the neutral type was considered. In

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[2], the robust stability of uncertain linear neutral systems with time-varying discrete delay was investigated. Combining the parameterized model transformation method with a free-weighting matrices technique, Wu *et al.* obtained some stability conditions and an approach for stabilizing neutral systems in [3]. For neutral type systems with mixed delays and time-varying structured uncertainties, some new delay-dependent robust stability criteria were presented in [4].

On the other hand, the absolute stability analysis of the Lurie system has received much attention; see [5-10] and the references therein. The Aizerman and Kalman conjectures have played an important role in the development of absolute stability theory. Some related interesting discussions about the classical conjectures were done in [11-12]. The absolute stability analysis in terms of full-block multipliers was presented in [13]. During the last two decades, various efficient methods have been employed by researchers to deeply study the absolute stability of neutral Lurie systems, and a number of valuable results have been achieved. Based on the model transformation and bounding techniques for cross terms, respectively, some absolute stability conditions for neutral Lurie indirect control systems have been derived in [14-15]. Using a free-weighting matrix approach and the extended Jensen inequality, delay-dependent robust absolute stability criteria were presented in [16]. By introducing some triple-integral terms in LKF, Duan *et al.* considered the stability problem of uncertain neutral-type Lur'e systems with time-varying delays in [17], and the derived criteria were less conservative than some proposed in [16, 18-20]. Additionally, robust absolute stability for uncertain Lurie interval time-varying delay systems of neutral type has been addressed and some stability results have been reported; see [21-24] and the references therein.

In [14-15,18,25-27], they require that the coefficient matrices of neutral Lurie systems are constants. In [9,16-17,19-20,21-24], they require the system coefficients to be uncertain but norm-bounded. Until now, most results concerning neutral Lurie systems required that the system coefficients be bounded. However, most of these systems are time-varying in engineering problems, and thus the above theoretical results are conservative to some extent. Moreover, the above-mentioned conclusions are not applicable to norm-unbounded systems. Motivated by this fact, the absolute stability of time-varying neutral Lurie indirect control systems is studied in this paper. In particular, the system coefficients can be norm-unbounded. As with most similar research, the Lyapunov second method is used in this paper. By choosing the proper Lyapunov-Krasovskii functional, some absolute stability criteria are established.

Notation. Throughout this paper, $P>0\,(P<0)$ means that P is positive (negative) definite and $\lambda(A)$ stands for any eigenvalue of the square matrix A. For vector $x=\begin{bmatrix}x_1 & x_2 & \cdots & x_m\end{bmatrix}^T$, we use the usual Euclidean norm, i.e., $\|x\|=\sqrt{\sum\limits_{i=1}^{m}x_i^2}$. For matrix $A=(a_{ij})_{m\times n}$, $\|A\|$ denotes the matrix norm induced by the Euclidean vector norm. So it is easy to verify that $\|A\|=\sqrt{\lambda_{\max}(A^TA)}$. In the derivation of this paper, $\|A\|$ can be replaced by $\|A\|_F$, $\|A\|_F=\sqrt{\sum\limits_{j=1}^{n}\sum\limits_{k=1}^{m}|a_{kj}|^2}$. This is because $\|A\|\le\|A\|_F$ and the calculation of $\|A\|_F$ is is not hard. $\overline{\lim\limits_{t\to\infty}}$ represents the upper limit. For simplicity, denote $\phi_t(s)=\begin{bmatrix}x(t+s)\\ \sigma(t)\end{bmatrix}, s\in[-\tau,0], t\ge 0$, $\|\phi\|_{L_2}=\sqrt{\int_{-\tau}^{0}\|\phi_t(s)\|^2ds}$.

First, a time-varying neutral Lurie system with single nonlinearity is considered. Subsequently, the obtained results are further extended to multiple nonlinearities. The Lyapunov stability theorem used in this paper is given in [28-29]. For the case of multiple nonlinearities, $\sigma(t)$ in $\phi_t(s)$ can be regarded as a vector.

2. Absolute stability of neutral Lurie systems with single nonlinearity

Consider the following time-varying neutral Lurie indirect control system with single nonlinearity

$$\begin{cases}\dot{x}(t)-D\dot{x}(t-\tau)=A(t)x(t)+B(t)x(t-\tau)+b(t)f(\sigma(t)),\\ \dot{\sigma}(t)=c^T(t)x(t)-\rho(t)f(\sigma(t)),\\ x(t)=\varphi(t), t\in[-\tau,0],\end{cases}\tag{2.1}$$

where $x(t)\in R^n$, $\sigma(t)\in R$, $b(t), c(t)$ are n dimensional column vectors, $A(t), B(t), D$ are $n\times n$ matrices, $\tau>0$ is a constant time delay, $\rho(t)\ge\rho>0$, ρ is a constant. $A(t), B(t), b(t), c(t), \rho(t)$ are continuous in $[0,\infty)$. $f(\cdot)$ is a nonlinear function satisfying the following sector condition:

$$F_{[k_1,k_2]}=\left\{f(\cdot)\,|\,f(0)=0; k_1\sigma^2(t)\le\sigma(t)f(\sigma(t))\le k_2\sigma^2(t), \sigma(t)\ne 0\right\},$$

where k_1, k_2 are constants and $k_2>k_1>0$.

For system (2.1), define operator $\bar{\mathscr{D}}: C\left([-\tau,0], R^{n+1}\right)\to R^{n+1}$ as

$$\bar{\mathscr{D}}\phi_t=\phi_t(0)-\bar{D}\phi_t(-\tau),$$

where $\bar{D}=\begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix}$ and $\phi_t(s)$ is defined in the notations above.

For the sake of brevity, let us decompose operator $\bar{\mathscr{D}}$:

$$\bar{\mathscr{D}}\phi_t=\begin{bmatrix} \mathscr{D}x_t \\ \sigma(t) \end{bmatrix}=\begin{bmatrix} x(t)-Dx(t-\tau) \\ \sigma(t) \end{bmatrix}.$$

Definition 2.1. [30] System (1) is said to be absolutely stable if its zero solution is globally asymptotically stable for any nonlinearity $f(\cdot)\in F_{[k_1,k_2]}$.

The following assumptions are made for system (2.1).

A1: All the eigenvalues of matrix D are inside the open unit circle, i.e. $|\lambda_i(D)|<1(i=1,2,\cdots,n)$.

A2: For any $t\in[0,\infty)$, there exist $P>0$ and $G>0$ such that

$$\lambda\left(PA(t)+A^T(t)P+2G\right)\le-\delta(t)\le-\delta,$$

$$2D^TGD-G=-R,$$

where $\delta>0$ is a constant and $R>0$.

A3: For any $t\in[0,\infty)$,

$$\frac{\|PA(t)D+PB(t)+2GD\|}{\sqrt{\delta(t)\lambda_{\min}(R)}}\le\alpha,\frac{\left\|Pb(t)+\frac{1}{2}c(t)\right\|}{\sqrt{\delta(t)\rho(t)}}\le\beta,\frac{\left\|\frac{1}{2}D^Tc(t)\right\|}{\sqrt{\lambda_{\min}(R)\rho(t)}}\le\gamma,$$

where α,β,γ are constants.

Remark 2.2. It should be noted that A1 guarantees that the zero solution of homogeneous difference equation $x(t)-Dx(t-\tau)=0$ is uniformly asymptotically stable, that is, $\mathscr{D}$ is stable [18].

Theorem 2.3. *Under A1-A3, system (2.1) is absolutely stable if*

$$2\alpha\beta\gamma+\left(\alpha^2+\beta^2+\gamma^2\right)<1. \tag{2.2}$$

Proof. According to Remark 1 and $\bar{D}=\begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix}$, it is clear that the operator $\bar{\mathscr{D}}$ is stable by A1. By employing matrices P and G, a Lyapunov-Krasovskii functional is chosen as

$$V(t,\phi_t)=(x(t)-Dx(t-\tau))^TP(x(t)-Dx(t-\tau))+\int_{t-\tau}^{t}x^T(s)Gx(s)ds+\int_0^{\sigma(t)}f(s)ds.$$

Notice that $V(t,\phi_t)$ consists of three terms. The first term deals with the general neutral systems. The second one focuses on the time delay term on the right-hand side of system (1), and considering the sector constraint on $f(\sigma)$ the third one is proposed. It can be proved that if $f\in F_{[k_1,k_2]}$, then

$$\frac{1}{2}k_1\sigma(t)^2\le\int_0^{\sigma(t)}f(s)\,ds\le\frac{1}{2}k_2\sigma(t)^2.$$

Thus V satisfies

$$\begin{aligned}&\lambda_{\min}(P)\|x(t)-Dx(t-\tau)\|^2+\frac{1}{2}k_1\sigma^2(t)\\&\le V(t,\phi_t)\\&\le\lambda_{\max}(P)\|x(t)-Dx(t-\tau)\|^2+\frac{1}{2}k_2\sigma^2(t)+\lambda_{\max}(G)\int_{-\tau}^0\|x(t+s)\|^2ds.\end{aligned}$$

Furthermore, one has

$$\begin{aligned}&\min\left\{\lambda_{\min}(P),\frac{k_1}{2}\right\}\left\|\begin{bmatrix}x(t)-Dx(t-\tau)\\\sigma(t)\end{bmatrix}\right\|^2\\&\le V(t,\phi_t)\\&\le\max\left\{\lambda_{\max}(P),\frac{k_2}{2}\right\}\left\|\begin{bmatrix}x(t)-Dx(t-\tau)\\\sigma(t)\end{bmatrix}\right\|^2+\lambda_{\max}(G)\int_{-\tau}^0\|x(t+s)\|^2ds,\end{aligned}$$

that is,

$$\begin{aligned}&\min\{\lambda_{\min}(P),\frac{1}{2}k_1\}\|\bar{\mathscr{D}}\phi_t\|^2\\&\le V(t,\phi_t)\\&\le\max\{\lambda_{\max}(P),\frac{1}{2}k_2\}\|\bar{\mathscr{D}}\phi_t\|^2+\lambda_{\max}(G)\int_{-\tau}^0\|x(t+s)\|^2ds.\end{aligned}$$

Letting

$$u(s)=\min\left\{\lambda_{\min}(P),\frac{1}{2}k_1\right\}s^2,v_1(s)=\max\left\{\lambda_{\max}(P),\frac{1}{2}k_2\right\}s^2,v_2(s)=\lambda_{\max}(G)s^2,$$

we have, for $t\ge0$, that

$$u\left(\|\bar{\mathscr{D}}\phi_t\|\right)\le V(t,\phi_t)\le v_1\left(\|\bar{\mathscr{D}}\phi_t\|\right)+v_2\left(\|\phi\|_{L_2}\right)$$

Thus, $V(t,\phi_t)$ satisfies the conditions of the Lyapunov theorem [28, Theorem 2 and 29, Theorem 3.2].

Now let us calculate the derivative of $V(t,\phi_t)$ along the trajectory of system (2.1)

$$\begin{aligned}
\dot{V}(t,\phi_t)\big|_{(1)} &= 2(x(t)-Dx(t-\tau))^T P(A(t)x(t)+B(t)x(t-\tau)+b(t)f(\sigma(t))) \\
&+x^T(t)2Gx(t)-x^T(t-\tau)Gx(t-\tau) \\
&+\left(c^T(t)x(t)-\rho(t)f(\sigma(t))\right)f(\sigma(t))-x^T(t)Gx(t) \\
&=2(x(t)-Dx(t-\tau))^T PA(t)x(t)+2(x(t)-Dx(t-\tau))^T PB(t)x(t-\tau) \\
&+2(x(t)-Dx(t-\tau))^T Pb(t)f(\sigma(t))+(x(t)-Dx(t-\tau))^T 2G(x(t)-Dx(t-\tau)) \\
&-x^T(t-\tau)D^T 2GDx(t-\tau)+4x^T(t)GDx(t-\tau)-x^T(t-\tau)Gx(t-\tau) \\
&+x^T(t)c(t)f(\sigma(t))-\rho(t)f^2(\sigma(t))-x^T(t)Gx(t) \\
&=2(x(t)-Dx(t-\tau))^T PA(t)(x(t)-Dx(t-\tau)) \\
&+2(x(t)-Dx(t-\tau))^T PA(t)Dx(t-\tau) \\
&+2(x(t)-Dx(t-\tau))^T PB(t)x(t-\tau)+2(x(t)-Dx(t-\tau))^T Pb(t)f(\sigma(t)) \\
&+(x(t)-Dx(t-\tau))^T 2G(x(t)-Dx(t-\tau))-x^T(t-\tau)D^T 2GDx(t-\tau) \\
&+4(x(t)-Dx(t-\tau))^T GDx(t-\tau)+4x^T(t-\tau)D^T GDx(t-\tau) \\
&-x^T(t-\tau)Gx(t-\tau)+(x(t)-Dx(t-\tau))^T c(t)f(\sigma(t)) \\
&+x^T(t-\tau)D^T c(t)f(\sigma(t))-\rho(t)f^2(\sigma(t))-x^T(t)Gx(t)
\end{aligned}$$

From the operator form $\mathscr{D}x_t = x(t)-Dx(t-\tau)$ and A2, it follows that

$$\begin{aligned}
\dot{V}(t,\phi_t)\big|_{(1)} &= (\mathscr{D}x_t)^T\left(PA(t)+A^T(t)P+2G\right)(\mathscr{D}x_t) \\
&+2(\mathscr{D}x_t)^T(PA(t)D+PB(t)+2GD)x(t-\tau) \\
&+2(\mathscr{D}x_t)^T\left(Pb(t)+\frac{1}{2}c(t)\right)f(\sigma(t))+x^T(t-\tau)\left(2D^T GD-G\right)x(t-\tau) \\
&+x^T(t-\tau)D^T c(t)f(\sigma(t))-\rho(t)f^2(\sigma(t))-x^T(t)Gx(t) \\
&=(\mathscr{D}x_t)^T\left(PA(t)+A^T(t)P+2G\right)(\mathscr{D}x_t) \\
&+2(\mathscr{D}x_t)^T(PA(t)D+PB(t)+2GD)x(t-\tau) \\
&+2(\mathscr{D}x_t)^T\left(Pb(t)+\frac{1}{2}c(t)\right)f(\sigma(t))-x^T(t-\tau)Rx(t-\tau) \\
&+x^T(t-\tau)D^T c(t)f(\sigma(t))-\rho(t)f^2(\sigma(t))-x^T(t)Gx(t).
\end{aligned}$$

Taking A2 and the property of matrix norm into account, we have

$$\begin{aligned}\dot{V}(t,\phi_t)\big|_{(1)} \leq & -\delta(t)\|\mathscr{D}x_t\|^2 + 2\|PA(t)D + PB(t) + 2GD\|\,\|\mathscr{D}x_t\|\,\|x(t-\tau)\| \\ & + 2\left\|Pb(t) + \frac{1}{2}c(t)\right\|\|\mathscr{D}x_t\|\,|f(\sigma(t))| - \lambda_{\min}(R)\|x(t-\tau)\|^2 \\ & + \left\|D^T c(t)\right\|\|x(t-\tau)\|\,|f(\sigma(t))| - \rho(t) f^2(\sigma(t)) - \lambda_{\min}(G)\|x(t)\|^2.\end{aligned}$$

In order to make full use of A3 and the unbounded terms in the coefficients of system (2.1), $\sqrt{\delta(t)}\|\mathscr{D}x_t\|$, $\sqrt{\lambda_{\min}(R)}\|x(t-\tau(t))\|$, $\sqrt{\rho(t)}|f(\sigma(t))|$ and $\sqrt{\lambda_{\min}(G)}\|x(t)\|$ are taken as the following variables of the quadratic form. To this end, further estimating the upper bound of $\dot{V}(t,\phi_t)\big|_{(1)}$ by A3 yields that

$$\begin{aligned}\dot{V}(t,\phi_t)\big|_{(1)} \leq & -\delta(t)\|\mathscr{D}x_t\|^2 \\ & + 2\frac{\|PA(t)D + PB(t) + 2GD\|}{\sqrt{\delta(t)\lambda_{\min}(R)}}\left[\sqrt{\delta(t)}\|\mathscr{D}x_t\|\right]\cdot\left[\sqrt{\lambda_{\min}(R)}\|x(t-\tau)\|\right] \\ & + 2\frac{\left\|Pb(t) + \frac{1}{2}c(t)\right\|}{\sqrt{\delta(t)\rho(t)}}\left[\sqrt{\delta(t)}\|\mathscr{D}x_t\|\right]\cdot\left[\sqrt{\rho(t)}f(\sigma(t))\right] - \lambda_{\min}(R)\|x(t-\tau)\|^2 \\ & + 2\frac{\left\|\frac{1}{2}D^T c(t)\right\|}{\sqrt{\lambda_{\min}(R)\rho(t)}}\left[\sqrt{\lambda_{\min}(R)}\|x(t-\tau)\|\right]\cdot\left[\sqrt{\rho(t)}f(\sigma(t))\right] \\ & - \rho(t) f^2(\sigma(t)) - \lambda_{\min}(G)\|x(t)\|^2 \\ \leq & -\delta(t)\|\mathscr{D}x_t\|^2 + 2\alpha\left[\sqrt{\delta(t)}\|\mathscr{D}x_t\|\right]\cdot\left[\sqrt{\lambda_{\min}(R)}\|x(t-\tau)\|\right] \\ & + 2\beta\left[\sqrt{\delta(t)}\|\mathscr{D}x_t\|\right]\cdot\left[\sqrt{\rho(t)}f(\sigma(t))\right] - \lambda_{\min}(R)\|x(t-\tau)\|^2 \\ & + 2\gamma\left[\sqrt{\lambda_{\min}(R)}\|x(t-\tau)\|\right]\cdot\left[\sqrt{\rho(t)}f(\sigma(t))\right] \\ & - \rho(t) f^2(\sigma(t)) - \lambda_{\min}(G)\|x(t)\|^2.\end{aligned}$$

Then, the right-hand side of the above inequality is re-written as

$$\dot{V}(t,\phi_t)\big|_{(1)} \leq \begin{bmatrix}\sqrt{\delta(t)}\|\mathscr{D}x_t\| \\ \sqrt{\lambda_{\min}(R)}\|x(t-\tau)\| \\ \sqrt{\rho(t)}|f(\sigma(t))| \\ \sqrt{\lambda_{\min}(G)}\|x(t)\|\end{bmatrix}^T M \begin{bmatrix}\sqrt{\delta(t)}\|\mathscr{D}x_t\| \\ \sqrt{\lambda_{\min}(R)}\|x(t-\tau)\| \\ \sqrt{\rho(t)}|f(\sigma(t))| \\ \sqrt{\lambda_{\min}(G)}\|x(t)\|\end{bmatrix},$$

where

$$M=\begin{bmatrix} -1 & \alpha & \beta & 0 \\ \alpha & -1 & \gamma & 0 \\ \beta & \gamma & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

Let M_i stand for the leading principal minors of order i. For matrix M, there are four leading principal minors which are listed in the following:

$$M_1 = -1,$$

$$M_2 = 1-\alpha^2,$$

$$M_3 = -1+2\alpha\beta\gamma+\left(\alpha^2+\beta^2+\gamma^2\right),$$

$$M_4 = 1-2\alpha\beta\gamma-\left(\alpha^2+\beta^2+\gamma^2\right).$$

Due to $\alpha,\beta,\gamma \geq 0$ and condition (2.2), we obtain $M_1<0, M_2>0, M_3<0, M_4>0$. According to the matrix theory, it can be concluded that matrix M is negative definite. Then, let $-\mu\,(\mu>0)$ denotes the largest eigenvalue of M, and we obtain

$$\begin{aligned}\dot{V}(t,\phi_t)\big|_{(1)} &\leq -\mu\left[\delta\|\mathscr{D}x_t\|^2+\rho|f(\sigma(t))|^2+\lambda_{\min}(G)\|x(t)\|^2\right] \\ &\leq -\mu\left[\delta\|\mathscr{D}x_t\|^2+\rho k_1^2\sigma^2(t)+\lambda_{\min}(G)\|x(t)\|^2\right] \\ &\leq -\mu\left[\min\left(\delta,\frac{1}{2}\rho k_1^2\right)\left\|\begin{bmatrix}\mathscr{D}x_t \\ \sigma(t)\end{bmatrix}\right\|^2+\min\left(\frac{1}{2}\rho k_1^2,\lambda_{\min}(G)\right)\left\|\begin{bmatrix}x(t) \\ \sigma(t)\end{bmatrix}\right\|^2\right].\end{aligned}$$

Accordingly, if we let $w_1(s)=\mu\min\left(\delta,\frac{1}{2}\rho k_1^2\right)s^2$, $w_2(s)=\mu\min\left(\frac{1}{2}\rho k_1^2,\lambda_{\min}(G)\right)s^2$, then we obtain the following

$$\dot{V}(t,\phi_t)\big|_{(1)} \leq -\left[w_1\left(\left\|\bar{\mathscr{D}}\phi_t\right\|\right)+w_2\left(\|\phi_t(0)\|\right)\right].$$

Thus system (2.1) is absolutely stable by Lyapunov theorem [28, Theorem 2 and 29, Theorem 3.2]. This completes the proof.

The following corollaries are more applicable, although the required conditions are slightly stronger.

Corollary 2.4. *Under A1-A3, system (2.1) is absolutely stable if*

$$\alpha+\beta<1, \alpha+\gamma<1, \beta+\gamma<1 \tag{2.3}$$

Proof. From (2.3), we see that

$$\alpha+\beta(\beta+\gamma)<\alpha+\beta<1, \alpha+\gamma(\beta+\gamma)<\alpha+\gamma<1,$$

that is, $\alpha+\beta^2+\beta\gamma<1, \alpha+\gamma^2+\beta\gamma<1$. Accordingly, one has $\alpha+\beta\gamma<1-\beta^2, \alpha+\beta\gamma<1-\gamma^2$. In the light of $\alpha,\beta,\gamma\geq 0$, we obtain $(\alpha+\beta\gamma)^2<\left(1-\beta^2\right)\left(1-\gamma^2\right)$. Simplifying the above inequality yields that $\alpha^2+\beta^2+\gamma^2+2\alpha\beta\gamma<1$. This suggests that condition (2.2) in Theorem 2.1 holds if (2.3) are satisfied. Thus Corollary 2.4 is valid by Theorem 2.1.

The following Corollary can be easily derived by Corollary 2.4.

Corollary 2.5. *Under A1-A3, system (2.1) is absolutely stable if* $\alpha+\beta+\gamma<1$.

In fact, three inequalities required in Corollary 2.4 are all satisfied if $\alpha+\beta+\gamma<1$. Hence Corollary 2.5 holds.

Note that the asymptotical stability focuses on the behavior of dynamical systems as time tends to infinity. When studying the absolute stability of system (2.1), we only need to ensure that the above conditions are satisfied as time tends to infinity. Therefore, the aforementioned "$t\in[0,\infty)$" in A2 and A3 can be replaced by "$t\in[T,\infty), T\geq 0$". Furthermore, A3 can be rewritten as a new form of the upper limit, i.e., A4.

A4: Assume

$$\overline{\lim_{t\to\infty}}\frac{\|PA(t)D+PB(t)+2GD\|}{\sqrt{\delta(t)\lambda_{\min}(R)}}=\bar{\alpha}, \overline{\lim_{t\to\infty}}\frac{\left\|Pb(t)+\frac{1}{2}c(t)\right\|}{\sqrt{\delta(t)\rho(t)}}=\bar{\beta}, \overline{\lim_{t\to\infty}}\frac{\left\|\frac{1}{2}D^Tc(t)\right\|}{\sqrt{\lambda_{\min}(R)\rho(t)}}=\bar{\gamma},$$

where $\bar{\alpha},\bar{\beta},\bar{\gamma}$ are constants.

Corollary 2.6. *Under A1, A2 and A4, system (2.1) is absolutely stable if*

$$\bar{\alpha}^2+\bar{\beta}^2+\bar{\gamma}^2+2\bar{\alpha}\bar{\beta}\bar{\gamma}<1.$$

Proof. Define a continuous function as

$$\psi(\varepsilon)=(\bar{\alpha}+\varepsilon)^2+\left(\bar{\beta}+\varepsilon\right)^2+(\bar{\gamma}+\varepsilon)^2+2(\bar{\alpha}+\varepsilon)\left(\bar{\beta}+\varepsilon\right)(\bar{\gamma}+\varepsilon).$$

Since this function is nonnegative continuous and $\psi(0)=\bar{\alpha}^2+\bar{\beta}^2+\bar{\gamma}^2+2\bar{\alpha}\bar{\beta}\bar{\gamma}<1$, we find $\varepsilon>0$ such that $\psi(\varepsilon)<1$. By the property of the upper limit, if A4 is satisfied, for the already found ε, there exists $T(T\geq 0)$ such that

$$\frac{\|PA(t)D+PB(t)+2GD\|}{\sqrt{\delta(t)\lambda_{\min}(R)}}\leq\bar{\alpha}+\varepsilon, \frac{\left\|Pb(t)+\frac{1}{2}c(t)\right\|}{\sqrt{\delta(t)\rho(t)}}\leq\bar{\beta}+\varepsilon, \frac{\left\|\frac{1}{2}D^Tc(t)\right\|}{\sqrt{\rho(t)\lambda_{\min}(R)}}\leq\bar{\gamma}+\varepsilon,$$

for $t > T$. Letting

$$\alpha = \bar{\alpha} + \varepsilon, \beta = \bar{\beta} + \varepsilon, \gamma = \bar{\gamma} + \varepsilon,$$

we find that $\psi(\varepsilon) < 1$ is exactly required in Theorem 2.1. This completes the proof.

Remark 2.7. Define $\delta = 1 - \left(\bar{\alpha}^2 + \bar{\beta}^2 + \bar{\gamma}^2 + 2\bar{\alpha}\bar{\beta}\bar{\gamma}\right) = 1 - \psi(0)$. As to ensure $\psi(\varepsilon) < 1$, the chosen ε should satisfy $\varepsilon < 1$. Additionally, we notice that $0 \le \bar{\alpha}, \bar{\beta}, \bar{\gamma} < 1$. By means of the Lagrange Mean Value Theorem, we have $\psi(\varepsilon) = \psi(0) + \psi'(\theta\varepsilon)\varepsilon\,(0 < \theta < 1)$. Note that

$$\begin{aligned}\psi'(\theta\varepsilon) &= 2(\bar{\alpha} + \theta\varepsilon) + 2\left(\bar{\beta} + \theta\varepsilon\right) + 2(\bar{\gamma} + \theta\varepsilon) \\ &\quad + 2\left[\left(\bar{\beta} + \theta\varepsilon\right)(\bar{\gamma} + \theta\varepsilon) + (\bar{\alpha} + \theta\varepsilon)(\bar{\gamma} + \theta\varepsilon) + (\bar{\alpha} + \theta\varepsilon)\left(\bar{\beta} + \theta\varepsilon\right)\right].\end{aligned}$$

Since all the terms of $\psi'(\theta\varepsilon)$ and its coefficients are positive, one sees that

$$\psi'(\theta\varepsilon) < \left[\psi'(\theta\varepsilon)\right]_{\bar{\alpha}=\bar{\beta}=\bar{\gamma}=\theta=\varepsilon=1} = 36.$$

Hence, $\psi(\varepsilon) < \psi(0) + 36\varepsilon$. Letting $\varepsilon = \frac{\delta}{36} = \frac{1-\psi(0)}{36}$, we find that $\psi(\varepsilon) < 1$ holds.

Corollary 2.8. Under A1, A2 and A4, system (2.1) is absolutely stable if

$$\bar{\alpha} + \bar{\beta} < 1, \bar{\alpha} + \bar{\gamma} < 1, \bar{\beta} + \bar{\gamma} < 1$$

Corollary 2.9. Under A1, A2 and A4, system (2.1) is absolutely stable if

$$\bar{\alpha} + \bar{\beta} + \bar{\gamma} < 1$$

Remark 2.10. In order to guarantee the stability of operator $\bar{\mathscr{D}}$, only the coefficient matrix D is required to satisfy A1 (hence norm-bounded), while the other coefficients can be norm-unbounded.

Remark 2.11. If $D = 0$, then system (2.1) reduces to the following retarded Lurie indirect control system:

$$\begin{cases} \dot{x}(t) &= A(t)x(t) + B(t)x(t-\tau) + b(t)f(\sigma(t)), \\ \dot{\sigma}(t) &= c^T(t)x(t) - \rho(t)f(\sigma(t)), \\ x(t) &= \varphi(t), t \in [-\tau, 0]. \end{cases}$$

It is worth pointing out that, in this case, the results proposed in this paper are consistent with those in [31]. Thus this work extends the corresponding criteria in [31]. In addition, the absolute stability problem of Lur'e indirect control systems was also considered in [32]. But the study is only applicable to the systems with constant coefficients, and it can not deal with the delay of neutral type. Therefore, the results in this paper have a greater range of applications.

In the following, the above results are extended to multiple nonlinearities.

3. Absolute stability of neutral Lurie systems with multiple nonlinearities

Consider the following time-varying neutral Lurie indirect control system with multiple nonlinearities

$$\begin{cases} \dot{x}(t) - D\dot{x}(t-\tau) = A(t)x(t) + B(t)x(t-\tau) + \sum\limits_{i=1}^{m} b_i(t) f_i(\sigma_i(t)), \\ \dot{\sigma}_i(t) = c_i^T(t)x(t) - \rho_i(t) f_i(\sigma_i(t)) \ \ (i = 1,2,\cdots,m), \\ x(t) = \varphi(t), t \in [-\tau, 0], \end{cases} \tag{3.1}$$

where $x(t) \in R^n$, $\sigma_i(t) \in R\,(i = 1,2,\cdots,m)$, $b_i(t), c_i(t)\,(i = 1,2,\cdots,m)$ are n dimensional column vectors, $A(t), B(t), D$ are $n \times n$ matrices, $\tau > 0$ is a constant time delay, $\rho_i(t) \geq \rho_i > 0$, ρ_i is a constant, $A(t), B(t), b_i(t), c_i(t), \rho_i(t)$ are continuous in $[0,\infty)$, $f_i(\cdot)(i = 1,2,\cdots,m)$ are nonlinear continuous functions satisfying the following sector condition:

$$F_{[k_{i1},k_{i2}]} = \left\{ f_i(\cdot) \,|\, f_i(0) = 0; k_{i1}\sigma_i^2(t) \leq \sigma_i(t) f_i(\sigma_i(t)) \leq k_{i2}\sigma_i^2(t), \sigma_i(t) \neq 0 \right\},$$

where k_{i1}, k_{i2} are constants and $k_{i2} > k_{i1} > 0$.

For system (3.1), we introduce operator $\bar{\mathscr{D}} : C([-\tau,0], R^{n+m}) \to R^{n+m}$ as

$$\bar{\mathscr{D}}\phi_t = \phi_t(0) - \bar{D}\phi_t(-\tau), \bar{D} = \begin{bmatrix} D & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}.$$

For simplicity, let us decompose $\bar{\mathscr{D}}$:

$$\bar{\mathscr{D}}\phi_t = \begin{bmatrix} \mathscr{D}x_t \\ \sigma_1(t) \\ \vdots \\ \sigma_m(t) \end{bmatrix} = \begin{bmatrix} x(t) - Dx(t-\tau) \\ \sigma_1(t) \\ \vdots \\ \sigma_m(t) \end{bmatrix}.$$

Definition 3.1. [30] System (3.1) is said to be absolutely stable if its zero solution is globally asymptotically stable for any nonlinearities $f_i(\cdot) \in F_{[k_{i1},k_{i2}]}\,(i = 1,2,\cdots,m)$.

Besides A1 and A2, the following assumptions are needed for system (3.1).

A5: For any $t \in [0,\infty)$,

$$\frac{\|PA(t)D+PB(t)+2GD\|}{\sqrt{\delta(t)\lambda_{\min}(R)}} \le \alpha, \frac{\left\|Pb_i(t)+\frac{1}{2}c_i(t)\right\|}{\sqrt{\delta(t)\rho_i(t)}} \le \beta_i, \frac{\left\|\frac{1}{2}D^T c_i(t)\right\|}{\sqrt{\lambda_{\min}(R)\rho_i(t)}} \le \gamma_i,$$

where $\alpha,\beta_i,\gamma_i\,(i=1,2,\cdots,m)$ are constants.

Theorem 3.2. *Under A1, A2 and A5, system (3.1) is absolutely stable if*

$$\sum_{i=1}^{m} {\beta_i}^2 < 1, \tag{3.2}$$

$$\left(\alpha+\sum_{i=1}^{m}\beta_i\gamma_i\right)^2 < \left(-1+\sum_{i=1}^{m}\beta_i^2\right)\left(-1+\sum_{i=1}^{m}\gamma_i^2\right). \tag{3.3}$$

Proof. A Lyapunov-Krasovskii candidate is constructed as

$$V(t,\phi_t) = (x(t)-Dx(t-\tau))^T P(x(t)-Dx(t-\tau)) + \int_{t-\tau}^{t} x^T(s)\,Gx(s)ds + \sum_{i=1}^{m}\int_0^{\sigma_i(t)} f_i(s)ds.$$

From Theorem 2.3, we can obtain the desired conclusion immediately.

In particular, if $m=1$, Theorem 3.2 is reduced to Theorem 2.3.

Corollary 3.3. Under A1, A2 and A5, system (3.1) is absolutely stable if

$$\alpha+\sum_{i=1}^{m}\beta_i<1, \alpha+\sum_{i=1}^{m}\gamma_i<1, \beta_i+\gamma_i<1, \quad (i=1,2,\cdots,m). \tag{3.4}$$

Proof. It is noted that condition (3.4) means $\sum\limits_{i=1}^{m}\beta_i^2 < \sum\limits_{i=1}^{m}\beta_i < 1$. Hence, condition (3.2) in Theorem 3.1 is satisfied. In addition, under the condition (7), one has

$$\alpha+\sum_{i=1}^{m}\beta_i(\beta_i+\gamma_i) < \alpha+\sum_{i=1}^{m}\beta_i < 1, \alpha+\sum_{i=1}^{m}\gamma_i(\beta_i+\gamma_i) < \alpha+\sum_{i=1}^{m}\gamma_i < 1,$$

that is,

$$\alpha+\sum_{i=1}^{m}\beta_i^2+\sum_{i=1}^{m}\beta_i\gamma_i<1, \alpha+\sum_{i=1}^{m}\gamma_i^2+\sum_{i=1}^{m}\beta_i\gamma_i<1.$$

Namely,

$$\alpha+\sum_{i=1}^{m}\beta_i\gamma_i < 1-\sum_{i=1}^{m}\beta_i^2, \alpha+\sum_{i=1}^{m}\beta_i\gamma_i < 1-\sum_{i=1}^{m}\gamma_i^2.$$

In view of the non-negativeness of α,β_i,γ_i, one has

$$\left(\alpha+\sum_{i=1}^{m}\beta_i\gamma_i\right)^2 < \left(1-\sum_{i=1}^{m}\beta_i^2\right)\left(1-\sum_{i=1}^{m}\gamma_i^2\right).$$

This implies that condition (3.3) in Theorem 3.1 is satisfied. Hence, the conditions (3.2) and (3.3) of Theorem 3.1 are satisfied if the condition (3.4) holds. The result then follows immediately from Theorem 3.1.

The following results are easily derived by Corollary 3.3.

Corollary 3.4. Under A1, A2 and A5, system (3.1) is absolutely stable if

$$\alpha+\sum_{i=1}^{m}\beta_i+\sum_{i=1}^{m}\gamma_i<1.$$

A6: Assume that

$$\overline{\lim_{t\to\infty}}\frac{\|PA(t)D+PB(t)+2GD\|}{\sqrt{\delta(t)\lambda_{\min}(R)}}=\bar{\alpha},$$

$$\overline{\lim_{t\to\infty}}\frac{\left\|Pb_i(t)+\frac{1}{2}c_i(t)\right\|}{\sqrt{\delta(t)\rho_i(t)}}=\bar{\beta}_i,$$

$$\overline{\lim_{t\to\infty}}\frac{\left\|\frac{1}{2}D^Tc_i(t)\right\|}{\sqrt{\lambda_{\min}(R)\rho_i(t)}}=\bar{\gamma}_i,$$

where $\bar{\alpha},\bar{\beta}_i,\bar{\gamma}_i\,(i=1,2,\cdots,m)$ are constants.

Corollary 3.5. Under A1, A2 and A6, system (3.1) is absolutely stable if

$$\sum_{i=1}^{m}\bar{\beta}_i^2<1,\left(\bar{\alpha}+\sum_{i=1}^{m}\bar{\beta}_i\bar{\gamma}_i\right)^2<\left(-1+\sum_{i=1}^{m}\bar{\beta}_i^2\right)\left(-1+\sum_{i=1}^{m}\bar{\gamma}_i^2\right).$$

Corollary 3.6. Under A1, A2 and A6, system (3.1) is absolutely stable if

$$\bar{\alpha}+\sum_{i=1}^{m}\bar{\beta}_i<1,\bar{\alpha}+\sum_{i=1}^{m}\bar{\gamma}_i<1,\bar{\beta}_i+\bar{\gamma}_i<1\,(i=1,2,\cdots,m).$$

The following results is easily produced by Corollary 3.6.

Corollary 3.7. Under A1, A2 and A6, system (3.1) is absolutely stable if

$$\bar{\alpha}+\sum_{i=1}^{m}\bar{\beta}_i+\sum_{i=1}^{m}\bar{\gamma}_i<1.$$

4. Numerical Simulation

In this section, two examples are presented to show the effectiveness of the proposed criteria.

Example 4.1. Consider the following time-varying neutral Lurie indirect control systems with single nonlinearity

$$\begin{cases} \dot{x}(t) - \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix} \dot{x}(t-4) = \begin{bmatrix} -4t-1 & 1 \\ t & -3t-1 \end{bmatrix} x(t) \\ \qquad\qquad + \begin{bmatrix} 0.4t & 1 \\ -0.1t & 0.6t \end{bmatrix} x(t-4) + \begin{bmatrix} -2 \\ -\sqrt{t} \end{bmatrix} f(\sigma(t)), \\ \dot{\sigma}(t) = \begin{bmatrix} 4 \\ 2\sqrt{t} \end{bmatrix}^T x(t) - \left(t^2+1\right) f(\sigma(t)),\ f(\cdot) \in F_{[0.01,100]}. \end{cases} \tag{4.1}$$

Compared with system (2.1), the corresponding coefficient matrices are as follows:

$$D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, A(t) = \begin{bmatrix} -4t-1 & 1 \\ t & -3t-1 \end{bmatrix}, B(t) = \begin{bmatrix} 0.4t & 1 \\ -0.1t & 0.6t \end{bmatrix},$$

$$b(t) = \begin{bmatrix} -2 \\ -\sqrt{t} \end{bmatrix}, c(t) = \begin{bmatrix} 4 \\ 2\sqrt{t} \end{bmatrix}, \rho(t) = t^2+1.$$

Next, we prove the absolute stability of this system by Corollary 2.9.

Obviously, the eigenvalues of D are 0.1 and 0.2, which are all inside the open unit circle. Hence A1 is satisfied. Letting $P = G = I$, we get

$$PA(t) + A^T(t)P + 2G = \begin{bmatrix} -8t & t+1 \\ t+1 & -6t \end{bmatrix}.$$

It is easy to calculate

$$\lambda_{\max}\left(PA(t) + A^T(t)P + 2G\right) = -7t + \sqrt{t^2 + (t+1)^2} = -\delta(t),$$

where $\delta(t) = 7t - \sqrt{2t^2+2t+1}$. It is clear that $\lim_{t\to\infty} \delta(t) = \infty$. Letting $\delta = 1$, we find that there exists T such that $\lambda\left(PA(t) + A^T(t)P + 2G\right) < -\delta$, whenever $t > T$. Note that $2D^TGD - G = -R$, where

$$R = \begin{bmatrix} 0.98 & 0 \\ 0 & 0.92 \end{bmatrix}.$$

Thus A2 is satisfied. Moreover,

$$\lim_{t\to\infty} \frac{\|PA(t)D + PB(t) + 2GD\|}{\sqrt{\delta(t)\lambda_{\min}(R)}} = 0, \lim_{t\to\infty} \frac{\left\|Pb(t) + \frac{1}{2}c(t)\right\|}{\sqrt{\delta(t)\rho(t)}} = 0, \lim_{t\to\infty} \frac{\left\|\frac{1}{2}D^T c(t)\right\|}{\sqrt{\lambda_{\min}(R)\rho(t)}} = 0.$$

Note that the upper limit exists and it is equal to the limit value here. Thus A4 is satisfied with $\bar{\alpha} = \bar{\beta} = \bar{\gamma} = 0$. Clearly, $\bar{\alpha} + \bar{\beta} + \bar{\gamma} = 0 < 1$, that is, all the conditions of Corollary 5 can be verified, which means system (9) is absolutely stable. Take $f(\sigma(t)) = 2\sigma(t) + \sin\sigma(t)$ and the initial condition $\begin{bmatrix} x_1(t) & x_2(t) & \sigma(0) \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T, t \in [-4, 0]$. The simulation result is displayed in Figure 1.

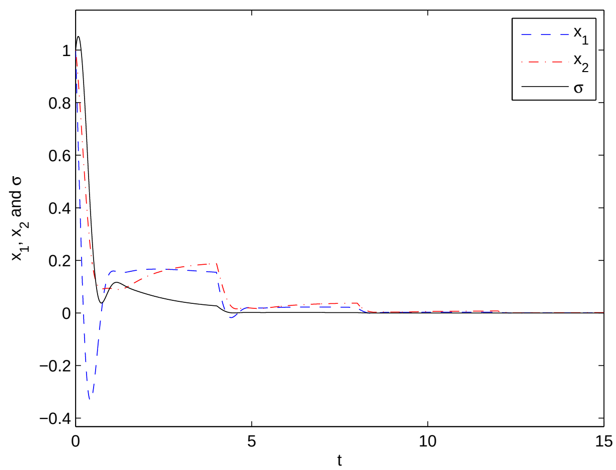

FIGURE 1. The state response of system (4.1)

As we can see clearly in Figure 1, the zero solution of system (4.1) is asymptotically stable. Simulating with different $f(\sigma)$ yields that the zero solution of system (4.1) is always asymptotically stable for any initial condition as long as $f(\cdot) \in F_{[0.01,100]}$. Hence, system (4.1) is absolutely stable. Therefore, the absolute stability criteria proposed in this paper are effective for neutral Lurie systems with unbounded coefficients.

It is worth pointing out that, for this example, only the coefficient matrix D is norm-bounded, while the coefficients $A(t), B(t), b(t), c(t), \rho(t)$ are norm-unbounded. This is a main feature of this paper. All the theorems and corollaries are applicable to the systems with unbounded coefficients.

Next, an example with multiple nonlinearities is presented.

Example 4.2. Consider the following time-varying neutral Lurie indirect control system with two nonlinearities

$$\left\{\begin{aligned} &\dot{x}(t)-\begin{bmatrix}0.1 & 0\\ 0 & 0.2\end{bmatrix}\dot{x}(t-4)=\begin{bmatrix}-4t-1 & 0\\ -1 & -3t-1\end{bmatrix}x(t)\\ &\qquad\qquad+\begin{bmatrix}0.4t & 0.2\\ 0 & 0.6t\end{bmatrix}x(t-4)+\begin{bmatrix}-2\\ -0.1\sqrt{t}\end{bmatrix}f_1(\sigma_1(t))+\begin{bmatrix}-\sqrt{t}\\ 1\end{bmatrix}f_2(\sigma_2(t))\\ &\dot{\sigma}_1(t)=\begin{bmatrix}0\\ 0.2\sqrt{t}\end{bmatrix}^T x(t)-0.09tf_1(\sigma_1(t))\\ &\dot{\sigma}_2(t)=\begin{bmatrix}2\sqrt{t}\\ 1\end{bmatrix}^T x(t)-0.09tf_2(\sigma_2(t)),f_i(\cdot)\in F_{[0.01,100]}\ (i=1,2).\end{aligned}\right. \tag{4.2}$$

It follows that

$$D=\begin{bmatrix}0.1 & 0\\ 0 & 0.2\end{bmatrix},A(t)=\begin{bmatrix}-4t-1 & 0\\ -1 & -3t-1\end{bmatrix},B(t)=\begin{bmatrix}0.4t & 0.2\\ 0 & 0.6t\end{bmatrix},b_1(t)=\begin{bmatrix}-2\\ -0.1\sqrt{t}\end{bmatrix},$$

$$b_2(t)=\begin{bmatrix}-\sqrt{t}\\ 1\end{bmatrix},c_1(t)=\begin{bmatrix}0\\ 0.2\sqrt{t}\end{bmatrix},c_2(t)=\begin{bmatrix}2\sqrt{t}\\ 1\end{bmatrix},\rho_1(t)=\rho_2(t)=0.09t.$$

Next, we test the absolute stability of system (4.2) by using the proposed criteria of this paper.

Obviously, D has two eigenvalues 0.1 and 0.2, which are all inside the open unit circle. Thus A1 is satisfied. By letting $P=G=I$, it is not difficult to compute that

$$PA(t)+A^T(t)P+2G=\begin{bmatrix}-8t & -1\\ -1 & -6t\end{bmatrix}$$

and

$$\lambda\left(PA(t)+A^T(t)P+2G\right)\leq -7t+\sqrt{t^2+1}.$$

Furthermore, if $t>T=1$, we have

$$\lambda\left(PA(t)+A^T(t)P+2G\right)\leq -\left(7-\sqrt{2}\right)t<-\left(7-\sqrt{2}\right).$$

Note that there exists $R = \begin{bmatrix} 0.98 & 0 \\ 0 & 0.92 \end{bmatrix}$ such that $2D^T GD - G = -R$, thus, A2 is satisfied with $\delta(t) = \left(7-\sqrt{2}\right)t, \delta = 7-\sqrt{2}$. In addition, one can derive that

$$\lim_{t\to\infty} \frac{\|PA(t)D + PB(t) + 2GD\|}{\sqrt{\delta(t)\lambda_{\min}(R)}} = 0,$$

$$\lim_{t\to\infty} \frac{\left\|Pb_1(t) + \frac{1}{2}c_1(t)\right\|}{\sqrt{\delta(t)\rho_1(t)}} = 0, \lim_{t\to\infty} \frac{\left\|Pb_2(t) + \frac{1}{2}c_2(t)\right\|}{\sqrt{\delta(t)\rho_2(t)}} = 0,$$

$$\lim_{t\to\infty} \frac{\left\|\frac{1}{2}D^T c_1(t)\right\|}{\sqrt{\lambda_{\min}(R)\rho_1(t)}} = \frac{1}{\sqrt{207}}, \lim_{t\to\infty} \frac{\left\|\frac{1}{2}D^T c_2(t)\right\|}{\sqrt{\lambda_{\min}(R)\rho_2(t)}} = \frac{5}{3\sqrt{23}}.$$

Let us recall the fact that if a limit value exists, then it is equal to the upper limit value. Thus, A6 is satisfied with $\bar{\alpha} = \bar{\beta}_1 = \bar{\beta}_2 = 0, \bar{\gamma}_1 = \frac{1}{\sqrt{207}}, \bar{\gamma}_2 = \frac{5}{3\sqrt{23}}$.

Finally, $\bar{\alpha} + \bar{\beta}_1 + \bar{\beta}_2 + \bar{\gamma}_1 + \bar{\gamma}_2 < 1$, which means the conditions of Corollary 3.7 are satisfied. Consequently, we claim that system (4.2) is absolutely stable. Let

$$f_1(\sigma(t)) = 2\sigma(t) + \sin\sigma(t), f_2(\sigma(t)) = \begin{cases} \sigma(t), & |\sigma(t)| < 1, \\ \sigma^3(t), & 1 \le |\sigma(t)| \le 2, \\ 4\sigma(t), & |\sigma(t)| > 2, \end{cases}$$

and suppose $\begin{bmatrix} x_1(t) & x_2(t) & \sigma_1(0) & \sigma_2(0) \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T, t \in [-4, 0]$. The simulation result is shown in Figure 2.

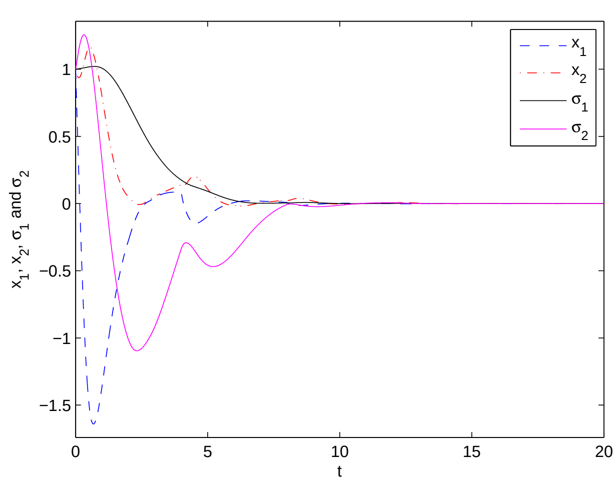

FIGURE 2. The state response of system (4.2)

According to Figure 2, one can see that the state trajectories of system (4.2) approach zero asymptotically, that is, system (4.2) is asymptotically stable. The numerical simulation shows the effectiveness and correctness of the theoretical results.

5. Conclusion

In this paper, the absolute stability of time-varying neutral Lurie indirect control systems with single nonlinearity and multiple nonlinearities has been investigated. Based on the Lyapunov stability theory, two theorems and several corollaries have been obtained. The results in this paper are particularly applicable to neutral Lurie indirect control systems with unbounded coefficients, and the time delay itself can be very large. The effectiveness of the proposed methods was illustrated via numerical examples.

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