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RANDOM FIXED POINTS OF MULTIFUNCTIONS ON METRIC SPACES

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Abstract. Sufficient conditions for the existence of random fixed points of Suzuki type random multifunctions and hemiconvex multifunctions are obtained. Our results generalize some known results in the literature.

Keywords. Random fixed point; Contraction multifunction; Hemiconvex multifunction.

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1. Introduction and preliminaries

Random fixed point theory is a stochastic generalization of classical fixed point theory for deterministic mappings. Recently, several authors have investigated the existence and applications of random fixed points; see [2, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17]) and the references therein. In this article, we obtain random fixed point theorems for generalized contractive Suzuki type multifunctions and hemiconvex multifunctions. In particular, the results presented in this paper improve and extend the results of Xu and Beg [19, Theorems 2.3 and 3.1] and Benavides, Acedo and H.K. Xu [9]. Throughout this paper, we assume that (X,d) is a metric space, M a subset of X and (Ω,Σ) is a measurable space, where Σ is a sigma-algebra of subsets of the nonempty set Ω. We denote the family of all nonempty subsets of X by 2^X and the family of all closed and bounded subsets of X by $CB(X)$. Also, we denote the Hausdorff metric on $CB(X)$

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by H, i.e.

$$H(A,B) = \max\{\sup_{a\in A} d(a,B), \sup_{b\in B} d(b,A)\} \tag{1.1}$$

for all $A,B \in CB(X)$, where $d(x,A) = \inf_{a\in A} \|x-a\|$. Let $T : M \longrightarrow 2^X$ be a multifunction. The multifunction T is called upper semi-continuous (lower semi-continuous) whenever

$$\{x \in M : T(x) \subset A\} \tag{1.2}$$

($\{x \in M : T(x) \bigcap A \neq \emptyset\}$) is an open subset of M for every open subset A of X. A multifunction $T : \Omega \longrightarrow 2^X$ is called Σ-measurable if $\{x \in \Omega : T(x) \bigcap A \neq \emptyset\} \in \Sigma$ for every open subset A of X. A measurable function $f : \Omega \longrightarrow X$ is called a measurable selector of the measurable multifunction $T : \Omega \longrightarrow 2^X$ whenever $f(\omega) \in T(\omega)$ for all $\omega \in \Omega$. Also, the multifunction $T : \Omega \times M \longrightarrow 2^X$ is called a random multifunction whenever for each fixed $x \in M$, the multifunction $T(.,x) : \Omega \longrightarrow 2^X$ is measurable. An element $x \in M$ is called a deterministic fixed point of random multifunction $T : \Omega \longrightarrow 2^M$ whenever $x \in T(\omega,x)$ for all $\omega \in \Omega$. A measurable function $f : \Omega \longrightarrow M$ is said to be a random fixed point of a random multifunction $T : \Omega \times M \longrightarrow 2^M$ if $f(\omega) \in T(\omega, f(\omega))$ for all $\omega \in \Omega$. When X is a Banach space, a multifunction $T : M \longrightarrow CB(M)$ is said to be demiclosed at $\mathbf{0}$ if the conditions $x_n \in M$ for all $n \geq 1$, $x_n \stackrel{w}{\to} x$, $y_n \in Tx_n$ and $y_n \to \mathbf{0}$ imply that $\mathbf{0} \in Tx$. Also, we say that multifunction $T : M \longrightarrow 2^X$ has the property (D) if for every closed ball B in M with radius $r > 0$ and any sequence $\{x_n\}$ in M for which $d(x_n,B) \to 0$ and $d(x_n,Tx_n) \to 0$, there exists $x_0 \in B$ such that $x_0 \in Tx_0$. We appeal the following results in the sequel.

Proposition 1.1. [4, p.569] *Let $T : X \longrightarrow 2^Y$ be a lower semi continuous between topological space and let the function $f : GrT \longrightarrow \mathbb{R}$ be lower semi continuous. Define the extended real function $m : X \longrightarrow \mathbb{R}$ by*

$$m(x) = \sup_{y\in T(x)} f(x,y). \tag{1.3}$$

Then function m is lower semi continuous.

Corollary 1.2. *Let $T : X \longrightarrow 2^Y$ be a lower semi continuous between Banach spaces X,Y. Then $f : X \longrightarrow \mathbb{R}$ defined by*

$$f(x) = d(x,T(x)). \tag{1.4}$$

is upper-semi continuous.

Proof. Note that $-d(x,y) : GrT \longrightarrow \mathbb{R}$ is lower semi continuous and

$$f(x) = \inf_{y \in T(x)} d(x,y) = -\sup_{y \in T(x)} -d(x,y).$$

Using the Proposition 1.1, we find that f is upper semi continuous.

Proposition 1.3. [1, p. 191] *Let M be a nonempty subset of a metric space (X,d). Suppose that mapping $T : M \longrightarrow CB(M)$ is upper semi continuous. Then mapping $f(x) = d(x,Tx)$ is lower semi continuous.*

Corollary 1.4. *Let $T : X \longrightarrow 2^Y$ be continuous between Banach spaces X,Y. Then $f : X \longrightarrow \mathbb{R}$ defined by $f(x) = d(x,T(x))$ is continuous.*

Proof. In view of (1.2) and (1.3), we find the desired conclusion immediately.

Proposition 1.5. [18] *Let M be a separable metric space, (Ω,Σ) be a measurable space and $f : \Omega \times M \longrightarrow \mathbb{R}$ be a Caratheodory map, i.e., for every $x \in M$, map $f(.,x) : \Omega \longrightarrow \mathbb{R}$ is measurable and for every $\omega \in \Omega$ map $f(\omega,.) : M \longrightarrow \mathbb{R}$ is continuous. Then, for each $s > 0$, operator $G_s : \Omega \longrightarrow M$ given by*

$$G_s(\omega) = \{x \in M : f(\omega,x) < s\} \ (\omega \in \Omega) \tag{1.5}$$

is measurable.

Corollary 1.6. *Let M be a separable metric space , (Ω,Σ) be a measurable space and $T : \Omega \times M \longrightarrow CB(M)$ be a continuous random multifunction. Then $f(\omega,x) = d(x,T(\omega,x))$ is Carathedory map.*

Proof. By Corollary 1.4 and the Lemma 18.5 of [4], we have that f is Carathedory map.

Corollary 1.7. *Let M be a separable metric space, (Ω,Σ) be a measurable space and $T : \Omega \times M \longrightarrow CB(M)$ be a random continuous multifunction. Then, for each $s > 0$, operator $G_s : \Omega \longrightarrow M$ given by*

$$G_s(\omega) = \{x \in M : d(x,T(\omega,x)) < s\} \ (\omega \in \Omega) \tag{1.6}$$

is measurable and so is operator $clG(\omega)$ of the closure of $G(\omega)$.

Proof. Note that T is a random continuous multifunction. Using Corollary 1.6, we see that $f(\omega,x) : \Omega \times M \longrightarrow \mathbb{R}$ with $f(\omega,x) = d(x,T(\omega,x))$, is a Caratheodory map. Using Proposition 1.5, G_s is measurable and so is the operator $clG(\omega)$ of the closure of $G(\omega)$ (Lemma 18.3 [4]).

Proposition 1.8. [4] *If $T : \Omega \longrightarrow 2^X$ is a closed valued map, then T has a measurable selector.*

Proposition 1.9. [4, p. 561] *Let Y be a regular topological space. If $T : X \longrightarrow 2^Y$ is upper-semi continuous and closed-valued, then T is closed.*

Proposition 1.10. [19] *Assume $F(\omega)$ is closed-value mapping and $\{F_n(\omega)\}$ is a sequence of measurable mappings. If*

$$\lim_{n\to\infty} H(\overline{F_n(\omega)}, F(\omega)) = 0 \tag{1.7}$$

for all $\omega \in \Omega$, then F is measurable.

2. Random fixed points of the Suzuki-contraction

Now, we are ready to state and prove our main results.

Theorem 2.1. [8] *Let (X,d) be a complete metric space and let $T : X \longrightarrow CB(X)$ be a multifunction. Suppose that there exist $\alpha, \beta \in (0,1)$ such that $\alpha(\beta+1) \leq 1$ and $\alpha d(x,Tx) \leq d(x,y)$ implies $H(Tx,Ty) \leq \beta d(x,y)$ for all $x,y \in X$. Then T has a fixed point.*

Lemma 2.2. [8] *Let (X,d) be a complete metric space, $T : X \longrightarrow CB(X)$ a multifunction and $\delta > 0$. Suppose that there exist $\alpha, \beta \in (0,1)$ such that $\alpha(\beta+1) \leq 1$ and $\alpha d(x,Tx) \leq d(x,y)$ implies $H(Tx,Ty) \leq \beta d(x,y)$ for all $x,y \in X$. Put*

$$F = \{x \in X : x \in Tx\} \text{ and } F_\delta = \{x \in X : d(x,Tx) < \delta\}. \tag{2.1}$$

Then $H(\overline{F_\delta}, F) \leq \frac{\delta}{1-\beta}$.

Theorem 2.3. *Let (X,d) be a complete separable metric space and let $T : \Omega \times C \longrightarrow CB(X)$ be a random continuous multifunction. Suppose that for each $\omega \in \Omega$ there exist positive numbers $\alpha_\omega, \beta_\omega \in (0,1)$ such that $\alpha_\omega(\beta_\omega + 1) \leq 1$ and*

$$\alpha_\omega d(x, T(\omega,x)) \leq d(x,y) \text{ implies that } H(T(\omega,x), T(\omega,y)) \leq \beta_\omega d(x,y) \tag{2.2}$$

for all $x,y \in X$. Then T has a random fixed point.

Proof. By using Theorem 2.1 and Proposition 1.9, we see that $F(\omega) = \{x \in X : x \in T(\omega,x)\}$ is nonempty and closed for all $\omega \in \Omega$. For each $n \geq 1$ and $\omega \in \Omega$, put $F_n(\omega) = \{x \in X : d(x, T(\omega,x)) < \frac{1}{n}\}$. By Corollary 1.7, each $F_n(\omega)$ is measurable and so is $\overline{F_n(\omega)}$. By Lemma 2.2, we have $H(\overline{F_n(\omega)}, F(\omega)) \leq \frac{1}{n(1-\beta_\omega)}$ for all $n \geq 1$. It follows from Proposition 1.10 that

$F(\omega)$ is measurable. Hence, it has a measurable selector $x(\omega)$ by Proposition 1.8. This implies that $x(\omega) \in T(\omega, x(\omega))$ for all $\omega \in \Omega$, that is, T has a random fixed point.

Theorem 2.4. *Let M be a closed separable subset of a complete metric space X and let T : $\Omega \times M \longrightarrow CB(M)$ be a random multifunction. Suppose that T is lower semi-continuous and for each $\omega \in \Omega$ there exists a positive number $\alpha_\omega \in (0, \frac{1}{2}]$ such that*

$$\alpha_\omega d(x, T(\omega, x)) < d(x, y) \text{ implies that } H(T(\omega, x), T(\omega, y)) < d(x, y) \tag{2.3}$$

for all $x, y \in X$. Then T has a random fixed point.

Proof. We first show that T has property (D). Let B be a closed ball of M , $\{x_n\}$ be a sequence in M with $d(x_n, B) \to 0$ and $d(x_n, T(\omega, x_n)) \to 0$ as $n \to \infty$. Since Tx_n is a compact set, we find that there exists a sequence $\{y_n\}$ in $T(\omega, x_n)$ with $d(x_n, y_n) \to 0$. As M is compact, we may, without loss of generality, assume that $\{x_n\}$ and $\{y_n\}$ converge to some x_0 and y_0, respectively. Next we show that $x_0 \in T(\omega, x_0)$. Since $T(\omega, y_n)$ is compact for all $n \geq 1$, we find that there exists a sequence $\{z_n\}$ in M such that $z_n \in T(\omega, y_n)$ for all $n \geq 1$ and $\lim_{n\to\infty} z_n = x_0$. Now, for each $n \geq 1$ either $\alpha_\omega d(x_n, T(\omega, x_n)) < d(x_n, x_0)$ or $\alpha_\omega d(y_n, T(\omega, y_n)) < d(x_0, y_n)$. If $\alpha_\omega d(x_n, T(\omega, x_n)) \geq d(x_n, x_0)$ and $\alpha_\omega d(y_n, T(\omega, y_n)) \geq d(x_0, y_n)$ for some $n \geq 1$, then

$$\begin{aligned} d(x_n, y_n) &\leq d(x_n, x_0) + d(x_0, y_n) \leq \alpha_\omega d(x_n, T(\omega, x_n)) + \alpha_\omega d(y_n, T(\omega, y_n)) \\ &\leq \alpha_\omega d(x_n, y_n) + \alpha_\omega H(T(\omega, x_n), T(\omega, y_n)) < 2\alpha_\omega d(x_n, y_n). \end{aligned}$$

Thus, we obtain $\alpha_\omega \geq \frac{1}{2}$. This is a contradiction. Thus, by using the assumption, for each $n \geq 1$ either $H(T(\omega, x_n), T(\omega, x_0)) < d(x_n, x_0)$ or $H(T(\omega, y_n), T(\omega, x_0)) < d(y_n, x_0)$ hold. Therefore, one of the following cases holds:

(i)- There exists an infinite subset $I \subseteq \mathbb{N}$ such that $H(T(\omega, x_n), T(\omega, x_0)) < d(x_n, x_0)$ for all $n \in I$,

(ii)- There exists an infinite subset $J \subseteq \mathbb{N}$ such that $H(T(\omega, y_n), T(\omega, x_0)) < d(y_n, x_0)$ for all $n \in J$.

In case (i), we obtain

$$d(x_0, T(\omega, x_0)) = \lim_{n\to\infty} d(y_n, T(\omega, x_0)) \leq \lim_{n\to\infty} H(T(\omega, y_n), T(\omega, x_0)) \leq \lim_{n\to\infty} d(x_n, x_0) = 0.$$

Hence, $x_0 \in T(\omega, x_0)$. In case (ii), we obtain

$$d(x_0, T(\omega, x_0)) = \lim_{n\to\infty} d(z_n, T(\omega, x_0)) \leq \lim_{n\to\infty} H(T(\omega, y_n), T(\omega, x_0)) \leq \lim_{n\to\infty} d(y_n, x_0) = 0.$$

Hence, $x_0 \in T(\omega, x_0)$. Since B is closed and

$$d(x_0, B) = d(\lim_{n\to\infty} x_n, B) = \lim_{n\to\infty} d(x_n, B) = 0,$$

we have $x_0 \in B$. On the other hands, $F(\omega) = \{x \in M : x \in T(\omega, x)\} \neq \emptyset$. Using Proposition 1.8, we find that T has a random fixed point.

Theorem 2.5. *Let M be a weakly compact convex separable subset of a Banach space X and let $T : \Omega \times M \longrightarrow CB(M)$ be a continuous random multifunction. Suppose that, for each $\omega \in \Omega$, there exists a positive number $\alpha_\omega \in (0, \frac{1}{2}]$ such that*

$$\alpha_\omega d(x, T(\omega, x)) < d(x, y) \text{ implies that } H(T(\omega, x), T(\omega, y)) < d(x, y) \tag{2.4}$$

for all $x, y \in X$. If, in addiction, for each fixed $\omega \in \Omega$, $I - T(\omega, 0)$ is demiclosed at $\mathbf{0}$, then T has a random fixed point.

Proof. Let $\xi_0 : \Omega \to M$ be a fixed measurable mapping. For each n, define $T_n : \Omega \times X \longrightarrow CB(M)$ by

$$T_n(\omega, x) = \frac{1}{n}\xi_0(\omega) + (1 - \frac{1}{n})T(\omega, x). \tag{2.5}$$

If $\alpha d(x, Tx) < d(x, y)$, we have

$$H(T_n(\omega, x), T_n(\omega, y)) \leq (1 - \frac{1}{n})\|x - y\|.$$

Since $\alpha(2 - \frac{1}{n}) \leq \frac{1}{2}(2 - \frac{1}{n}) = \frac{2n-1}{2n} < 1$, $T_n(\omega, x)$ is applied to relations in Theorem2.1. Hence each T_n has a random fixed point ξ_n by Theorem 2.3. Since M is weakly compact, we find that there exist $\xi(\omega) \in M$ that $\xi_n(\omega) \xrightarrow{w} \xi(\omega)$. From (1) we have

$$\xi_n(\omega) = \frac{1}{n}\xi_0(\omega) + (1 - \frac{1}{n})z_n(\omega),$$

where $z_n(\omega) \in T(\omega, \xi_n(\omega)$. Observe that

$$\|\xi_n(\omega) - z_n(\omega)\| = \frac{1}{n}\|\xi_0(\omega) - z_n(\omega)\|.$$

Hence $\xi_n(\omega) - z_n(\omega) \longrightarrow 0$. On the other hand

$$\xi_n(\omega) - z_n(\omega) \in I(\xi_n(\omega)) - T(\omega, \xi_n(\omega))$$

and since $I - T$ is demiclosed, $0 \in I(\xi(\omega)) - T(\omega, \xi(\omega))$, we obtain that $\xi(\omega)) \in T(\omega, \xi(\omega))$.

3. Random fixed points of hemi-convex multifunctions

In [3], Aleomraninejad, Rezapour and Shahzad extended the concept of convex multifunctions by following definition:

Definition 3.1. [3] *Let M be a convex subset of a Banach space X and $r > 0$. We say that the multifunction $T : M \longrightarrow 2^M$ is r-hemi-convex whenever*

$$d(\lambda x + (1-\lambda)y, T(\lambda x + (1-\lambda)y)) \leq r \tag{3.1}$$

for all $\lambda \in [0,1]$ and $x, y \in M$ with $d(x, T(x)) < r$ and $d(y, T(y)) < r$. We say that T is hemi-convex whenever T is r-hemi-convex for all $r > 0$.

It is clear that each convex multifunction on a Banach space is a hemi-convex multifunction. The following example shows that the converse is not true.

Example 3.2. [3] *Define multifunction $T : \mathbb{R} \longrightarrow 2^{\mathbb{R}}$ by $T(x) = [2x, 3x]$ if $x \geq 0$ and $T(x) = [3x, 2x]$ if $x < 0$. Then T is not convex whereas T is hemi-convex. In fact, $(1,2), (-1,-3) \in Gr(T)$, but for $\lambda = \frac{1}{2}$ we have*

$$\lambda(1,2) + (1-\lambda)(-1,-3) \notin Gr(T). \tag{3.2}$$

Since $d(x, T(x)) = |x|$ for all $x \in \mathbb{R}$, we find that T is a hemi-convex multifunction.

They also obtained the following results.

Theorem 3.3. [3] *Let $T : M \longrightarrow CB(M)$ be an upper-semi continuous hemi-convex multifunction. Then the set of fixed points of T is convex and closed.*

Theorem 3.4. [3] *Let M be a weakly compact subset of X and let $T : M \longrightarrow CB(M)$ be a multifunction and* $\inf_{x \in M} d(x, T(x)) = 0$. *If $f : X \longrightarrow [0, \infty)$ defined by $f(x) = d(x, T(x))$, is lower-semi continuous and hemi-convex on M, then T has a fixed point in M.*

Lemma 3.5. *(*[4]*.page 594) Let X be a separable metric space and let (Ω, Σ) be measurable space. If each $\varphi_n : \Omega \longrightarrow 2^X$ is measurable with closed-valued and for each $\omega \in \Omega$ and there is some k such that $\varphi_k(\omega)$ is compact, then the intersection multifunction $\theta : \Omega \longrightarrow 2^X$ defined by $\theta(\omega) = \bigcap_{n=1}^{\infty} \varphi_n(\omega)$ is measurable.*

Theorem 3.6. *Let M be a convex, weakly compact separable subset of a Banach space X and let $T : \Omega \times M \longrightarrow CB(M)$ be a continuous random hemi-convex multifunction. If $F(\omega) = \{x : x \in T(\omega, x)\} \neq \emptyset$, then T has a random fixed point.*

Proof. For each $n \in \mathbb{N}$ and $\omega \in \Omega$, put $F_n(\omega) = \{x : d(x, T(\omega, x)) \leq \frac{1}{n}\}$ and define the function $f_\omega : X \to \mathbb{R}$ by $f_\omega(x) = d(x, T(\omega, x))$. Then f_ω is lower-semi continuous. So, each $F_n(\omega)$ is measurable and closed. We claim that $\bigcap_{n=1}^{\infty} F_n(\omega) = F(\omega)$. It is trivial that $F(\omega) \subset F_n(\omega)$. Now, suppose that $x \in \bigcap_{n=1}^{\infty} F_n(\omega)$. Then for each $n \geq 1$, we can choose $x_n \in F_n(\omega)$ such that $\|x - x_n\| \leq \frac{1}{n}$. Since $x_n \longrightarrow x$ and $T(\omega, x)$ is closed, $x \in T(\omega, x)$ and consequently $x \in F(\omega)$. Thus, the claim is proved. Note that $F_n(\omega)$ and $F(\omega)$ are convex and closed. Hence

$$\bigcap_{n=1}^{\infty} \overline{F_n(\omega)^w} = \overline{F(\omega)^w},$$

where $\overline{F(\omega)^w}$ is the weak closure of $F(\omega)$. Since each $\overline{F_n(\omega)^w}$ is weakly compact, we find from Lemma 3.3 that F is weakly measurable and so F is measurable. Consequently F has a measurable selector such that $x(\omega) \in F(\omega)$ for all $\omega \in \Omega$. Thus, $x(\omega) \in T(\omega, x(\omega))$ for all $\omega \in \Omega$, that is, T has a random fixed point.

Note that Theorem 3.6 generalizes following theorem.

Theorem 3.7. *(Theorem 3.2 of* [**?**]*) Let M be a convex, weakly compact separable subset of a Banach space X and $T : \Omega \times M \longrightarrow CB(M)$ a continuous random and convex multifunction. If $F(\omega) = \{x : x \in T(\omega, x)\} \neq \emptyset$, then T has a random fixed point.*

Proof. Every convex multifunction is a hemi-convex multifunction. Using Theorem 3.6, we find that T has a random fixed point.

Theorem 3.8. *Let M be a convex weakly compact separable subset of Banach space X and let $T : \Omega \times M \longrightarrow CB(M)$ be a continuous random hemi-convex multifunction. For any $\omega \in \Omega$, $\inf_{x \in M} d(x, T(\omega, x)) = 0$. Then T has a random fixed point.*

Proof. In view of Theorem 3.4, the fixed point set of T is nonempty. Using Theorem 3.6, one obtains that T has a random fixed point.

The following example shows that there are many multifunctions which satisfy condition $\inf_{x \in M} d(x, T(\omega, x)) = 0$.

Example 3.9. *Let M be a convex and bounded subset of a Banach space X, $u \in M$ a fixed element and $T : \Omega \times M \to CB(M)$ a nonexpansive multifunction. For each $n \geq 2$, define $T_n : \Omega \times M \to CB(M)$ by $T_n(\omega, x) = \frac{1}{n}u + (1 - \frac{1}{n})T(\omega, x)$. Since $H(T_n(\omega, x), T_n(\omega, y)) \leq (1 - \frac{1}{n})\|x - y\|$ for all $x, y \in M$ and $n \geq 2$, T_n is a contraction multifunction and so for each $n \geq 2$ there exists $x_n \in M$ such that $x_n \in T_n(x_n)$. Note that $d(x_n, T(\omega, x_n)) \to 0$ and so $\inf_{x \in M} d(x, T(\omega, x)) = 0$.*

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