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STRONG CONVERGENCE OF A SHRINKING PROJECTION ALGORITHM FOR A SPLIT FEASIBILITY PROBLEM

VAHID DADASHI

Department of Mathematics, Sari Branch, Islamic Azad University, Sari, Iran

Abstract. In this paper, we investigate a split feasibility problem based on a shrinking projection method. We prove that the sequence generated in our shrinking projection algorithm converges strongly to a solution of the split feasibility problem.

Keywords. Projection operator; Shrinking projection algorithm; Split feasibility problem.

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1. INTRODUCTION

Let H_1 and H_2 be two real Hilbert spaces. Let $T : H_1 \to H_2$ be a bounded linear operator. Suppose that C and D be nonempty, closed and convex subsets of H_1 and H_2, respectively. Recall that the split feasibility problem is to find $z \in H_1$ such that $z \in C \cap T^{-1}D$. Censor and Elfving [1] first introduced the split feasibility problem in finite dimensional Hilbert spaces which arises from phase retrievals and in medical image reconstruction; see [2] and the references therein. The split feasibility problem can also be used in various disciplines such as image restoration, computed tomography and radiation therapy treatment planning; see [3, 4, 5] and the references therein. It is known that, if $C \cap T^{-1}D \neq \emptyset$, a solution $z \in C \cap T^{-1}D$ to the split feasibility problem is equivalent to $z = P_C(I - rT^*(I - P_D)T)z$, where T^* is the adjoint operator of T and P_C and P_D are the metric projections from H_1 onto C and from H_2 onto D, respectively, $r > 0$ is a positive constant; see, for more details, [6] and the references therein. Byrne [2, 7] considered the following CQ algorithm to solve the split feasibility problem as

$$x_{n+1} = P_C(x_n - \gamma T^*(x_n - P_D T x_n)).$$

where $\gamma \in (0, \frac{1}{\lambda})$ with λ being the spectral radius of operator T^*T. Xu [8] studied the convergence of CQ algorithms and applied Mann's algorithm to the split feasibility problem.

Recently, many authors studied the split feasibility problem so that strong convergence is guaranteed; see [9]-[16] and references therein. In 2015, Takahashi and Yao [17] proved the following result.

E-mail address: vahid.dadashi@iausari.ac.ir (V. Dadashi).

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Theorem 1.1. *Let H be a Hilbert space and let F be a strictly convex, reflexive and smooth Banach space. Assume that J_F is the duality mapping on F. Let C and D be nonempty, closed, and convex subsets of H and F, respectively. Let P_C and P_D be the metric projections of H onto C and F onto D, respectively. Let $T : H \to F$ be a bounded linear operator such that $T \neq 0$ and let T^* be the adjoint operator of T. Suppose that $C \cap T^{-1}D \neq \emptyset$. Let $x_1 \in H$ and $\{x_n\}$ be a sequence generated by*

$$\begin{cases} z_n = P_C(x_n - rT^*J_F(Tx_n - P_DTx_n)) \\ y_n = \alpha_n x_n + (1-\alpha_n)z_n, \\ C_n = \{z \in H : \ \|y_n - z\| \leq \|x_n - z\|\}, \\ Q_n = \{z \in H : \ \langle x_n - z, x_1 - x_n \rangle \geq 0\}, \\ x_{n+1} = P_{C_n \cap Q_n}(x_1), \end{cases}$$

where $0 \leq \alpha_n < 1$ and $0 < r\|T\|^2 < 2$. Then $\{x_n\}$ converges strongly to $z_0 \in C \cap T^{-1}D$, where $z_0 = P_{C\cap T^{-1}D}x_1$.

In this paper, inspired and motivated by the above result, we introduce and study a new shrinking projection algorithm for finding a solution of the split feasibility problem. The results presented in this paper mainly improve the results in [17] and other related results announced recently.

2. Preliminaries

Throughout the present paper, let X be a real Banach space. We write $x_n \to x$ to indicate that $\{x_n\}$ strongly converges to x. The normalized duality mapping J from X into the family of nonempty w^*-compact subsets of its dual X^* is defined by

$$J(x) = \{x^* \in X^* \ : \ \langle x^*, x\rangle = \|x\|^2 = \|x^*\|^2\}, \quad \forall x \in E. \tag{2.1}$$

Recall that a Banach space X is said to be uniformly convex if for any $\varepsilon \in (0,2]$ there exists a $\delta = \delta(\varepsilon) > 0$ such that if $x, y \in X$ with $\|x\| = 1$, $\|y\| = 1$ and $\|x - y\| \geq \varepsilon$, then $\|\frac{1}{2}(x+y)\| \leq 1 - \delta$. The norm of X is said to be Gateaux differentiable (X is said to be smooth) if

$$\lim_{t\to 0} \frac{\|x+ty\| - \|x\|}{t} \tag{2.2}$$

exists for each $x,\ y \in U := \{z \in X : \|z\| = 1\}$. It is known that if X is Gateaux differentiable, then J is single-valued. The norm is said to be a uniformly Gateaux differentiable if for $y \in U$, the limit is attained uniformly for $x \in U$. The space X is said to have a uniformly Frechet differentiable norm (X is said to be uniformly smooth) if limit (2.2) is attained uniformly for $(x, y) \in U \times U$. It is known that X is smooth if and only if each duality mapping J is single-valued.

Let C be a convex closed subset of X. Recall that an operator P_C is called a metric projection operator if it assigns to each $x \in X$ its nearest point $y \in C$ such that

$$\|x - y\| = \min\{\|x - z\| : z \in C\}.$$

It is known that the metric projection operator P_C is continuous in a uniformly convex Banach space X and uniformly continuous on each bounded set of X if, in addition, X is uniformly smooth. An element y is called the metric projection of X onto C and denoted by P_Cx. It exists and is unique at any point of the reflexive strictly convex space.

Lemma 2.1. [18] *Let X be a reflexive and strictly convex Banach space and let C be a nonempty, closed and convex subset of X. Then, for all $x \in X$, $z = P_C x$ if and only if*

$$\langle J(x-z), z-y\rangle \geq 0,\ \forall y \in C.$$

Lemma 2.2. [19] *Let X be a smooth, strictly convex and reflexive Banach space and let C be a nonempty, closed and convex subset of X. Suppose that P_C is the metric projections of X onto C. If $\{x_n\}$ is a sequence in X such that $x_n \rightharpoonup p$ and $x_n - P_C x_n \to 0$, then $P_C p = p$.*

Let C be a convex closed subset of a Hilbert space H and $x \in H$. Then the metric projection satisfies in the following inequality:

$$\|P_C x - P_C y\|^2 \leq \langle P_C x - P_C y, x-y\rangle,\ \forall x,y \in H, \tag{2.3}$$

that is, metric projections are firmly nonexpansive. For a sequence $\{C_n\}$ of nonempty, closed and convex subsets of a Banach space X, define $\mathrm{s\text{-}Li_n C_n}$ and $\mathrm{w\text{-}Ls_n C_n}$ as follows: $x \in \mathrm{s\text{-}Li_n C_n}$ if and only if there exists $\{x_n\} \subset X$ such that $\{x_n\}$ converges strongly to x and $x_n \in C_n$ for all $n \in \mathbb{N}$. Similarly, $y \in \mathrm{w\text{-}Ls_n C_n}$ if and only if there exist a subsequence $\{C_{n_i}\}$ of $\{C_n\}$ and a sequence $\{y_i\} \subset X$ such that $\{y_i\}$ converges weakly to y and $y_i \in C_{n_i}$ for all $i \in \mathbb{N}$. If C_0 satisfies in $C_0 = \mathrm{s\text{-}Li_n C_n} = \mathrm{w\text{-}Ls_n C_n}$, it is said that $\{C_n\}$ converges to C_0 in the sense of Mosco [20] and we write $C_0 = \mathrm{M}\text{-}\lim_{n\to\infty} C_n$. It is easy to show that if $\{C_n\}$ is nonincreasing with respect to inclusion, then $\{C_n\}$ converges to $\cap_{n=1}^{\infty} C_n$ in the sense of Mosco; see [20] and the references therein.

The following lemma was obtained by Tsukada [21].

Lemma 2.3. [21] *Let X be a uniformly convex Banach space. Let $\{C_n\}$ be a sequence of nonempty, closed, and convex subsets of X. If $C_0 = \mathrm{M}\text{-}\lim_{n\to\infty} C_n$ exists and nonempty, then for each $x \in X$, $P_{C_n} x$ converges strongly to $P_{C_0} x$, where P_{C_n} and P_{C_0} are the metric projections of X onto C_n and C_0, respectively.*

3. MAIN RESULTS

In this section, based on a shrinking projection method, we prove strong convergence theorems for finding a solution of the split feasibility problem.

Theorem 3.1. *Let X be a uniformly convex and smooth Banach space with duality mapping J_X. Let H be a Hilbert space. Let $T : H \to X$ be a bounded linear operator such that $T \neq 0$ and let T^* be the adjoint operator of T. Let C and D be nonempty, closed and convex subsets of H and X, respectively, such that $C \cap T^{-1} D \neq \emptyset$. Assume that P_C and P_D be the metric projections of H onto C and X onto D, respectively. Let $\{u_n\}$ be a sequence in H such that $u_n \to u$ and let $\{x_n\}$ be a sequence generated in the following algorithm*

$$\begin{cases} z_n = P_C(x_n - rT^* J_X(Tx_n - P_D T x_n)) \\ y_n = \alpha_n x_n + (1-\alpha_n) z_n, \\ C_{n+1} = \{z \in C_n :\ \langle y_n - z, x_n - y_n\rangle \geq 0\}, \\ x_{n+1} = P_{C_{n+1}}(u_{n+1}), \end{cases} \tag{3.1}$$

where $C_1 = C$ and $x_1 \in H$. If $0 < \frac{\|T\|}{2} \leq \alpha_n \leq a < 1$ and $0 < r < 1$, then $\{x_n\}$ converges strongly to a point $z_0 \in C \cap T^{-1} D$, where $z_0 = P_{C\cap T^{-1}D} u$.

Proof. First, we prove that sequence $\{x_n\}$ generated in (3.1) is well-defined. It is easy to check that C_n is closed and convex for each $n \in \mathbb{N}$. We show that $C \cap T^{-1}D \subset C_n$, for each $n \in \mathbb{N}$. It is clear that $C \cap T^{-1}D \subset C_1$. Assume that $C \cap T^{-1}D \subset C_n$ for some $n \in \mathbb{N}$. Let $p \in C \cap T^{-1}D$. Then $p \in C$, $Tp \in D$ and $p \in C_n$. It follows that $P_C p = p$ and

$$\langle J_X(Tx_n - P_D Tx_n), P_D Tx_n - Tp \rangle \geq 0.$$

From the definition of z_n, we find from (2.3) that

$$\begin{aligned}
\langle x_n - z_n, z_n - p \rangle &= \langle x_n - z_n, P_C(x_n - rT^* J_X(Tx_n - P_D Tx_n)) - P_C p \rangle \\
&= \langle x_n - rT^* J_X(Tx_n - P_D Tx_n) - p, P_C(x_n - rT^* J_X(Tx_n - P_D Tx_n)) - P_C p \rangle \\
&\quad + \langle rT^* J_X(Tx_n - P_D Tx_n), P_C(x_n - rT^* J_X(Tx_n - P_D Tx_n)) - P_C p \rangle \\
&\quad - \langle z_n - p, P_C(x_n - rT^* J_X(Tx_n - P_D Tx_n)) - P_C p \rangle \\
&\geq \|P_C(x_n - rT^* J_X(Tx_n - P_D Tx_n)) - P_C p\|^2 \\
&\quad + \langle rT^* J_X(Tx_n - P_D Tx_n), z_n - p \rangle - \|z_n - p\|^2 \\
&= \langle rT^* J_X(Tx_n - P_D Tx_n), z_n - p \rangle \\
&= r\langle J_X(Tx_n - P_D Tx_n), Tz_n - Tp \rangle \\
&= r\langle J_X(Tx_n - P_D Tx_n), Tz_n - Tx_n \rangle + r\langle J_X(Tx_n - P_D Tx_n), Tx_n - P_D Tx_n \rangle \\
&\quad + r\langle J_X(Tx_n - P_D Tx_n), P_D Tx_n - Tp \rangle \\
&\geq r\langle J_X(Tx_n - P_D Tx_n), Tz_n - Tx_n \rangle + r\|Tx_n - P_D Tx_n\|^2. \qquad (3.2)
\end{aligned}$$

Taking into account the definition of y_n, we obtain that

$$\begin{aligned}
\langle y_n - p, x_n - y_n \rangle &= \langle \alpha_n(x_n - p) + (1-\alpha_n)(z_n - p), (1-\alpha_n)(x_n - z_n) \rangle \\
&= \alpha_n(1-\alpha_n)\langle x_n - z_n + z_n - p, x_n - z_n \rangle + (1-\alpha_n)^2 \langle z_n - p, x_n - z_n \rangle \\
&= \alpha_n(1-\alpha_n)\|x_n - z_n\|^2 + \alpha_n(1-\alpha_n)\langle z_n - p, x_n - z_n \rangle \\
&\quad + (1-\alpha_n)^2 \langle z_n - p, x_n - z_n \rangle \\
&= \alpha_n(1-\alpha_n)\|x_n - z_n\|^2 + (1-\alpha_n)\langle z_n - p, x_n - z_n \rangle. \qquad (3.3)
\end{aligned}$$

From (3.2) and (3.3), we obtain that

$$\begin{aligned}
\langle y_n - p, x_n - y_n \rangle &\geq \alpha_n^2(1-\alpha_n)\|x_n - z_n\|^2 \\
&\quad + r(1-\alpha_n)\langle J_X(Tx_n - P_D Tx_n), Tz_n - Tx_n \rangle \\
&\quad + r(1-\alpha_n)\|Tx_n - P_D Tx_n\|^2 \\
&\geq (1-\alpha_n)[\alpha_n^2\|x_n - z_n\|^2 \\
&\quad - r\|T\|\|Tx_n - P_D Tx_n\|\|x_n - z_n\| + r\|Tx_n - P_D Tx_n\|^2] \\
&\geq (1-\alpha_n)[\alpha_n^2\|x_n - z_n\|^2 \\
&\quad - 2r\alpha_n\|Tx_n - P_D Tx_n\|\|x_n - z_n\| + r^2\|Tx_n - P_D Tx_n\|^2] \\
&= (1-\alpha_n)(\alpha_n\|x_n - z_n\| - r\|Tx_n - P_D Tx_n\|)^2 \geq 0, \qquad (3.4)
\end{aligned}$$

which implies that $p \in C_{n+1}$. By mathematical induction, we see that $C \cap T^{-1}D \subset C_n$ for every $n \in \mathbb{N}$. Therefore, $\{x_n\}$ is well-defined.

Let $C_0 = \cap_{n=1}^{\infty} C_n$. Since $C \cap T^{-1}D \subset C_0$, we have $C_0 \neq \emptyset$. Suppose that $w_n = P_{C_n}u$ for every $n \in \mathbb{N}$. By Lemma 2.3, we get that $w_n \to w_0 = P_{C_0}u$. Note that

$$\begin{aligned}\|x_n - w_0\| &\leq \|x_n - w_n\| + \|w_n - w_0\| \\ &= \|P_{C_n}u_n - P_{C_n}u\| + \|w_n - w_0\| \\ &\leq \|u_n - u\| + \|w_n - w_0\|,\end{aligned}$$

which yields that $x_n \to w_0$. Since $w_0 \in C_0 \subset C_{n+1}$, we have

$$0 \leq \langle y_n - w_0, x_n - y_n\rangle = -\|x_n - y_n\|^2 + \langle x_n - w_0, x_n - y_n\rangle,$$

therefore,

$$\|x_n - y_n\| \leq \|x_n - w_0\|.$$

It follows that

$$\|x_n - y_n\| \to 0 \text{ and } y_n \to w_0. \tag{3.5}$$

On the other hand, one has

$$\|x_n - z_n\| = \frac{1}{1-\alpha_n}\|x_n - y_n\| \to 0. \tag{3.6}$$

We know that for $p \in C \cap T^{-1}D$, the following inequality is satisfied by (3.4),

$$0 \leq (1-\alpha_n)(\alpha_n\|x_n - z_n\| - r\|Tx_n - P_DTx_n\|)^2 \leq \langle y_n - p, x_n - y_n\rangle. \tag{3.7}$$

Then, by (3.5), (3.6) and (3.7), we get

$$\|Tx_n - P_DTx_n\| \to 0. \tag{3.8}$$

Since $\{x_n\}$ converges strongly to w_0 and T is a bounded linear operator, we get that $\{Tx_n\}$ converges strongly to Tw_0. Using (3.8) and Lemma 2.2, we imply that $Tw_0 = P_DTw_0$. So, $w_0 \in T^{-1}D$. Note that $w_0 \in C$, because $z_n \in C$, $z_n \to w_0$ and C is closed. Therefore, $w_0 \in C \cap T^{-1}D$.

Since $C \cap T^{-1}D$ is nonempty, closed and convex, we see that there exists a unique element $z_0 \in C \cap T^{-1}D \subset C_{n+1}$ such that $z_0 = P_{C\cap T^{-1}D}u$. From $x_{n+1} = P_{C_{n+1}}u$, we get that

$$\|x_{n+1} - u\| \leq \|u - z_0\|,$$

for every $n \in \mathbb{N}$. Since $x_n \to w_0$, we get

$$\|w_0 - u\| \leq \|u - z_0\|. \tag{3.9}$$

It follows from $z_0 = P_{C\cap T^{-1}D}u$, $w_0 \in C \cap T^{-1}D$ and (3.9) that $w_0 = z_0$. Therefore, we have that $\{x_n\}$ converges strongly to $w_0 = z_0$. This completes the proof. □

From Theorem 3.1, we have the following results.

Corollary 3.2. *Let X be a uniformly convex and smooth Banach space with duality mapping J_X. Let H be a Hilbert space. Let $T : H \to X$ be a bounded linear operator such that $T \neq 0$ and let T^* be the adjoint operator of T. Let C and D be nonempty, closed and convex subsets of H and X, respectively,* *such that* $C \cap T^{-1}D \neq \emptyset$. *Assume that* P_C *and* P_D *are the metric projections of* H *onto* C *and* X *onto* D, *respectively. Let the sequence* $\{x_n\}$ *be generated by the following algorithm*

$$\begin{cases} z_n = P_C(x_n - rT^*J_X(Tx_n - P_DTx_n)) \\ y_n = \alpha_n x_n + (1-\alpha_n)z_n, \\ C_{n+1} = \{z \in C_n : \langle y_n - z, x_n - y_n\rangle \geq 0\}, \\ x_{n+1} = P_{C_{n+1}}(x_1), \end{cases}$$

where $C_1 = C$ *and* $x_1 \in H$. *If* $0 < \frac{\|T\|}{2} \leq \alpha_n \leq a < 1$ *and* $0 < r < 1$, *then* $\{x_n\}$ *converges strongly to a point* $z_0 \in C \cap T^{-1}D$, *where* $z_0 = P_{C\cap T^{-1}D}x_1$.

Corollary 3.3. *Let* H *be a Hilbert space. Let* $T : H \to H$ *be a bounded linear operator such that* $T \neq 0$ *and let* T^* *be the adjoint operator of* T. *Let* C *and* D *be nonempty, closed and convex subsets of* H *such that* $C \cap T^{-1}D \neq \emptyset$. *Assume that* P_C *and* P_D *be the metric projections of* H *onto* C *and* D. *Let the sequence* $\{x_n\}$ *be generated by the following algorithm*

$$\begin{cases} z_n = P_C(x_n - rT^*(Tx_n - P_DTx_n)) \\ y_n = \alpha_n x_n + (1-\alpha_n)z_n, \\ C_{n+1} = \{z \in C_n : \langle y_n - z, x_n - y_n\rangle \geq 0\}, \\ x_{n+1} = P_{C_{n+1}}(x_1), \end{cases}$$

where $C_1 = C$ *and* $x_1 \in H$. *If* $0 < \frac{\|T\|}{2} \leq \alpha_n \leq a < 1$ *and* $0 < r < 1$, *then* $\{x_n\}$ *converges strongly to a point* $z_0 \in C \cap T^{-1}D$, *where* $z_0 = P_{C\cap T^{-1}D}x_1$.

Corollary 3.4. *Let* H *be a Hilbert space. Let* C *and* D *be nonempty, closed and convex subsets of* H *such that* $C \cap D \neq \emptyset$. *Let* P_C *and* P_D *be the metric projections of* H *onto* C *and* H *onto* D, *respectively. Let the sequence* $\{x_n\}$ *be generated by the following algorithm*

$$\begin{cases} z_n = P_C((1-r)x_n + rP_Dx_n) \\ y_n = \alpha_n x_n + (1-\alpha_n)z_n, \\ C_{n+1} = \{z \in C_n : \langle y_n - z, x_n - y_n\rangle \geq 0\}, \\ x_{n+1} = P_{C_{n+1}}(x_1), \end{cases}$$

where $C_1 = C$ *and* $x_1 \in H$. *If* $\frac{1}{2} \leq \alpha_n \leq a < 1$ *and* $0 < r < 1$, *then* $\{x_n\}$ *converges strongly to a point* $z_0 \in C \cap D$, *where* $z_0 = P_{C\cap D}x_1$.

In order to illustrate the performance of the proposed algorithm, we give the following numerical example, which is performed using Wolfram Mathematica 11 on an Intel Core i7-7500U running 64-bit Windows.

Example 3.5. Let $H = \mathbb{R}^3$ and $X = \mathbb{R}^2$. We take

$$C = \left\{(x,y,z) \in \mathbb{R}^3 : (x,y,z) \in [0,3] \times [-1,1] \times [0,5]\right\},$$

and

$$D = \left\{(x,y) \in \mathbb{R}^2 : (x,y) \in [0,1] \times [0,2]\right\}.$$

Suppose that a bounded linear operator $T : \mathbb{R}^3 \to \mathbb{R}^2$ is defined by $T(x,y,z) = (\frac{1}{2}x, \frac{1}{2}z)$, for all $(x,y,z) \in \mathbb{R}^3$. It is easy to see that $\|T\| = \frac{1}{2}$ and the adjoint operator $T^* : \mathbb{R}^2 \to \mathbb{R}^3$ is defined by $T^*(x,y) =$

TABLE 1. Results for start point $x_1 = (4,-2,3)$ and $u_n = (\frac{3n}{n+1}, 4+\frac{1}{n}, 2-\frac{1}{n})$

α_n	r	Iters	Time(Sec)	Sol
$0.25+\frac{1}{100n}$	$\frac{3}{4}$	22	8.27498	(2.01487,1.,1.95454)
$0.25+\frac{1}{100n}$	$\frac{7}{8}$	20	7.09318	(2.0127,1.,1.95)
$0.5+\frac{1}{100n}$	$\frac{3}{4}$	25	10.0192	(2.02947,1.,1.96)
$0.5+\frac{1}{100n}$	$\frac{7}{8}$	24	9.57135	(2.02278,1.,1.95833)

TABLE 2. Results for start point $x_1 = (-3,1,4)$ and $u_n = (\frac{5n}{2n+1}, -3, 2-\frac{n^2}{n^2+1})$

α_n	r	Iters	Time(Sec)	Sol
$0.25+\frac{1}{100n}$	$\frac{3}{4}$	17	5.53923	(2.01744,-1.,1.00345)
$0.25+\frac{1}{100n}$	$\frac{7}{8}$	16	5.08731	(2.01421,-1.,1.00389)
$0.5+\frac{1}{100n}$	$\frac{3}{4}$	20	6.97691	(2.02728,-1.,1.00249)
$0.5+\frac{1}{100n}$	$\frac{7}{8}$	19	6.44502	(2.02283,-1.,1.00276)

$(\frac{1}{2}x, 0, \frac{1}{2}y)$, for all $(x,y) \in \mathbb{R}^2$. We are now in a position to show that algorithm (3.1) converges to a point $z \in \mathbb{R}^3$ such that $z \in C \cap T^{-1}D$. It is easy to see that

$$C \cap T^{-1}D = \left\{(x,y,z) \in \mathbb{R}^3 : (x,y,z) \in [0,2] \times [-1,1] \times [0,4]\right\}.$$

In this experiment, we consider the stopping criterion by $\frac{\|x_{n+1} - x_n\|}{\max\{1, \|x_n\|\}} \leq 10^{-3}$ and the following cases of the step size parameters r and α_n with two initial points x_1 and sequences $\{u_n\}$. The numerical results are showed in Table 1 and 2. The generated sequences by our algorithm with the initial points converge to a point in $C \cap T^{-1}D$.

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