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SOLVING ABSOLUTE VALUE EQUATIONS VIA COMPLEMENTARITY AND INTERIOR-POINT METHODS

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Abstract. In this paper, an infeasible path-following interior-point algorithm is proposed for solving the NP-hard absolute value equations (AVE) of the type $Ax - B|x| = b$. Under the condition that the minimal singular value of A is strictly greater than the maximal singular value of B, the unique solvability theorem of AVE is presented by formulating the AVE as a monotone horizontal linear complementary problem (HLCP). We also propose an infeasible primal-dual interior-point algorithm for solving the AVE across the HLCP. Some numerical results are provided to show the efficiency of the proposed algorithm.

Keywords. Absolute value equation; Complementarity; Interior-point method; Singular value.

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1. INTRODUCTION

Given two matrices $A, B \in \mathbb{R}^{n\times n}, B \neq 0$, and a vector $b \in \mathbb{R}^n$, the absolute value equations (AVE) is defined as follows:

$$Ax - B|x| = b, \tag{1.1}$$

where, $|x|$ denotes the componentwise absolute value of vector $x \in \mathbb{R}^n$. When $B = I$, where I is the identity matrix, then the AVE (1.1) is reduced to the following type

$$Ax - |x| = b. \tag{1.2}$$

Recently, the AVE (1.1)-(1.2) have attracted considerable attention because they have many applications, such as linear and quadratic programming, linear complementarity problems, mixed-integer programming and interval linear systems; see, Rohn [1, 2] and Hladik [3]. Since the general NP-hard linear complementarity can be formulated as the AVE (1.1)-(1.2), it is an NP-hard in its general form. The AVE (1.1) was first introduced by Rohn [1], and recently has been extensively investigated by many authors, such as, Mangasarian and Meyer [4], Prokopyev [5] and Lotfi and Veiseh [6]. The research effort can be summarized to the following two aspects. One is purely theoretical analysis. Authors focus on

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the reformulations of the AVE (1.1)-(1.2) as different equivalent problems, and the existence and nonexistence of solutions. The other one is the numerical solvability of AVEs. Recently, a series of numerical methods have been proposed to solve the AVE (1.1)-(1.2); see e.g., [7, 8, 3, 9, 19, 10, 11, 17, 13, 14, 15] and the references therein. In the literatures, most numerical methods directly tackled the AVE based on smoothing and non-smoothing nonlinear optimization techniques. However, other numerical approaches focus on reformulating the AVE (1.1)-(1.2) as complementarity problems; see e.g., Yong *et al.* [16]. They also computed under a suitable condition, that is, the singular values of A exceed 1, the complexity analysis of the feasible short-step interior-point algorithm corresponding to the standard LCP, which is linked to the AVEs (1.2).

In this paper, we solve the AVE (1.1) via primal-dual interior-point methods. First, we transform the AVE (1.1) into an horizontal linear complementarity problem (HLCP) and show, under the condition that the minimal singular value of A is strictly greater than the maximal singular value of B, that the HLCP has a unique solution, so is the AVE (1.1) for every $b \in \mathbb{R}^n$. The main result is a generalization of an earlier result by Mangasarian and Meyer [4]. For the special case (1.2), where the singular values of A exceed 1, Rohn's proof [1] is based on the alternative theorem. Recently, Jiang and Zhang [17] gave a simple proof of the unique solvability based on the same assumption. Lotfi and Veiseh [6] studied the unique solvability of the AVE (1.1) that their investigation is based on the sufficient regularity conditions to the interval matrix. It is worth mentioning that our proof is close to Xiang's and totally differs to the one given by Rohn, which directly tackled the AVE (1.1) rather than using the theory of linear complementarity problems. Second, across the HLCP, we numerically solve the AVE (1.1) by introducing an infeasible path-following interior-point algorithm. Finally, numerical results are provided to illustrate the efficiency of this algorithm for solving the AVE (1.1). In addition, to show the performance of our method in practice, a comparison of our obtained numerical results with those obtained via an available feasible interior-point method is made.

The rest of the paper is organized as follows. In Section 2, the reformulation of AVE (1.1) as horizontal linear complementarity problem and the unique solvability of AVE (1.1) are stated. In Section 3, an infeasible path-following interior-point algorithm for solving the AVE (1.1) via the HLCP is proposed. In Section 4, the last section, some numerical results on some AVE problems are reported.

Throughout the paper, we use the following notations. $\mathbb{R}^n_+$ denotes the nonnegative orthant of $\mathbb{R}^n$. For a matrix $A \in \mathbb{R}^{n\times n}$, we denote by $\sigma_{\min}(A)$ ($\lambda_{\min}(A)$) and $\sigma_{\max}(A)$ ($\lambda_{\max}(A)$) the smallest singular value (the smallest eigenvalue) and the maximal singular value (the maximal eigenvalue) of A. The identity matrix and the vector column of ones are denoted by I and e, respectively. For a vector x, X:=Diag(x) is the diagonal matrix whose diagonal elements are $X_{ii} = x_i$. $x \geq 0$ ($x > 0$) means that the components of x are greater or equal to 0 (> 0). For x, $y \in \mathbb{R}^n$, their usual inner product and the Euclidean norm of a vector x are denoted by $x^T y = \langle x, y\rangle$ and $\|x\|$, respectively.

2. Reformulation of the AVE as an HLCP

In this section, some necessary definitions and lemmas are required. A matrix $M \in \mathbb{R}^{n\times n}$ is said to be positive definite if $\langle x, Mx\rangle > 0$ for all nonzero $x \in \mathbb{R}^n$. $M \in \mathbb{R}^{n\times n}$ is called a $\mathscr{P}$-matrix if all its minors are positive. As a consequence, if M is positive definite, then M is a $\mathscr{P}$-matrix.

Lemma 2.1. *A symmetric matrix $M \in \mathbb{R}^{n\times n}$ is positive definite if and only if $\lambda_{\min}(M) > 0$.*

Next, we recall the mathematical reformulation of an HLCP and a standard LCP and state a result for the unique solution of an LCP.

Definition 2.2. The horizontal linear complementarity problem consists of finding (x,y) in $\mathbb{R}^n \times \mathbb{R}^n$ such that

$$x \geq 0,\ y \geq 0,\ Ny - Mx = q,\ \langle x,y \rangle = 0, \tag{2.1}$$

where $N, M \in \mathbb{R}^{n\times n}$ and $q \in \mathbb{R}^n$ are given.

Definition 2.3. The standard linear complementarity problem consists of finding (x,y) in $\mathbb{R}^n \times \mathbb{R}^n$ such that

$$x \geq 0,\ y \geq 0,\ y - Mx = q,\ \langle x,y \rangle = 0. \tag{2.2}$$

Remark 2.4. If N is invertible, then the HLCP is reduced to a standard LCP.

Definition 2.5. An HLCP is said to be monotone (strictly monotone) if the pair of matrices $[N, M]$ satisfies

$$Nu - Mv = 0 \Rightarrow u^T v \geq 0, \text{ for any } u, v \in \mathbb{R}^n (u^T v > 0 \text{ for any nonzero } u, v).$$

Remark 2.6. In the case of LCP standard, the monotony (strictly monotony) is reduced to the positive semidefinite (positive definite) of the matrix M.

The following result was proved by Cottle, Pang and Stone [18].

Theorem 2.7 (Theorem 3.3.7, [18]). *A matrix $M \in \mathbb{R}^{n\times n}$ is a $\mathscr{P}$-matrix if and only if the LCP (2.2) has a unique solution for every $q \in \mathbb{R}^n$. In this case, the LCP with $\mathscr{P}$-matrix is denoted by $\mathscr{P}$-LCP.*

Definition 2.8. For $x \in \mathbb{R}^n$, the vectors x^+ and x^- are defined such that

$$x_i^+ = \max_i(0, x_i) \text{ and } x_i^- = \max_i(0, -x_i).$$

Then

$$x^+ \geq 0,\ x^- \geq 0,\ x = x^+ - x^-,\ |x| = x^+ + x^-,\ \langle x^+, x^- \rangle = 0. \tag{2.3}$$

The following Proposition gives the equivalence between the AVE (1.1) and the HLCP. This latter is similar to the one given in [19].

Proposition 2.9. *The AVE (1.1) is as an HLCP.*

Proof. According to the decomposition of x and $|x|$ in (2.3), the AVE (1.1) can be reduced to the following equivalent HLCP (2.1)

$$\begin{cases} Nx^+ - Mx^- = q, \\ x^+ \geq 0, \quad x^- \geq 0, \\ \langle x^+, x^- \rangle = 0, \end{cases} \tag{2.4}$$

where

$$N = (A - B), M = (A + B) \text{ and } q = b.$$

This completes the proof. □

Unique solvability of the AVE

We assume that the AVE (1.1) satisfies the following condition

- **Assumption 1.** The pair of the matrices $[N, M]$ satisfies

$$\sigma_{\min}(A) > \sigma_{\max}(B).$$

From the theory of the HLCP and Assumption 1, we will show, based on the following theorem, that the AVE (1.1) is uniquely solvable for every $b \in \mathbb{R}^n$.

Theorem 2.10. *Under Assumption 1, the AVE (1.1) is uniquely solvable for every $b \in \mathbb{R}^n$.*

Proof. We prove under Assumption 1 that HLCP (2.4) is reduced to a standard $\mathscr{P}$-LCP with $M = (A - B)^{-1}(A + B)$ and $q = (A - B)^{-1}b$. Then we only prove that M is positive definite. First, we show that $(A - B)$ is invertible. If not, for some nonzero $x \in \mathbb{R}^n$, we have that $(A - B)x = 0$, which derives a contradiction. This implies that $Ax = Bx$. Hence,

$$\begin{aligned}
\sigma_{\min}(A) &= \min_{\|z\|=1} \langle A^T Az, z\rangle \leq \langle A^T Ax, x\rangle = \langle B^T Bx, x\rangle \\
&\leq \max_{\|z\|=1} \langle B^T Bz, z\rangle = \sigma_{\max}(B)
\end{aligned}$$

which contradicts our condition. Hence $(A - B)^{-1}$ is invertible. Now, we prove that M is positive definite. To this end, we have, for all nonzero $x \in \mathbb{R}^n$,

$$\langle (A - B)^{-1}(A + B)x, x\rangle = \langle (A + B)x, (A^T - B^T)^{-1}x\rangle.$$

Letting $(A^T - B^T)^{-1}x = z$, one has

$$\begin{aligned}
\langle (A - B)^{-1}(A + B)x, x\rangle &= \langle (A + B)(A^T - B^T)z, z\rangle \\
&= \langle (AA^T - BB^T - AB^T + BA^T)z, z\rangle \\
&= \langle (AA^T - BB^T)z, z\rangle + \langle (BA^T - AB^T)z, z\rangle.
\end{aligned}$$

Note that $\langle (BA^T - AB^T)z, z\rangle = 0$. $(A - B)^{-1}(A + B)$ is positive definite if and only if $(AA^T - BB^T)$ is positive definite. Indeed, by Weyl's inequalities [Matrix Analysis: Theorem 4.3.1 in [20]], we find that

$$\lambda_{\min}(AA^T - BB^T) \geq \lambda_{\min}(AA^T) + \lambda_{\min}(-BB^T) = \lambda_{\min}(AA^T) - \lambda_{\max}(BB^T).$$

From Assumption 1, $\lambda_{\min}(AA^T) - \lambda_{\max}(BB^T) > 0$, $(AA^T - BB^T)$ is positive definite. Hence $(A - B)^{-1}(A + B)$ is positive definite. Consequently, M is a $\mathscr{P}$-matrix. Thus, from Theorem 2.7, the standard LCP has a unique solution for any $q \in \mathbb{R}^n$, so is the AVE (1.1) for any $b \in \mathbb{R}^n$. This completes the proof. □

Remark 2.11. For AVE (1.2), Assumption 1 becomes $\sigma_{min}(A) > 1$ and the unique solvability was investigated by Mangasarian and Meyer [21].

Corollary 2.12. *Under Assumption 1, the vector $x^* = x_*^+ - x_*^-$ is the unique solution of AVE (1.1) if and only if the pair of vectors (x_*^+, x_*^-) is the unique solution of HLCP (2.4).*

Proof. Note that AVE (1.1) is uniquely solvable for every b. Let x^* be its unique solution, i.e., x^* satisfies

$$Ax^* - B|x^*| = b.$$

From (2.3), $x^* = x_*^+ - x_*^-$ and $|x^*| = x_*^+ + x_*^-$, we arrive at

$$A(x_+^* - x_-^*) - B(x_+^* + x_-^*) = b,$$

which is equivalent to

$$(A-B)x_+^* - (A+B)x_-^* = b.$$

Let $A-B=N, A+B=M$ and $q=b$. Hence $Nx_+^* - Mx_-^* = q$. From (2.3), we have $x_+^*, x_-^* \geq 0$ and $\langle x_+^*, x_-^* \rangle = 0$. Consequently, the pair (x_+^*, x_-^*) is a solution of HLCP (2.4). Note that the HLCP is reduced to an $\mathscr{P}$-LCP. From Theorem 2.7, we see that this solution is unique. Conversely, with the same manner, we can prove the inverse implication. This completes the proof. □

3. Solving AVE (1.1) via interior-point methods

In this section, we numerically solve the AVE (1.1) by the application of an infeasible primal-dual version of interior-point methods for solving the equivalent HLCP (2.4). We need to put it in the framework of interior-point methods. Here, we prefer directly tackle the HLCP rather than the standard LCP because the matrix inversion may be ill-conditioned. It is known that solving HLCP is equivalent to solve the following nonlinear system of equations

$$(S) \qquad \begin{cases} Nx^+ - Mx^- = q, x^+ \geq 0, x^- \geq 0 \\ X^+x^- = 0, \end{cases} \tag{3.1}$$

where $X^+ := \mathrm{Diag}(x^+)$. In system (3.1), the first equation denotes the feasibility of HLCP and the second one is called the complementarity condition of HLCP. Next, the feasible set and the strictly feasible set of LCP are denoted by

$$\mathscr{F} = \left\{(x^+, x^-) \in \mathbb{R}_+^n \times \mathbb{R}_+^n : \; Nx^+ = Mx^- + q\right\},$$

and

$$\mathscr{F}^0 = \left\{(x^+, x^-) \in \mathscr{F}, x^+ > 0, x^- > 0\right\},$$

respectively. In addition to the strict monotony of HLCP (2.4) (Theorem 2.10), we assume in the sequel that $\mathscr{F}^0 \neq \varnothing$ this assumption is known as the interior-point condition (IPC). These two conditions together play an important role for the analysis of the convergence of infeasible interior-point algorithms for solving HLCP (see, e.g., [22, 23, 24]). The path-following interior-point methods [24] are based on replacing the second equation in (3.1) by the parameterized equation $X^+x^- = \mu e$, where $\mu > 0$. Thus, we consider the following perturbed system (S_μ):

$$(S_\mu) \qquad F_\mu(x^+, x^-) = \begin{pmatrix} Nx^+ - Mx^- - b \\ X^+x^- - \mu e \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{3.2}$$

Under the condition of IPC, S_μ has a unique solution denoted by $((x^+(\mu), x^-(\mu))$ for all $\mu > 0$. The set

$$\mathscr{C} = \left\{(x^+(\mu), x^-(\mu)) : \mu > 0\right\}$$

is called the central-path of HLCP. If $\mu \to 0$, then the limit of the central-path exists. Since the limit point satisfies the complementarity condition, the limit yields a solution of HLCP [23]. The central-path has the property, that is, a smooth trajectory and runs through the set $\mathscr{F}^0$ and keeps the iterates at an adequate distance from the non-optimal solutions and ends at an optimal solution of the HLCP. Therefore, almost all primal-dual path-following interior-point methods use the central-path as a guide for obtaining a solution of HLCP. Now, we are in a position to define search directions $(\Delta x^+, \Delta x^-)$ that move in the direction of the central-path $\mathscr{C}$. Applying Newton's method for S_μ in (3.2), we get

$$\nabla F_\mu(x^+, x^-) \begin{pmatrix} \Delta x^+ \\ \Delta x^- \end{pmatrix} = -F_\mu(x^+, x^-),$$

which is equivalent to the following linear system

$$\begin{cases} N\Delta x^+ - M\Delta x^- = r_f, \\ X^-\Delta x^+ + X^+\Delta x^- = r_c, \end{cases} \tag{3.3}$$

where X^-=Diag(x^-), $r_f = Mx^- + b - Nx^+$ and $r_c = \mu e - X^+x^-$. Note that if x^+ and x^- are in $\mathscr{F}^0$, strictly feasible, then $r_f = 0$. The unique solution of (3.3) is guaranteed by assumptions $\mathscr{F}^0 \neq \varnothing$, and the strict monotony of HLCP since the block matrix

$$\begin{pmatrix} N & -M \\ X^- & X^+ \end{pmatrix}$$

is invertible (see Proposition 3.1 in [23]).

3.1. **An infeasible path-following algorithm for AVE.** We present an infeasible path-following interior-point algorithm for computing the unique solution of HLCP (2.4) that uses the primal-dual interior-point framework considered by many authors. In each iteration, the algorithm starts with the initial (vectors) $x^+, x^- > 0$ not necessarily feasible. We would like to update these vectors until we are in our desired tolerance which satisfies (3.1). We stop our algorithm when the

$$\max\left(\left\|Nx^+ - Mx^- - b\right\|, \left\|X^+x^-\right\|\right)$$

is small enough (3.1). The outline of the algorithm is presented in Figure 1. In order to implement our algorithm, we need to compute a direction from (3.3) and a suitable step size $\alpha > 0$ in each iteration such that $x^+ + \alpha\Delta x^+ > 0$ and $x^- + \alpha\Delta x^- > 0$.

3.2. **The algorithm.**

Input:
An accuracy parameter $\varepsilon > 0$;
initial guesses $x^+, x^- > 0$ and $\mu > 0$;
A pair of matrices $[A, B]$, a vector b with $N = A - B$ and $M = A + B$;
While $\max(\|Nx^+ - Mx^- - b\|, \|X^+x^-\|) > \varepsilon$ **do**
begin
- Solve system (3.3) to obtain $(\Delta x^+, \Delta x^-)$;
- Determine a step size $\alpha > 0$ s.t. $x^+ + \alpha\Delta x^+ > 0$ and $x^- + \alpha\Delta x^- > 0$;
- Update $x^+ := x^+ + \alpha\Delta x^+$, $x^- := x^- + \alpha\Delta x^-$;
end

Figure 1. Infeasible interior-point algorithms for solving AVE.

The direction is determined by solving (3.3). We compute the damped Newton step-size $\alpha > 0$ so that $x^+ + \alpha\Delta x^+ > 0$ and $x^- + \alpha\Delta x^- > 0$. We need to determine the maximum possible step-size $\alpha_{\max}$ so that if

$0 < \alpha < \alpha_{\max}$, then $x^+ + \alpha\Delta x^+ > 0$ and $x^- + \alpha\Delta x^- > 0$. Let α_{x^+} and α_{x^-} be the maximum possible step-sizes in the directions Δx^+ and Δx^-, respectively. Then our maximum step-size is $\alpha_{\max} = \min(\alpha_{x^+}, \alpha_{x^-})$, where

$$\alpha_{x^+} = \begin{cases} \min_i -\frac{x_i^+}{\Delta x_i^+}, & \text{if} \quad \Delta x_i^+ < 0, i = 1, \dots, n, \\ 1, & \quad\quad \text{otherwise,} \end{cases}$$

and

$$\alpha_{x^-} = \begin{cases} \min_i -\frac{x_i^-}{\Delta x_i^-}, & \text{if} \quad \Delta x_i^- < 0, i = 1, \dots, n \\ 1, & \quad\quad \text{otherwise.} \end{cases}$$

Then, we compute our step-size as $\alpha = \min(\rho\alpha_{\max}, 1)$, for some $\rho \in (0, 1]$, which guaranties that our next iterates will be positive. For specifying the parameter $\mu > 0$, we easily see from the second equation in S_μ that $\mu = \frac{\langle x^+, x^- \rangle}{n}$, which is known as the duality gap of HLCP. The polynomial complexity of these algorithms is proved by many authors in the literature of infeasible interior-point methods; see, e.g., [22, 23, 24] and the references therein.

4. Numerical results

In this section, the pair $(x_0^+, x_0^-) > 0$ denotes the infeasible starting point for the algorithm and the pair (x_*^+, x_*^-) denotes the unique solution of the HLCP. The number of iterations and the solution for the AVE (1.1) are denoted by "**It**" and $x^* = x_*^+ - x_*^-$, respectively. The implementation of Algorithm 3.2 is running on MATLAB 7.1. The accuracy used in the two first problems is set to $\varepsilon = 10^{-6}$ and the parameter ρ is chosen such that $0.85 \le \rho \le 0.95$.

Problem 1. The matrices A and B, and vector $b \in \mathbb{R}^n$ are given by

$$A = (a_{ij}) = \begin{cases} 5, & \text{if} \quad i = j, \\ 1, & \text{if} \quad |i-j| = 1 \quad \forall i, j = 1, .., n, \\ 0, & \text{otherwise,} \end{cases}$$

$$B = (b_{ij}) = \begin{cases} 1, & \text{if} \quad i = j, \\ 1, & \text{if} \quad |i-j| = 1 \quad \forall i, j = 1, .., n, \\ 0, & \text{otherwise} \end{cases}$$

and

$$b = (-8, -10, ..., -10, -8)^T \in \mathbb{R}^n.$$

The pair of initial point for this problem is

$$x_0^- = x_0^+ = (1, 2, ..., n)^T,$$

and the exact solution is $x^* = e$. The numerical results for different size of n are summarized in Table 1.

n	**It**	**CPU**
3	22	0.0510
10	19	0.0592
100	15	0.8542
500	12	39.0083
1000	11	248.945

Table 1.

Algorithm 3.2 gives an approximate solution of Problem 1 as

$$x^* = (0.99995,\, 0.99995,\, ...,\, 0.99995)^T.$$

Problem 2. The data (A,B,b) of the AVE is given by

$$A = (a_{ij}) = \begin{cases} 10, & \text{if} \quad i = j \\ -1, & \text{if} \quad |i-j| = 1 \quad \forall i,j = 1,..,n, \\ 0, & \text{otherwise,} \end{cases}$$

$$B = (b_{ij}) = \begin{cases} 5, & \text{if} \quad i = j, \\ -1, & \text{if} \quad |i-j| = 1 \quad \forall i,j = 1,..,n, \\ 0, & \text{otherwise} \end{cases}$$

and $b = (A-I)\,e$. The exact solution is

$$x^* = (1.6, 1.4, ..., 1.4, 1.6)^T.$$

The numerical results are summarized in Table 2.

n	**It**	**CPU**
3	21	0.0506
10	20	0.0640
100	18	0.956
520	16	62.240
1000	16	385.66

Table 2.

Algorithm 3.2 gives an approximate solution of Problem 2 as:

$$x^* = (1.59999,\, 1.39999,\, ...,\, 1.59999)^T.$$

In the next example, we compare Algorithm 3.2 with a feasible primal-dual IPMs used to solve the AVE (1.2). This latter is developed by Long ([16]).

Problem 3 [16]. The data (A,B,b) of the AVE is given by

$$\begin{aligned} &rand('state',0); R = rand(n,n); \\ &A = R' * R + n * eye(n); b = rand(n,1); \\ &B = I; \\ &q = ((A+eye(n)) * (inv(A-eye(n))) - eye(n)) * b. \end{aligned}$$

The numerical results are summarized in Table 3, where Algorithm 3.2 is denoted by **Algo1** and the algorithm used in [16] is denoted by **Algo2**. From [16], we use the same tolerance $\varepsilon = 10^{-4}$.

Size $n\downarrow$	Algorithms $\downarrow$	**It**	**CPU**
8	**Algo**1	13	0.1103
	Algo2	19	8.1350
32	**Algo**1	12	0.1445
	Algo2	34	9.1270
128	**Algo**1	10	0.3802
	Algo2	94	15.0780
512	**Algo**1	7	18.3298
	Algo2	147	21.6560
1024	**Algo**1	5	94.8895
	Algo2	171	37.9070

Table 3.

Remark 4.1. From the numerical results stated in the above tables, we see that Algorithm 3.2 offers a solution of the absolute value equations. In addition, the numerical results, obtained for Problem 3, confirm that Algorithm 3.2 performs well in comparison with those obtained via the feasible interior-point methods since the number of iterations and CPU times are almost less. From above tables, we also mention that the advantage of interior-point methods for solving this problem is that the higher dimension of the problem and the lower number of iterations are obtained.

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