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FIXED POINTS OF PREŠIĆ-ĆIRIĆ TYPE FUZZY OPERATORS

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Abstract. In this paper, we introduce a Prešić-Ćirić type fuzzy mapping in product spaces. A fixed point result for the mapping in complete metric spaces is obtained. Our result extends and generalizes some known results of the literature for fuzzy mappings. An example is given to illustrate the main result of this paper.

Keywords. Fuzzy mapping; Prešić-Ćirić type fuzzy operator; Fixed point.

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1. Introduction and Preliminaries

The fixed point theory is the most useful tool in various mathematical models, which appear in the practical problems. In particular, the Banach contraction principle [1] provides a constructive method for finding a unique solution for models involving various types of differential and integral equations.

The Banach contraction principle has been generalized by several authors in various directions. Nadler [2] considered the operators, which map from a complete metric space to a collection of nonempty, closed and bounded subsets of the space. The notion of fuzzy mappings was introduced by Heilpern [3]. He defined mappings from an arbitrary set to one subfamily of fuzzy sets in a metric linear space X. The elements of this family are the approximate quantities. He also defined the distance between two approximate quantities and proved a fixed point result for the fuzzy mappings. The fixed point result of Heilpern [3] was a fuzzy analogue of the fixed point result of Nadler [2]. Subsequently, several authors obtained fixed point results for fuzzy mappings in various settings, see, e.g., [4, 5, 6, 7, 8, 9, 10] and the references therein.

On the other hand, Prešić [11, 12] generalized the Banach contraction principle in a product space and proved a fixed point result for self mappings defined on the product of space. Let (X,d) be a complete metric space. Let k be a positive integer and let $T\colon X^k \to X$ be a mapping. Prešić's theorem shows that under some restrictions the mapping T has a fixed point, i.e., a point $x \in X$ such that $T(x,x,\dots,x) = x$,

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and for arbitrary $x_1, x_2, \ldots, x_k \in X$, the iterative sequence $(x_n)_{n\in\mathbb{N}}$ converges to this fixed point of T, where

$$x_{n+k} = T(x_n, x_{n+1}, \ldots, x_{n+k-1}),\ n \in \mathbb{N}.$$

Ćirić and Prešić [13] generalized the result of Prešić by improving the contractive condition on mapping T. The result of Prešić and its generalizations have several applications, for instance, in finding global attractors of difference equations and matrix equations, in finding equilibrium points of difference inclusions, in finding solutions of differential equations etc. (see, [14, 15, 16, 17, 18, 19, 20]). Shukla, Sen and Radenović [21] extended Prešić's results for set-valued mappings and proved some fixed point results for set-valued Prešić type mappings in metric spaces. Recently, the result of Sen and Radenović [21] was further generalized in several directions, see, e.g., [18, 22, 23, 24, 25] and the references therein.

In this paper, we introduce the Prešić-Ćirić type fuzzy mappings in product spaces and extend the definition of fuzzy mappings into the product spaces. We also prove a fixed point result for Prešić-Ćirić type fuzzy mappings in product spaces which generalizes the results of Prešić [11, 12], Ćirić and Prešić [13], Shukla, Sen and Radenović [21] and Heilpern [3]. An example is provided which illustrates the main result of this paper.

First, we state some definitions and properties about fuzzy sets and fuzzy mappings which will be needed in the sequel. The following terminology is adapted from [3].

Let (X,d) be a metric space and 2^X denote the collection of all possible subsets of X. We use the following notations:

$$\begin{aligned} CL(X) &= \{A \subseteq X : A \text{ is nonempty and closed}\},\\ C(X) &= \{A \subseteq X : A \text{ is nonempty and compact}\},\\ CB(X) &= \{A \subseteq X : A \text{ is nonempty, closed and bounded}\}. \end{aligned}$$

For $A, B \in CB(X)$ we define

$$d(x,A) = \inf\{d(x,y) : y \in A\}.$$

For $A, B \in CB(X)$, we define

$$\delta(A,B) = \sup\{d(x,B) : x \in A\},$$

and

$$H(A,B) = \max\{\delta(A,B), \delta(B,A)\}.$$

Then H is a metric on $CB(X)$ and is called Hausdorff (Pompeiu-Hausdorff) metric.

A function F whose domain is X and values lie in the interval $[0,1]$ is called a fuzzy set in X. By $\mathfrak{F}(X)$, we denote the collection of all fuzzy sets in X. For $x \in X$ and $F \in \mathfrak{F}(X)$, the value $F(x)$ is called the grade of membership of x in F. For $\alpha \in (0,1]$, we denote by $[F]_\alpha$ the α-level set of $F \in \mathfrak{F}(X)$ and

$$[F]_\alpha = \{x \in X : F(x) \geq \alpha\}.$$

For $\alpha = 0$, we define

$$[F]_0 = \overline{\{x \in X : F(x) > 0\}},$$

where $\overline{A}$ denotes the closure of the set $A \subseteq X$. For $F, G \in \mathfrak{F}(X)$, the inclusion $F \subseteq G$ means $F(x) \leq G(x)$ for all $x \in X$. It is obvious that if $0 < \alpha \leq \beta \leq 1$, then $F_\beta \subseteq F_\alpha$. For each $\alpha \in (0,1]$ and $x \in X$, we define a fuzzy set $\{x\}_\alpha$ by

$$\{x\}_\alpha(y) = \begin{cases} \alpha, & \text{if } y = x; \\ 0, & \text{otherwise.} \end{cases}$$

In particular, for $\alpha = 1$, we obtain the characteristic function of the crisp set $\{x\} \subseteq X$

$$\chi_{\{x\}}(y) = \{x\}_1(y) = \begin{cases} 1, & \text{if } y = x; \\ 0, & \text{otherwise.} \end{cases}$$

For each $\alpha \in (0,1]$, a sub-collection $\mathscr{F}_\alpha(X) \subseteq \mathfrak{F}(X)$ is defined by:

$$\mathscr{F}_\alpha(X) = \{F \in \mathfrak{F}(X) \colon [F]_\alpha \text{ is nonempty bounded and closed}\}.$$

For fixed $\alpha \in [0,1]$ and $F, G \in \mathscr{F}_\alpha(X)$, we define

$$D_\alpha(F,G) = H([F]_\alpha, [G]_\alpha).$$

Let Y be an arbitrary nonempty subset of X and $\alpha \in [0,1]$. Then, a mapping $T \colon Y \to \mathscr{F}_\alpha(X)$ is called a fuzzy mapping over the set Y. More generally, (see [5]) for a nonempty set X, a mapping $T \colon X \to \mathfrak{F}(X)$ is called a fuzzy mapping. For a fixed $\alpha \in [0,1]$ and a point $x \in X$, the fuzzy set $\{x\}_\alpha$ is called a fixed point of the fuzzy mapping $T \colon X \to \mathfrak{F}(X)$ if $\{x\}_\alpha \subseteq Tx$.

The following lemmas will be used in the sequel.

Lemma 1.1 ([2]). *Let (X,d) be a metric space. Let A and B be two nonempty, closed and bounded subsets of a metric space X. If $a \in A$, then $d(a,B) \leq H(A,B)$.*

Lemma 1.2 ([2]). *Let (X,d) be a metric space, A and B be two nonempty, closed and bounded subsets of a metric space X and $\varepsilon > 0$. Then, for every $a \in A$ there exists $b \in B$ such that $d(a,b) \leq H(A,B) + \varepsilon$.*

Remark 1.3. Let $A, B \in CB(X)$ and let $h \in (1,\infty)$ be given. Then, for $a \in A$, there exists $b \in B$ such that $d(a,b) \leq hH(A,B)$.

It is well known that Lemma 1.2 and the Remark 1.3 are equivalent.

Remark 1.4. Let $A, B \in \mathscr{F}_\alpha(X)$ and let $h \in (1,\infty)$ be given. Then, for $\{a\}_\alpha \subseteq A$, there exists $b \in X$ such that

$$\{b\}_\alpha \subseteq B \text{ and } d(a,b) \leq hD_\alpha(A,B).$$

Proof. Let $\{a\}_\alpha \subseteq A$. Then $A(a) \geq \alpha$, i.e., $a \in [A]_\alpha$. Because $B \in \mathscr{F}_\alpha(X)$, we have that $[B]_\alpha$ is nonempty and closed. By Remark 1.3, there exists $b \in [B]_\alpha$ such that

$$d(a,b) \leq hH([A]_\alpha, [B]_\alpha).$$

From the definition, we have

$$d(a,b) \leq hD_\alpha(A,B).$$

□

Definition 1.5 ([13]). Let (X,d) be a metric space. Let k be a positive integer and let $T \colon X^k \to X$ be a mapping. Then, T is called a Prešić-Ćirić mapping if there exists $\lambda \in [0,1)$ such that

$$d(T(x_1,\ldots,x_k), T(x_2,\ldots,x_{k+1})) \leq \lambda \cdot \max\{d(x_i,x_{i+1}) \colon 1 \leq i \leq k\}$$

for all $x_1,\ldots,x_k,x_{k+1} \in X$.

2. Main Results

In what follows, we always assume that $\alpha \in [0,1]$ is a fixed number.

Definition 2.1. Let X be a nonempty set and k be a positive integer. Then, for $x \in X$, the fuzzy set $\{x\}_\alpha$ is called a fixed point of a fuzzy mapping $T\colon X^k \to \mathfrak{F}(X)$ if $\{x\}_\alpha \subseteq T(x,x,\dots,x)$, i.e.,

$$T(x,x,\dots,x)(x) \geq \alpha.$$

First, we define the Prešić-Ćirić type fuzzy operators on a metric space.

Definition 2.2. Let (X,d) be a metric space. Let k be a positive integer and let $T\colon X^k \to \mathscr{F}_\alpha(X)$ be a fuzzy mapping. Then, T is called a Prešić-Ćirić type fuzzy operator if

$$D_\alpha(T(x_1,\dots,x_k),T(x_2,\dots,x_{k+1})) \leq \lambda \cdot \max\{d(x_i,x_{i+1})\colon 1 \leq i \leq k\} \tag{2.1}$$

for all $x_1,\dots,x_k,x_{k+1} \in X$, where $\lambda \in (0,1)$.

Now, we prove a fixed point theorem for Prešić-Ćirić type fuzzy operators in complete metric spaces.

Theorem 2.3. *Let (X,d) be a complete metric space. Let k be a positive integer and let $T\colon X^k \to \mathscr{F}_\alpha(X)$ be a Prešić-Ćirić type fuzzy operator. Then T has a fixed point in X.*

Proof. Let $x_1,\dots,x_k$ be arbitrary points in X. Since $T(x_1,\dots,x_k) \in \mathscr{F}_\alpha(X)$, we can choose $x_{k+1} \in X$ such that

$$\{x_{k+1}\}_\alpha \subseteq T(x_1,\dots,x_k).$$

Again, using Remark 1.4 and

$$T(x_2,\dots,x_{k+1}) \in \mathscr{F}_\alpha(X),$$

we can choose $x_{k+2} \in X$ such that

$$\{x_{k+2}\}_\alpha \subseteq T(x_2,\dots,x_{k+1})$$

and

$$d(x_{k+1},x_{k+2}) \leq \frac{1}{\sqrt{\lambda}} D_\alpha(T(x_1,\dots,x_k),T(x_2,\dots,x_{k+1})).$$

Using (2.1), we have

$$\begin{aligned} d(x_{k+1},x_{k+2}) &\leq \frac{1}{\sqrt{\lambda}} \lambda \max\{d(x_i,x_{i+1})\colon 1 \leq i \leq k\} \\ &= \sqrt{\lambda} \max\{d(x_i,x_{i+1})\colon 1 \leq i \leq k\}. \end{aligned}$$

Similarly, there exists $x_{k+3} \in X$ such that

$$\{x_{k+3}\}_\alpha \subseteq T(x_3,\dots,x_{k+2})$$

and

$$\begin{aligned} d(x_{k+2},x_{k+3}) &\leq \frac{1}{\sqrt{\lambda}} D_\alpha(T(x_2,\dots,x_{k+1}),T(x_3,\dots,x_{k+2})) \\ &= \sqrt{\lambda} \max\{d(x_i,x_{i+1})\colon 2 \leq i \leq k+1\}. \end{aligned}$$

Continuing this process, we obtain that, for every $n \in \mathbb{N}$, there exists $x_{n+k} \in X$ such that

$$\{x_{n+k}\}_\alpha \subseteq T(x_n,\dots,x_{n+k-1})$$

and

$$d(x_{n+k},x_{n+k+1}) \leq \sqrt{\lambda}\max\{d(x_i,x_{i+1})\colon n\leq i\leq n+k-1\}. \tag{2.2}$$

We shall show that the sequence $(x_n)_n^\infty$ is a Cauchy sequence. For convenience, set $\xi = \lambda^{\frac{1}{2k}}$ and

$$\mu = \max\left\{\frac{d(x_1,x_2)}{\xi},\frac{d(x_2,x_3)}{\xi^2},\ldots,\frac{d(x_k,x_{k+1})}{\xi^k}\right\}.$$

We claim that

$$d(x_n,x_{n+1}) \leq \mu\xi^n \quad \text{for all} \quad n\in\mathbb{N}. \tag{2.3}$$

It is obvious that (2.3) is true for $n = 1,2,\ldots,k$. Suppose the following inequalities hold

$$\begin{aligned} d(x_n,x_{n+1}) &\leq \mu\xi^n, d(x_{n+1},x_{n+2}) \\ &\leq \mu\xi^{n+1},\ldots,d(x_{n+k-1},x_{n+k}) \\ &\leq \mu\xi^{n+k-1}. \end{aligned}$$

Using (2.2) and the above inequalities, we obtain

$$\begin{aligned} d(x_{n+k},x_{n+k+1}) &\leq \sqrt{\lambda}\max\{d(x_i,x_{i+1})\colon n\leq i\leq n+k-1\} \\ &\leq \sqrt{\lambda}\max\{\mu\xi^i\colon n\leq i\leq n+k-1\} \\ &= \sqrt{\lambda}\mu\xi^n \qquad (\text{since } \xi=\lambda^{\frac{1}{2k}}<1) \\ &= \mu\xi^{n+k}. \end{aligned}$$

Therefore, by induction, we see that inequality (2.3) is true for all $n\in\mathbb{N}$. Now, for $n,m\in\mathbb{N}$, $m>n$, we have

$$\begin{aligned} d(x_n,x_m) &\leq d(x_n,x_{n+1})+d(x_{n+1},x_{n+2})+\cdots+d(x_{m-1},x_m) \\ &\leq \mu\xi^n+\mu\xi^{n+1}+\cdots+\mu\xi^{m-1} \\ &\leq \frac{\mu\xi^n}{1-\xi}. \end{aligned}$$

As $\xi=\lambda^{\frac{1}{2k}}<1$, it follows from the above inequality that

$$\lim_{n,m\to\infty} d(x_n,x_m)=0.$$

Thus, $(x_n)_n^\infty$ is a Cauchy sequence. By the completeness of X, there exists $u\in X$ such that $\lim_{n\to\infty}x_n=u$.

We next show that $\{u\}_\alpha$ is a fixed point of T. Since

$$\{x_{n+k}\}_\alpha \subseteq T(x_n,\ldots,x_{n+k-1})$$

for all $n\in\mathbb{N}$, we have

$$x_{n+k}\in[T(x_n,\ldots,x_{n+k-1})]_\alpha \text{ for all } n\in\mathbb{N}.$$

Therefore, using Lemma 1.1 and the triangular inequality, we have

$$\begin{aligned}
& d\left(u,[T(u,\ldots,u)]_\alpha\right) \\
\leq\ & d(u,x_{n+k})+d\left(x_{n+k},[T(u,\ldots,u)]_\alpha\right) \\
\leq\ & d(u,x_{n+k})+H\left([T(x_n,\ldots,x_{n+k-1})]_\alpha,[T(u,\ldots,u)]_\alpha\right) \\
\leq\ & d(u,x_{n+k})+H\left([T(x_n,\ldots,x_{n+k-1})]_\alpha,[T(x_{n+1},\ldots,x_{n+k-1},u)]_\alpha\right) \\
& +H\left([T(x_{n+1},\ldots,x_{n+k-1},u)]_\alpha,[T(x_{n+2},\ldots,x_{n+k-1},u,u)]_\alpha\right) \\
& +\cdots+H\left([T(x_{n+k-1},u,\ldots,u)]_\alpha,[T(u,\ldots,u)]_\alpha\right).
\end{aligned}$$

Using the definition of D_α and (2.1) in the above inequality, we obtain

$$\begin{aligned}
& d\left(u,[T(u,\ldots,u)]_\alpha\right) \\
\leq\ & d(u,x_{n+k})+\lambda\max\{d(x_n,x_{n+1}),\ldots,d(x_{n+k-1},x_{n+k-1}),d(x_{n+k-1},u)\} \\
& +\lambda\max\{d(x_{n+1},x_{n+2}),\ldots,d(x_{n+k-1},x_{n+k-1}),d(x_{n+k-1},u)\} \\
& +\cdots+\lambda d(x_{n+k-1},u).
\end{aligned}$$

As $n\to\infty$ in the above inequality, we have

$$d\left(u,[T(u,\ldots,u)]_\alpha\right)=0,$$

i.e., $u\in[T(u,\ldots,u)]_\alpha$. Therefore, it follows from the definition of α-level set that

$$\{u\}_\alpha\subseteq T(u,\ldots,u).$$

Thus, u is a fixed point of T. □

Next, we give an example to illustrate the above theorem.

Example 2.4. Let $X=\{0,1,2\}$ and let $d\colon X\times X\to[0,\infty)$ be defined by: $d(x,y)=d(y,x), d(x,x)=0$ for all $x,y\in X$ and

$$d(0,1)=2,\ d(0,2)=3,\ d(1,2)=5.$$

Then (X,d) is a complete metric space. Define a mapping $T\colon X^2\to\mathscr{F}_\alpha(X)$ such that

$$T(0,0)(x)=T(1,1)(x)=\begin{cases}\alpha, & \text{if } x=0;\\ \alpha/2, & \text{if } x=1;\\ \alpha/3, & \text{is } x=2.\end{cases}\qquad T(0,1)(x)=\begin{cases}\alpha, & \text{if } x=0;\\ 0, & \text{if } x=1;\\ \alpha/3, & \text{is } x=2.\end{cases}$$

$$T(0,2)(x)=\begin{cases}\alpha, & \text{if } x=0;\\ \alpha/2, & \text{if } x=1;\\ 0, & \text{is } x=2.\end{cases}\qquad T(1,2)(x)=\begin{cases}\alpha, & \text{if } x=0;\\ 0, & \text{if } x=1;\\ 0, & \text{is } x=2.\end{cases}$$

$$T(2,2)(x)=\begin{cases}0, & \text{if } x=0;\\ \alpha, & \text{if } x=1;\\ 0, & \text{is } x=2.\end{cases}\qquad T(x,y)=T(y,x)\text{ for all } x,y\in X.$$

Then it is easy to see that

$$[T(0,0)]_\alpha=[T(1,1)]_\alpha=[T(1,2)]_\alpha=[T(0,2)]_\alpha=[T(0,1)]_\alpha=\{0\}$$

and $[T(2,2)]_\alpha=\{1\}$.

We show that condition (2.1) is satisfied. Then, we have to check the validity of condition (2.1) only when $x_1 \in \{0,2\}$ and $x_2, x_3 \in \{2\}$ and other cases are either similar or trivial. If $x_1 = 0, x_2 = x_3 = 2$, then

$$D_\alpha(T(x_1,x_2),T(x_2,x_3)) = H([T(0,2)]_\alpha,[T(2,2)]_\alpha) = d(0,1) = 2$$

and

$$\max\{d(x_1,x_2),d(x_2,x_3)\} = d(0,2) = 3.$$

Therefore, (2.1) is satisfied for $\lambda \in [2/3,1)$. If $x_1 = 1, x_2 = x_3 = 2$, then

$$D_\alpha(T(x_1,x_2),T(x_2,x_3)) = H([T(1,2)]_\alpha,[T(2,2)]_\alpha) = d(0,1) = 2$$

and

$$\max\{d(x_1,x_2),d(x_2,x_3)\} = d(1,2) = 5.$$

Therefore, (2.1) is satisfied for $\lambda \in [2/5,1)$. Thus, the condition (2.1) is satisfied with $\lambda \in [2/3,1)$ and T is a Prešić-Ćirić type fuzzy operator. All the other conditions of Theorem 2.3 are satisfied and T has a fixed point, namely, $\{0\}_\alpha \subseteq T(0,0)$ is the fixed point of T.

The following corollary is the fuzzy analogue of the result of Shukla et al. [21] and extends the result of Prešić [11, 12] to fuzzy mappings.

Corollary 2.5. *Let (X,d) be a complete metric space. Let k be a positive integer and let $T: X^k \to \mathscr{F}_\alpha(X)$ be a mapping satisfying the following condition:*

$$D_\alpha(T(x_1,\ldots,x_k),T(x_2,\ldots,x_{k+1})) \leq \sum_{i=i}^{k} q_i d(x_i,x_{i+1})$$

for all $x_1,\ldots,x_k,x_{k+1} \in X$, where q_i's are nonnegative constants such that $\sum_{i=1}^{k} q_i < 1$. Then T has a fixed point in X.

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