J. Nonlinear Funct. Anal. 2025 (2025) 30 https://doi.org/10.23952/jnfa.2025.30

Journal of Nonlinear Functional Analysis
Available online at http://jnfa.mathres.org

ON A GENERALIZATION OF PETRYSHYN'S FIXED POINT THEOREM

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Abstract. The Petryshyn's fixed point theorem is applied for the condensing maps with respect to Kuratowski measure of noncompactness satisfying a boundary condition. This paper extends this theorem by using a new boundary condition and the measure of noncompactness defined by the system of axioms.
Keywords. Condensing map, Darbo k-set contraction, Petryshyn's fixed point theorem.
2020 MSC. 47H09, 47H10.

1. Introduction

Fixed points of nonlinear operators are powerful in various research fields. The measure of noncompactness, which is a useful tool in nonlinear functional analysis, can used employed to investigate existence results for various nonlinear differential equations as well as nonlinear functional integral equations in various Banach spaces; see, e.g., [3, 4, 10, 11, 13]. Recently, authors investigate various nonlinear problem by using fixed points of nonlinear operators, which can be written as a condensing operator with respect to a suitable measure of noncompactness. Compact operators are important in obtaining strong convergence of various schemes in infinite dimensional spaces. For condensing operators, they are also known as the operators satisfying a certain condition concerning compactness. Condensing operators are a generalizations of compact operators. One mentions here that measure of noncompactness was first introduced as Kuratowski or Hausdorff measure of noncompactness. From [14], one sees measure of noncompactness is one of the main roles in proving the well-known Darbo's fixed point theorem. In [3, 4], the authors gaven an axiomatic definition of the measure of noncompactness. There are several systems of axioms which are not equivalent. However they have some axioms common. The Darbo's fixed point theorem, which finds extensively real applications in numerous fields, such as economics, was extended in different directions; see, e.g., [4, 6, 8, 9]. But most of these generalizations required a self-mapping condition for the operators involved, that is $F : D \to D$ while varying the conditions imposed on the condensing operator. In [12], Petryshyn replaced

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Received July 7, 2025; Accepted September 26, 2025.

the self-mapping condition by a weaker *boundary condition* which was first introduced in case that the domain D is a ball centered at the origin.

In this paper, we consider themeasure of noncompactness defined by a simple system of axioms to prove a generalization of the Darbo's fixed point theorem. W apply the fixed point theorem to solve a Hadamard fractional integral equation. Although this equation was investigated for two variables and a fixed initial value of the solution in [1], we emphasize that the initial value of the solution in our equation follows a compact operator.

2. Preliminaries

The following tools are needed for our main results.

Let $\mathscr{M}$ be a family of subsets of a Banach space E such that $\{u\} \in \mathscr{M}$ for all $u \in E$. If $\Omega \in \mathscr{M}$, then

(i) $\Omega \cup \{u\}, \Omega_1 \in \mathscr{M}$ whenever $\Omega_1 \subset \Omega$
(ii) $\overline{conv}(\Omega) \in \mathscr{M}$

Recall that an operator $\Phi : \mathscr{M} \to \mathbb{R}_+$ is said to be a measure of noncompactness (MNC for short) [10] if

$$\Phi(\overline{conv}(\Omega)) = \Phi(\Omega), \text{ for all } \Omega \in \mathscr{M}.$$

Example 2.1. [3] Let $X = C([a,b];\mathbb{R})$ be the space of continuous functions on $[a,b]$ with the supremum norm, and let $\mathscr{M}$ be the class of bounded subset in X. For $u \in X$ and $\delta > 0$, let

$$w(u,\delta) = \sup\{|u(t) - u(s)| : t,s \in [a,b], |t-s| \le \delta\}.$$

The *modulus of continuity* of the subset $\Omega \in \mathscr{M}$ is defined as

$$w(\Omega,\delta) = \sup\{w(u,\delta) : u \in \Omega\}.$$

Then the following operator is a MNC on $\mathscr{M}$

$$\mu(\Omega) = \lim_{\delta \to 0^+} w(\Omega,\delta).$$

The notion of Kuratowski MNC, which was introduced by Kuratowski, is defined by, for each bounded subset $\Omega \subset E$,

$$\alpha(\Omega) = \inf\{d > 0 : \Omega \subset \cup_{i=1}^n B_i, diam(B_i) \le d\},$$

where $diam(B_i) = \sup\{\|u - v\|_E : u, v \in E\}$; see [3].

Further more, we have the following example.

Example 2.2. [10] Let E be a Banach space and $X := C(I,E)$ be the space of continuous valued E functions on I with the supremum norm

$$|u| := \sup_{t \in I} \|u(t)\|, \ u \in X.$$

Let $\mathscr{M}$ stand for the class of bounded equicontinuous subsets in X. Then α is a MNC on $(X,\mathscr{M})$

Definition 2.3. The MNC Φ is said to be

(a) *regular* if it satisfies the following condition

$$\Phi(\Omega) = 0 \text{ if and only if } \Omega \text{ is relatively compact;}$$

(b) *monotone* if, for all $\Omega \in \mathscr{M}$ and $\Omega_1 \subset \Omega_2$, then $\Phi(\Omega_1) \leq \Phi(\Omega)$;
(c) *nonsingular* if, for all $\Omega \in \mathscr{M}$ and $u \in E$, then $\Phi(\Omega \cup \{u\}) = \Phi(\Omega)$;
(d) *positive-homogeneous* if, for all $\Omega \in \mathscr{M}$ and $k \geq 0$, then $\Phi(k\Omega) = k\Phi(\Omega)$;
(e) *semi-additive* if, for all $\Omega_1, \Omega_2 \in \mathscr{M}$, then $\Phi(\Omega_1 + \Omega_2) \leq \Phi(\Omega_1) + \Phi(\Omega_2)$.

Remark 2.4. Assume that the MNC Φ is regular, monotone, positive-homogeneous, and semi-additive. Let $u_0 \in E$. Then

$$\Omega_1 \subset \{k_1 u_0 + k_2 u : u_0 \in E, u \in \Omega_2, k_1, k_2 \geq 0\} \Rightarrow \Phi(\Omega_1) \leq k_2 \Phi(\Omega_2). \tag{2.1}$$

For the Kuratowski MNC, we can find the properties in [2]. In the following proposition, we list the ones which are used in the sequel.

Proposition 2.5. [2] *Let E be a Banach space and α be the Kuratowski MNC in E. Then*

(1) α is regular, monotone, nonsingular, homogeneous and semi-additive.
(2) If $A \subset C([0,T],E)$ is a equicontinuous set such that the set $A(t) = \{u(t) : u \in A\}$ is bounded for all $t \in [0,T]$, then the function $t \mapsto \alpha(A(t))$ is continuous and one has

$$\alpha\left(\left\{\int_0^t u(\tau)d\tau : u \in A\right\}\right) \leq \int_0^t \alpha(A(\tau))d\tau.$$

Definition 2.6. Let Φ be a MNC defined on a family $\mathscr{M}$ of subsets of the Banach space E. A continuous map $F : D \subset E \to E$ is said to be condensing with respect to Φ (or $\Phi-$*condensing*) if

(i) for every $\Omega \subset D$ such that $\Omega \in \mathscr{M}$, $F(\Omega) \in \mathscr{M}$;
(ii) for every $\Omega \subset D$ such that $\Omega \in \mathscr{M}$ and $\Phi(\Omega) > 0$, $\Phi(F(\Omega)) < \Phi(\Omega)$.

Some authors defined the condensing maps in the sense of k-set contractive maps. We next present the Darbo's fixed point theorem which proved the existence of a fixed point of the k-set contractive map with respect to Kuratowski MNC in the following theorem.

Theorem 2.7. [14] *Let C be a nonempty, convex, bounded, and closed set in a Banach space E, and let $F : C \to C$ be a k-set contractive mappings with respect to Kuratowski MNC α in E, that is, F is continuous and $\alpha(F(\Omega)) \leq k\alpha(\Omega)$ for all $\Omega \subset C$ and $\alpha(\Omega) > 0$, where $k \in (0,1)$. Then F has a fixed point theorem.*

We see that if F is k-set contractive with $k \in (0,1)$, then F is condensing. In the following theorem, Petryshyn proved the existence of fixed points of the condensing map F. However, the result was proved in the case that C is a ball and under the weaker *boundary condition* than the condition $F(C) \subset C$.

Theorem 2.8. [12] *Let B be an closed ball centered at the origin in a Banach space E with radius $r > 0$, and let $F : B \to E$ be a α-condensing mapping which satisfies the boundary condition*

$$(BC:) \text{ If } F(u) = ku \text{ for some } u \in \partial B, \text{ then } k \leq 1.$$

Then F has at least one fixed point in B.

3. Main Results

3.1. **The generalized Petryshyn's fixed point theorem.** We first prove the following theorem which was proved in [10] for another version.

Theorem 3.1. *Let E be a Fréchet space, $D \subset E$ be a nonempty, closed, and convex set, $F : D \to D$ be a continuous map, and Φ be a regular, nonsingular MNC defined on a family $\mathscr{M}$ of subsets of E. Moreover, let F be $\Phi-$condensing and $F(D) \in \mathscr{M}$. Then F has at least one fixed point in D.*

Proof. Let us choose a point $u \in \overline{conv}(F(D))$ and denote by $\mathscr{B}$ the class of all convex and closed subsets Ω of D such that $\Omega \in \mathscr{M}, u \in \Omega$ and $F(\Omega) \subset \Omega$. Set

$$B = \bigcap_{\Omega \in \mathscr{B}} \Omega, \;\; K = \overline{conv}\left(F(B) \cup \{u\}\right).$$

Since F is $\Phi-$condensing, one asserts taht $F(D) \in \mathscr{M}$. Thus $\overline{conv}(F(D)) \in \mathscr{B}$. Obviously, $B \in \mathscr{M}$. Furthermore, from $F(\Omega) \subset \Omega$ for all $\Omega \in \mathscr{B}$, it follows that $F(B) \subset B$. Hence $K \in \mathscr{M}$.

We proceed to show that $B = K$. Indeed, since $u \in B$ and $F(B) \subset B$, it follows that $K \subset B$, which implies $F(K) \subset F(B) \subset K$. Hence $K \in \mathscr{B}$, and $B \subset K$. Therefore the nonsingularity of Φ shows that

$$\Phi(B) = \Phi(K) = \Phi\left(F(B) \cup \{u\}\right) = \Phi\left(F(B)\right).$$

Since F is Φ-condensing, it follows that $\Phi(B) = 0$ and B is relative compact by the regularity of Φ. Thus, from the Schauder-Tychonoff theorem, one concludes that there exists a fixed point for the operator, $F : D \to D$. □

We are now in position to show the main result for this paper. It is a generalization of Petryshyn's fixed point theorem where the map acting in a nonempty convex closed subset in a Banach space and the (BC) condition has been changed to be suitable for the domain.

Theorem 3.2. *Let D be a nonempty, convex, and closed subset in a Banach space E. Let $F : D \to E$ be a continuous map and Φ be a regular, nonsingular MNC defined on a family $\mathscr{M}$ of subsets of E. Let Φ satisfy condition (2.1), F be Φ-condensing, and satisfy the boundary condition*

$$(BC*:) \text{ If } F(u) = u_0 + k(u - u_0) \text{ for some } u \in \partial D, k > 0, \text{ then } k \le 1, \tag{3.1}$$

where $u_0 \in int(D)$. Then F has at least one fixed point in D.

Proof. First, we define a map $f_1(u) \in D$ for each $u \notin D$ as follow. Set $g(k) = u_0 + k(u - u_0)$, where k is in $[0,1]$, and the sequences $\{\alpha_n\}$ and $\{\beta_n\}$ as

$$\begin{aligned}
&\alpha_0 = 0, \; \beta_0 = 1; \\
&\text{if } g\left(\frac{\alpha_n + \beta_n}{2}\right) \in D, \text{ then } \alpha_{n+1} = \frac{\alpha_n + \beta_n}{2}, \; \beta_{n+1} = \beta_n, \\
&\text{else } \alpha_{n+1} = \alpha_n, \; \beta_{n+1} = \frac{\alpha_n + \beta_n}{2}, \forall n > 0.
\end{aligned}$$

We immediately see that $\{g(\alpha_n)\}_n \subset D$, the sequence $\{\alpha_n\}_n \subset [0,1]$ is increasing, and consequently it converges to $\gamma \in (0,1)$, where $g(\gamma) \in D$. Similarly, sequence $\{\beta_n\}_n$ decreases to γ. In fact, if $\beta_n \to \gamma' \neq \gamma$ as $n \to \infty$, then $\gamma < \gamma'$ and there exist k such that $\frac{\alpha_k + \beta_k}{2} \in (\gamma, \gamma')$, which is impossible. Define $f_1(u) = g(k_0)$. Since $g(\alpha_n) \in D, g(\beta_n) \notin D$ for all n, and D is closed, we have $f_1(u) \in \partial D$.

We proceed to show that $f \circ F$ is condensing from D to D, where

$$f(u) = \begin{cases} u, & u \in D, \\ f_1(u), & u \notin D. \end{cases}$$

In fact, for $\Omega \subset E$, set $\Omega_2 = F(\Omega)$, $\Omega_1 = f(F(\Omega))$. In view of the definitions of f and f_1, one sees that

$$\Omega_1 \subset \{(1-k)u_0 + ku : u \in \Omega_2, k \in (0,1)\}.$$

One concludes from (2.1) that $\Phi(\Omega_1) \leq k\Phi(\Omega_2)$. That is,

$$\Phi(f \circ F(\Omega)) \leq k\Phi(F(\Omega)),\ k \in (0,1). \tag{3.2}$$

Since F is Φ-condensing, we have $\Phi(F(\Omega)) < \Phi(\Omega)$ whenever $\Phi(\Omega) > 0$. It follows that if $\Phi(\Omega) > 0$, then $\Phi(f \circ F(\Omega)) < \Phi(\Omega)$. We conclude that $f \circ F$ is Φ-condensing. From theorem (3.1), $f \circ F$ has at least one fixed point in D and we denote it by w.

We next claim $w = F(w)$. Assume that $w \neq F(w)$. Since $w = f(F(w)) \neq F(w)$, we have $F(w) \notin D$, $w = f_1(F(w)) \in \partial D$ and

$$w = u_0 + \gamma(F(w) - u_0),\ \gamma \in (0,1).$$

Hence

$$F(w) = u_0 + \frac{1}{\gamma}(w - u_0),$$

where $(1/\gamma) > 1$. This contradicts the assumption $(BC*)$. Thus w is a fixed point of F in D □

The important point to note here is the relation between the assumption $F : D \to D$ in Theorem (3.1) and the condition $BC*$ (3.1). We have the following remark.

Remark 3.3. If $D \subset E$ be a nonempty, strictly convex, and closed set, $F : D \to D$, then condition $BC*$ (3.1) holds true.

Indeed, we suppose that $F(u) = u_0 + k(u - u_0)$, where $u \in \partial D, u_0 \in int(D)$ and $k > 1$. Then $u = (1/k)F(u) + (1 - (1/k))u_0$, where $0 < (1/k) < 1$. Since $F : D \to D$, then $F(u) \in D$. Hence $u \in int(D)$ as D is strictly convex. This is impossible.

3.2. **Hadamard fractional integral equation.** Let $I = [1, T]$ and E be a Banach space. We first introduce the following definition.

Definition 3.4. [7] The Hadamard fractional integral of order $r > 0$ for a Bochner-integrable function $g \in L^1(I, E)$ is defined as

$$({}^H I_1^r g)(t) = \frac{1}{\Gamma(r)} \int_1^t \left(\ln \frac{t}{\tau}\right)^{r-1} \frac{g(\tau)}{\tau} d\tau.$$

Here and subsequently, $\|.\|$ is borrowed to stand for the norm in E and $X := C(I, E)$ is borrowed to denote the space of continuous valued E functions on I with the supremum norm

$$|u| := \sup_{t \in I} \|u(t)\|,\ u \in X.$$

Furthermore, α is borrowed to stand for the Kuratowski MNC in E which is defined on the class of bounded subsets and α_X stands for the Kuratowski MNC in X which is defined on the class of bounded equicontinuous subsets.

We next prove the existence result for the following equation

$$u(t) = (Bu)(t) + \frac{1}{\Gamma(r)} \int_1^t \left(\ln \frac{t}{\tau}\right)^{r-1} \frac{f(\tau, u(\tau))}{\tau} d\tau, t \in I, \tag{3.3}$$

where B is a continuous compact operator from $X \to X$ satisfying

$$\exists C : |Bu| \leq C, \forall u \in X,$$

and $f : I \times E \to E$ is given continuous function. Our arguments are based on the following assumptions.

(H1) There exits functions $p_1, p_2 \in C(I, \mathbb{R})$ such that, for all $x \in E$ and $t \in I$,

$$\|f(t,x)\| \leq p_1(t) + p_2(t)\|x\|;$$

(H2) Let $|.|$ denote the norm in $C(I, \mathbb{R})$. Then

$$M := \left| \frac{(\ln(T))^r |p_2|}{\Gamma(r+1)} \right| < 1.$$

Lemma 3.5. [5] *Assumption (H1) yields that if B is a bounded subset in E, then*

$$\alpha(\{f(t,x) : x \in B\}) \leq |p_2| \alpha(B).$$

Theorem 3.6. *Assume that (H1) and (H2) hold, then equation* (3.3) *has a solution in X.*

Proof. Let $R > 0$ be the large number which will be chosen later, and let $\mathbb{B}_R$ be the ball centered at the origin with the radius R in X. The idea of the proof is to use the Theorem (3.2) to prove the operator $F : \mathbb{B}_R \to \mathbb{B}_R$ defined by

$$(Fu)(t) = (Bu)(t) + \frac{1}{\Gamma(r)} \int_1^t \left(\ln \frac{t}{\tau}\right)^{r-1} \frac{f(\tau, u(\tau))}{\tau} d\tau, t \in I$$

has a fixed point in X.

We first prove that F is continuous. Indeed, we see that

$$0 \leq \frac{1}{\Gamma(r)} \int_1^t \left(\ln \frac{t}{\tau}\right)^{r-1} \frac{d\tau}{\tau} \leq \frac{1}{\Gamma(r+1)} (\ln(T))^r, \ \forall t \in I.$$

Therefore

$$\begin{aligned} \|Fu_n(t) - Fu(t)\| \leq & \|[B(u) - B(u_n)](t)\| \\ & + \frac{1}{\Gamma(r)} \int_1^t \left\| \left(\ln \frac{t}{\tau}\right)^{r-1} \frac{f(\tau, u_n(\tau)) - f(\tau, u(\tau))}{\tau} \right\| d\tau \\ & \leq \|[B(u) - B(u_n)](t)\| \\ & + \frac{1}{\Gamma(r+1)} (\ln(T))^r . \sup_{t \in J} \|f(\tau, u_n(\tau)) - f(\tau, u(\tau))\|, \end{aligned}$$

for all $u_n, u \in \mathbb{B}_R$, $t \in J$. From the dominated convergence theorem, the compactness of B and the continuity of f, we see the continuity of F. For any $u \in X$ and $t \in I$, we have

$$\begin{aligned}
\|(Fu)(t)\| &\leq \|(Bu)(t)\| + \frac{1}{\Gamma(r)}\int_1^t \left(\ln\frac{t}{\tau}\right)^{r-1}\frac{\|f(\tau,u(\tau))\|}{\tau}d\tau \\
&\leq C + \frac{1}{\Gamma(r)}\int_1^t \left(\ln\frac{t}{\tau}\right)^{r-1}\frac{|p_1|+|p_2|\|u(\tau)\|}{\tau}d\tau \\
&\leq C + \frac{(\ln(T))^r}{\Gamma(r+1)}(|p_1|+|p_2||u|).
\end{aligned}$$

By assumption (H2), we can choose R large enough such that $|F(u)| \leq R$ for all $|u| \leq R$. The result is $F : \mathbb{B}_R \to \mathbb{B}_R$ and boundary condition (3.1) is true with $u_0 = \theta_X$.

We next prove that $F(\mathbb{B}_R)$ is equicontinuous so that the condition (i) in the definition (2.6) holds true. Let $t_1 \leq t_2$, $t_1, t_2 \in J$ and $u \in \mathbb{B}_R$. Then

$$\begin{aligned}
\|Fu(t_2) - Fu(t_1)\| &\leq \|Bu(t_2)-Bu(t_1)\| + \frac{1}{\Gamma(r)}\int_{t_1}^{t_2}\left\|\left(\ln\frac{t_2}{\tau}\right)^{r-1}\frac{f(\tau,u(\tau))}{\tau}\right\|d\tau \\
&+ \frac{1}{\Gamma(r)}\int_1^{t_1}\left[\left(\ln\frac{t_2}{\tau}\right)^{r-1} - \left(\ln\frac{t_1}{\tau}\right)^{r-1}\right]\left\|\frac{f(\tau,u(\tau))}{\tau}\right\|d\tau \\
&\leq \|Bu(t_2)-Bu(t_1)\| + \frac{1}{\Gamma(r)}\int_{t_1}^{t_2}\left\|\left(\ln\frac{t_2}{\tau}\right)^{r-1}\frac{|p_1|+|p_2||u|}{\tau}\right\|d\tau \\
&+ \frac{1}{\Gamma(r)}\int_1^{t_1}\left[\left(\ln\frac{t_2}{\tau}\right)^{r-1} - \left(\ln\frac{t_1}{\tau}\right)^{r-1}\right]\left\|\frac{|p_1|+|p_2||u|}{\tau}\right\|d\tau
\end{aligned}$$

By integration, we have

$$\begin{aligned}
\|Fu(t_2) - Fu(t_1)\| &\leq \|Bu(t_2)-Bu(t_1)\| \\
&+ \frac{|p_1|+|p_2||u|}{\Gamma(r+1)}\left[\left(\ln\frac{t_2}{t_1}\right)^r - \left(\ln\frac{t_2}{t_1}\right)^r + (\ln(t_2))^r - (\ln(t_1))^r\right].
\end{aligned}$$

We see that the right-side of the above inequality tends to 0 as $t_2 \to t_1$. Hence $F(\mathbb{B}_R)$ is equicontinuous.

We finally prove that F is α_X-condensing. Let $\Omega \subset \mathbb{B}_R$, Ω be equicontinuous and $t \in I$. From the properties of Kuratowski MNC (see (2.5)) and lemma (3.5), we have

$$\begin{aligned}
\alpha(F(\Omega)(t)) &:= \alpha(\{(Fu)(t) : u \in \Omega\}) \\
&\leq \alpha(\{(Bu)(t) : u \in \Omega\}) \\
&+ \frac{1}{\Gamma(r)}\alpha\left\{\int_1^t \left(\ln\frac{t}{\tau}\right)^{r-1}\frac{f(\tau,u(\tau))}{\tau}d\tau : u \in \Omega\right\} \\
&\leq \frac{1}{\Gamma(r)}\int_1^t \left(\ln\frac{t}{\tau}\right)^{r-1}\frac{\alpha(\{f(\tau,u(\tau)) : u \in \Omega\})}{\tau}d\tau \\
&\leq \frac{1}{\Gamma(r)}\int_1^t \left(\ln\frac{t}{\tau}\right)^{r-1}\frac{|p_2|\alpha(\Omega(\tau))}{\tau}d\tau \\
&\leq \frac{|p_2|\alpha_X(\Omega)}{\Gamma(r)}\int_1^t \left(\ln\frac{t}{\tau}\right)^{r-1}\frac{d\tau}{\tau} \leq M\alpha_X(\Omega).
\end{aligned}$$

We conclude that F is k-set contractive with $k = M < 1$. Thus F is α_X-condensing. The proof is completed. $\square$

Acknowledgments

The authors are very grateful to the referees for their careful reading the work that improve the paper. This work was funded by Ho Chi Minh City University of Technology and Education, Vietnam.

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