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EXISTENCE AND STABILITY OF STRONG MINIMAX REGRET EQUILIBRIA WITH VECTOR-VALUED PAYOFFS

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Abstract. In this paper, we mainly discuss the strong minimax regret equilibrium with vector-valued payoffs (SMREVP for short). Firstly, we introduce the concept of SMREVP and establish its existence. Next, we obtain sufficient conditions for the approximate strong minimax regret equilibrium with vector-valued payoffs (ASMREVP for short) by applying a fixed-point principle. Finally, we investigate the essential stability of ASMREVP and provide several examples to illustrate these results.
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1. Introduction

Since Blackwell [1] and Shapley [2] independently proposed multi-objective games (MOG in short), scholars have paid attention to the study of MOG due to its wide applications in various fields. It is widely known that existence and stability are always two popular research topics in MOG theory. For example, Ding [3] considered a family of MOG in H-spaces and obtained existence results concerning weighted Nash and Pareto equilibrium solutions. Gong [4] established a minimax theorem with a strong vector order and obtained an equivalence between this theorem and corresponding saddle point result. Jia et al. [5] studied MOG with master-slave structure and obtained the existence of weakly Pareto-Nash equilibrium. Furthermore, they also established the generic stability of MOG with master-slave structure. Yang and Gong [6] discussed the games with infinitely many leaders and infinitely many followers, for which the definition of weakly cooperative was proposed, and proved the existence theorem. Hung and Keller [7] presented a class of multi-objective generalized game control systems and showed

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the existence by a fixed-point principle, and obtained the generic stability under some conditions. Chen et al. [8] introduced the notion of equilibrium notions including Nash and cooperative equilibria for population games with set payoffs and researched the existence by Ky Fan's section theorem and a generalized Scarf-type theorem. Recently, Song [9] generalized the existence results for the core of general cooperative games to the setting involving discontinuous and non-quasiconcave payoffs. For more results on existence of MOG, we refer to [10–16].

Furthermore, essential solutions are crucial to the stability analysis of MOG. Kohlberg and Mertens [17] proposed the concept of stable Nash equilibria and established that every game possesses at least one equilibrium set in this sense. Yu and Xiang [18] introduced the concept of essential components of the solution set of Ky Fan inequality and proved that every n-person noncooperative game admits some essential connected component in its equilibrium set. Song and Wang [19] discussed the stability of weakly Pareto-Nash equilibrium for MOG when the payoff was perturbed and showed the generic continuous results. Yang and Zhang [20] introduced the concept of cooperative equilibria in population games, and established corresponding existence and essentiality theorems. Hung et al. [21] investigated a class of generalized MOG defined by fuzzy mappings within infinite-dimensional frameworks, and established the existence of relevant equilibrium solutions. They further addressed the stability issue associated with the proposed game model. For a more comprehensive overview, we refer to [22–24].

The minimax regret equilibria is an important model in game theory since it can serve as an important theoretical basis for decision making. Renou and Schlag [25] proposed the concept of minimax regret equilibrium and gave some applications of this concept in economic. Later, they investigated the implementation in minimax regret equilibrium in [26]. Yang and Pu [11] derived sufficient conditions ensuring the existence of minimax regret equilibria, discussed the generic stability of these equilibria, and showed that the set of equilibria associated with most problems is a singleton under certain assumptions. Zhang et al. [13] established an existence result concerning minimax regret equilibria in the framework of scalar set-valued payoffs. Recently, Chen et al. [27] introduced concepts of multiobjective minimax regret equilibria by virtue of feasible criterion mappings of players, and established the existence of such equilibria. More results about minimax regret model can be found in [28–30].

The organization of the paper proceeds as follows: Section 2 introduces the preliminaries and the models discussed. Section 3 provides the sufficient conditions of SMREVP and ASMREVP, respectively. Section 4, the last section, obtains the generic stability of ASMREVP.

2. Preliminaries and Model

2.1. **Preliminaries.**

In this subsection, we first present some necessary definitions and results, which are needed later.

Let E, $E_i (i = 1,2,\ldots,n)$ and V be linear metric spaces. Let $S \in V$ be a closed pointed convex cone such that $int S \neq \emptyset$. The cone S leads to a partial order in V, defined as follows:

$$x \leq_S y \Leftrightarrow x \in y - S, \ \ \forall x, y \in V.$$

Definition 2.1. *[31] Let A be a non-empty subset of V.*

(i) The set of all strong minimal elements of A is denoted by

$$Min_s A := \{z \in A \mid A \subset \{z\} + S\}.$$

(ii) The set of all strong maximal elements of A is denoted by

$$Max_s A := \{z \in A \mid A \subset \{z\} - S\}.$$

Remark 2.2. *If $Min_s A \neq \emptyset$ and $Max_s A \neq \emptyset$, then $Min_s A$ and $Max_s A$ are both singleton sets. In fact, if $a_1, a_2 \in Min_s A$, according to the definition of $Min_s A$, we have that $a_1 \in a_2 + S$ and $a_2 \in a_1 + S$. Since S is pointed, we obtain that $a_1 = a_2$. Similarly, $Max_s A$ is also a singleton set.*

Next, we restate the concept of downward directed in [4].

Definition 2.3. *[4] Let $A \subset E$. A is downward directed if there exists some $x_0 \in A$ such that*

$$x_0 \in x_1 - S,\ x_0 \in x_2 - S,\ \ \forall x_1, x_2 \in A.$$

Lemma 2.4. *[4] If (i) A is compact and (ii)A is downward directed, then, $Min_s A \neq \emptyset$.*

In the following, we illustrate that the assumption (ii) of Lemma 2.4 is necessary by an example.

Example 2.5. *Let $V = \mathbb{R}^2$, $A = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$, and the cone $S = \mathbb{R}^2_+$. Obviously, A is compact, but A is not downward directed. Let $x_1 = (0,1)$ and $x_2 = (-1,0)$. For all $x_0 \in A$, it is not true that $x_0 \in x_1 - S$ and $x_0 \in x_2 - S$. Furthermore, $Min_s A = \emptyset$. So, the assumption (ii) in Lemma 2.1 is necessary.*

Definition 2.6. *[31] Let $F : E \to 2^V$. Suppose that F has nonempty compact-valued and $x_n, x_0 \in E$ with $x_n \to x_0 \in E$.*

(i) F is called upper semicontinuous (u.s.c. in short) at x_0, if $\forall y_n \in F(x_n)$, $\exists y_0 \in F(x_0)$ and $\{y_{n_k}\}$ s.t. $y_{n_k} \to y_0$.

(ii) F is called lower semicontinuous (u.s.c. in short) at x_0, if $\forall y_0 \in F(x_0)$, there is some $y_n \in F(x_n)$ s.t. $y_n \to y_0$.

Definition 2.7. *[31]Let $F : E \to 2^V$. F is called closed if $Graph(F) = \{(x,y) \in X \times Y : y \in F(x)\}$ is closed in $X \times Y$.*

Definition 2.8. *[32] Let X be a convex set and $f : X \to V$. U is called S-quasiconcave on X if, for all $\beta \in V$, $\{x \in X : U(x) \in \beta + S\}$ is a convex set.*

Theorem 2.9. *[33] Let $F : E \to 2^V$. If F is closed and Y is compact, then F is u.s.c. on X.*

Theorem 2.10. *[34] Let E be a locally convex linear topological space, $X \subset E$ be a compact convex set and $H : X \to 2^X$. If, for all $x \in X$, $H(x)$ is a nonempty convex compact set and H is u.s.c. on X, then there exists some $x^* \in X$ s.t. $x^* \in H(x^*)$.*

Lemma 2.11. *[35] Let X be a complete space and $F : X \to 2^Y$. If F is nonempty compact-valued and u.s.c. on X, then there exists some a dense residual set Z in X such that F is l.s.c. on Z, where a dense residual set is the arbitrary intersection of countably many open dense sets.*

2.2. **Model.** In this subsection, we propose the SMREVP. Let $\mathscr{N} = \{1, 2, \ldots, n\}$; $\forall i \in \mathscr{N}, X_i \subset E_i$, $U_i : \prod_{i \in \mathscr{N}} X_i \to V$ and $\mathscr{A}_i \subset X_{-i}$ be the set representing player i's conjecture about the strategies of the other players. Set

$$X = \prod_{i \in \mathscr{N}} X_i,\ X_{-i} = \prod_{j \in \mathscr{N}/i} X_j.$$

The regret function $\mathscr{R}_i : X \to V$ is defined by $\mathscr{R}_i(x) = Max_s U_i(X_i, x_{-i}) - U_i(x)$, for all $i \in \mathscr{N}$, where $x = (x_i, x_{-i})$.

Definition 2.12. *A strategy $x^* \in X$ is called a strong minimax regret equilibrium point of vector-valued payoffs with respect to $(\mathscr{A}_1, \mathscr{A}_2, \ldots, \mathscr{A}_n)$ if, for all $i \in \mathscr{N}$, there exists some $x^* \in X$ such that*

$$Max_s \bigcup_{x_{-i} \in \mathscr{A}_i} \mathscr{R}_i(x_i^*, x_{-i}) = Min_s \bigcup_{x_i \in X_i} Max_s \mathscr{R}_i(x_i, \mathscr{A}_i).$$

Next, we provide a special variant of strong minimax regret equilibrium, namely ASMREVP. In ASMREVP, player i believes that his opponent will choose x_{-i}^* with probability $1-\varepsilon$, and he has no idea of her opponent's choice with probability ε. For any $\varepsilon \in [0,1]$, we have

$$\mathscr{A}_i^{\varepsilon}(x_{-i}^*) = \left\{(1-\varepsilon)x_{-i}^* + \varepsilon x_{-i} \middle| x_{-i} \in X_{-i}\right\}.$$

Definition 2.13. *A strategy $x^* \in X$ is called a ε-strong minimax regret equilibrium point of vector-valued payoffs with respect to $(\mathscr{A}_1^{\varepsilon}(x_{-1}^*), \mathscr{A}_2^{\varepsilon}(x_{-2}^*), \ldots, \mathscr{A}_n^{\varepsilon}(x_{-n}^*))$ if there exists some $x^* \in X$ such that, for all $i \in \mathscr{N}$,*

$$Max_s \bigcup_{x_{-i} \in \mathscr{A}_i^{\varepsilon}(x_{-i}^*)} \mathscr{R}_i(x_i^*, x_{-i}) = Min_s \bigcup_{x_i \in X_i} Max_s \mathscr{R}_i(x_i, \mathscr{A}_i^{\varepsilon}(x_{-i}^*)).$$

Remark 2.14. *In [27], the authors also introduced similar notions of multiobjective strong minimax regret equilibria. However, it should be emphasized that the concepts proposed in [27] are formulated via the players' feasible criterion mappings, and the order relation involved in their multiobjective criteria is restricted exclusively to the Pareto order. By contrast, the multiobjective order relation adopted in this paper is induced by a convex cone S. The corresponding partial order extends the Pareto order but is not necessarily identical to it. Consequently, the equilibrium concepts developed in the present work are genuinely different from those in [27].*

Remark 2.15. *If $V = \mathbb{R}$ and $S = \mathbb{R}_+$, the concepts reduce into the case in [11].*

Theorem 2.16. *[13] Let $i \in \mathscr{N}$ and $0 \le \varepsilon \le 1$. If $X_i \subset E_i$ is a compact and convex set, then A_i^{ε} is continuous on X_{-i} with compact values.*

3. EXISTENCE

3.1. Strong minimax regret equilibria of vector-valued payoffs.

Lemma 3.1. *Let $i \in \mathscr{N}$ and X_i be a compact subset of E_i. If U_i is continuous on X and $-U_i(\cdot, x_{-i})$ is downward directed, then $\mathscr{R}_i$ is continuous on X.*

Proof. For every $i \in \mathscr{N}$, because $-U_i(\cdot, x_{-i})$ is downward directed, $\mathscr{R}_i$ is reasonable. Next, we claim that $\mathscr{R}_i$ is continuous on X. In fact, it sufficient to show that the function

$$\mathscr{T}_i(x_{-i}) = Max_s U_i(X_i, x_{-i})$$

is continuous on X_{-i}. Let $x_{-i}^n \to x_{-i}^0$. Since X_{-i} is compact, it is enough to prove that $Graph\mathscr{T}_i$ is a closed set, where

$$Graph\mathscr{T}_i := \left\{(x_{-i}, z_i) \in X : z_i = Max_s U_i(X_i, x_{-i}),\ \ x_{-i} \in X_{-i}\right\}.$$

Assume that $Graph\mathscr{T}_i$ is not closed. Thus, there exist $(x_{-i}^n, z_i^n) \in Graph\mathscr{T}_i$ such that $(x_{-i}^n, z_i^n) \to (x_{-i}^0, z_i^0)$, but $(x_{-i}^0, z_i^0) \notin Graph\mathscr{T}_i$. Since $(x_{-i}^0, z_i^0) \notin Graph\mathscr{T}_i$, there exists $y_i \in X_i$ such that

$U_i(y_i, x^0_{-i}) \notin z^0_i - S$, which implies $U_i(y_i, x^0_{-i}) - z^0_i \in V \setminus -S$. We define the function $g_i : X_{-i} \to Y$ by the rule:

$$g_i(x_{-i}, z^0_i) = U_i(y_i, x^0_{-i}) - z^0_i, \quad \forall x^0_{-i} \in X_i.$$

Since $U_i(y_i, \cdot)$ is continuous, g_i is continuous. Furthermore, S is closed, so there exists N such that, $n > N$, $g_i(x^n_{-i}, z^n_i) = U_i(y_i, x^n_{-i}) - z^n_i \in V \setminus -S$, which is equivalent to $U_i(y_i, x^n_{-i}) \notin z^n_i - S$. Note that $y_i \in X_i$. From the above, we have $z^n_i \neq Max_s \bigcup_{y_i \in X_i} U_i(y_i, x^n_{-i})$, i.e., $(x^n_{-i}, z^n_i) \notin Graph\mathscr{T}_i$. This contradicts with our assumption. Thus $\mathscr{T}_i$ is continuous on X_{-i}. □

Theorem 3.2. *Let $i \in \mathscr{N}$, and X_i and A_i be compact subsets of E_i, respectively. If*

(i) U_i is continuous on X,

(ii) $-U_i(\cdot, x_{-i})$ is downward directed,

(iii) $U_i(x_i, \cdot)$ is downward directed,

(iv) $-Max_s U_i(x_i, \cdot)$ is downward directed,

then there exists $x^* \in X$ such that, for any $i \in \mathscr{N}$,

$$Max_s \bigcup_{x_{-i} \in \mathscr{A}_i} \mathscr{R}_i(x^*_i, x_{-i}) = Min_s \bigcup_{x_i \in X_i} Max_s \mathscr{R}_i(x_i, \mathscr{A}_i),$$

i.e., $x^* \in X$ is SMREVP with respect to $(A_1, A_2, \ldots, A_n)$.

Proof. By assumptions (i), (ii) and Lemma 3.1, one sees that $\mathscr{R}_i$ is continuous on X. Due to the compactness of A_i, for every $x_i \in X_i$, $\mathscr{R}_i(x_i, A_i)$ is compact. Fix x_i. By assumptions (iii) and (iv), we can easily have that $-\mathscr{R}_i(x_i, A_i)$ is downward directed. According to Lemma 2.4, $Max_s \mathscr{R}_i(x_i, A_i) \neq \emptyset$.

Next, we prove $Min_s \bigcup_{x_i \in X_i} Max_s \mathscr{R}_i(x_i, A_i) \neq \emptyset$, for all $i \in \mathscr{N}$. By Lemma 2.4, it is sufficient to prove that $Max_s \mathscr{R}_i(x_i, A_i)$ is downward directed. In fact, due to the continuity of $\mathscr{R}_i$ and the compactness of A_i, for any $x_i \in X_i$, $\mathscr{R}_i(x_i, A_i) \neq \emptyset$. Thus, for any $x^1_i, x^2_i \in X_i$, there exist $y^1_{-i}, y^2_{-i} \in A_i$ such that

$$R_i(x^1_i, y^1_{-i}) = Max_s R_i(x^1_i, A_i), \quad R_i(x^2_i, y^2_{-i}) = Max_s R_i(x^2_i, A_i).$$

Moreover, for any $y_{-i} \in A_i$, we have

$$\mathscr{R}_i(x^1_i, y_{-i}) \in Max_s \mathscr{R}_i(x^1_i, A_i) - S = \mathscr{R}_i(x^1_i, y^1_{-i}) - S,$$
$$\mathscr{R}_i(x^2_i, y_{-i}) \in Max_s \mathscr{R}_i(x^2_i, A_i) - S = \mathscr{R}_i(x^2_i, y^2_{-i}) - S.$$

For any $y_{-i} \in A_i$, by assumption (ii), for any $x^1_i, x^2_i \in X_i$, there exists some $\bar{x}_i \in X_i$ such that

$$\mathscr{R}_i(\bar{x}_i, y_{-i}) \in \mathscr{R}_i(x^1_i, y_{-i}) - S, \quad \mathscr{R}_i(\bar{x}_i, y_{-i}) \in \mathscr{R}_i(x^2_i, y_{-i}) - S.$$

Combining the convexity of S, for all $y_{-i} \in A_i$, we have

$$\mathscr{R}_i(\bar{x}_i, y_{-i}) \in Max_s \mathscr{R}_i(x^1_i, A_i) - S, \quad \mathscr{R}_i(\bar{x}_i, y_{-i}) \in Max_s \mathscr{R}_i(x^2_i, A_i) - S.$$

From the arbitrariness of y_{-i}, we obtain that

$$Max_s \mathscr{R}_i(\bar{x}_i, A_i) \in Max_s \mathscr{R}_i(x^1_i, A_i) - S, \quad Max_s \mathscr{R}_i(\bar{x}_i, A_i) \in Max_s \mathscr{R}_i(x^2_i, A_i) - S,$$

which indicates that $Max_s \mathscr{R}_i(\cdot, A_i)$ is downward directed. In other words, there exists $x^* \in X$ such that, for any $i \in \mathscr{N}$,

$$Max_s \bigcup_{x_{-i} \in \mathscr{A}_i} \mathscr{R}_i(x^*_i, x_{-i}) = Min_s \bigcup_{x_i \in X_i} Max_s \mathscr{R}_i(x_i, \mathscr{A}_i).$$

□

Remark 3.3. *If the utility functions in Theorem 3.2 are restricted to be single-valued, then the results in this paper reduce to those in Theorem 3.2 of [11]. Therefore, the models considered in this paper are more general than those in [11], leading to different existence results. The next examples illustrate the conclusion of Theorem 3.2*

Example 3.4. *Let $\mathcal{N} = \{1,2\}$, $X_1 = X_2 = [0,1] \subset \mathbb{R}^2$ be the pure strategy sets for the players, and let $A_1 = [0,1] \subset X_2$, $A_2 = [0,1] \subset X_1$ be the conjecture sets for the players with the cone $S = \mathbb{R}^2_+$. Define the vector payoff functions $U_1, U_2 : X_1 \times X_2 \to 2^{\mathbb{R}}$ as follows:*

$$U_1(x) = x_2(x_1, 1) \quad and \quad U_2(x) = x_1(1, x_2), \quad \forall x \in X.$$

Based on the definitions of U_1, U_2, we have

$$Max_s U_1(X_1, x_2) = (x_2, x_2), \quad Max_s U_2(x_1, X_2) = (x_1, x_1).$$

Therefore, the regret functions for the players are:

$$\begin{aligned} \mathscr{R}_1(x_1, x_2) &= Max_s U_1(X_1, x_2) - U_1(x_1, x_2) \\ &= (x_2, x_2) - x_2(x_1, 1) \\ &= (x_2 - x_1 x_2, 0) \end{aligned}$$

and

$$\begin{aligned} \mathscr{R}_2(x_1, x_2) &= Max_s U_2(x_1, X_2) - U_2(x_1, x_2) \\ &= (x_1, x_1) - x_1(1, x_2) \\ &= (0, x_1 - x_1 x_2). \end{aligned}$$

Next, we verify whether all the assumptions of Theorem 3.2 are fulfilled.

(i) Clearly, U_1 and U_2 are continuous.

(ii) Let $i = 1$ and $x_2 \in X_2$, $-U_1(\cdot, x_2)$ be downward directed. In fact, $-U_1(x_1, x_2) = x_2(-x_1, -1)$, for any $x_1^1, x_1^2 \in X_1$, there exists $\bar{x_1} = 0$ such that

$$-U_1(x_1^1, x_2) \in -U_1(\bar{x_1}, x_2) - S\ ,\ -U_1(x_1^2, x_2) \in -U_1(\bar{x_1}, x_2) - S.$$

Similarly, -$U_2(x_1, x_2) = x_1(-1, -x_2)$. For any $x_1^1, x_1^2 \in X_1$, there exists $\bar{x_1} = 0$ such that

$$-U_2(x_1^1, x_2) \in -U_2(\bar{x_1}, x_2) - S\ ,\ -U_2(x_1^2, x_2) \in -U_2(\bar{x_1}, x_2) - S.$$

This shows that $-U_2(\cdot, x_2)$ is also downward directed.

In conclusion, for any $i \in \mathcal{N}$, $-U_i(\cdot, x_{-i})$ is downward directed.

(iii) Let $i = 1$ and $x_1 \in X_1$, $U_1(x_1, \cdot)$ be downward directed. In fact, $U_1(x_1, x_2) = x_2(x_1, 1)$, for any $x_2^1, x_2^2 \in X_2$, there exists $\bar{x_2} = 1$ such that

$$U_1(x_1, x_2^1) \in U_1(x_1, \bar{x_2}) - S\ ,\ U_1(x_1, x_2^2) \in U_1(x_1, \bar{x_2}) - S.$$

Similarly, since $U_2(x_1, x_2) = x_1(1, x_2)$, for any $x_2^1, x_2^2 \in X_2$, there exists $\bar{x_1} = 1$ such that

$$U_2(x_1, x_2^1) \in U_2(x_1, \bar{x_2}) - S\ ,\ U_2(x_1, x_2^2) \in U_2(x_1, \bar{x_2}) - S.$$

This shows that $U_2(x_1, x_2)$ is also downward directed. Thus, for any $i \in \mathcal{N}$, $U_i(x_i, \cdot)$ is downward directed.

(iv) Let $i=1$ and $x_2\in X_2$, $-Max_s\bigcup_{x_1\in X_1}U_1(x_1,\cdot)$ be downward directed. In fact,

$$-Max_s\bigcup_{x_1\in X_1}U_1(x_1,x_2)=-(x_2,x_2).$$

Thus, for any $x_2^1,x_2^2\in X_2$, there exists $\bar{x_2}=0$ such that

$$-Max_sU_1(X_1,x_2^1)\in -Max_sU_1(X_1,\bar{x_2})-S,\quad -Max_sU_1(X_1,x_2^2)\in -Max_sU_1(X_1,\bar{x_2})-S.$$

In addition, $-Max_s\bigcup_{x_1\in X_1}U_2(x_1,x_2)=-(x_2,x_2)$ and for any $x_1^1,x_1^2\in X_1$, there exists $\bar{x_1}=0$ such that

$$-Max_sU_2(x_1^1,X_2)\in -Max_sU_2(\bar{x_1},X_2)-S,\quad -Max_sU_2(x_1^2,X_2)\in -Max_sU_2(\bar{x_1},X_2)-S.$$

Thus, $-Max_s\bigcup_{x_1\in X_1}U_2(x_1,x_2)$ is downward directed. Therefore, for any $i\in\mathcal{N}$, $-Max_sU_i(x_i,\cdot)$ is downward directed. In a word, all the assumptions of Theorem 3.2 hold and Theorem 3.2 is feasible. Let $x^*=(1,1)$ satisfy Theorem 3.2. We now substitute to verify

$$\begin{aligned}Max_s\bigcup_{x_2\in\mathscr{A}_1}\mathscr{R}_1(x_1^*,x_2)&=Min_s\bigcup_{x_1\in X_1}Max_s\mathscr{R}_1(x_1,\mathscr{A}_1)\\&=Min_s\bigcup_{x_1\in X_1}(1-x_1,0)\\&=(0,0)\end{aligned}$$

and

$$\begin{aligned}Max_s\bigcup_{x_1\in\mathscr{A}_2}\mathscr{R}_2(x_1,x_2^*)&=Min_s\bigcup_{x_2\in X_2}Max_s\mathscr{R}_2(\mathscr{A}_2,x_2)\\&=Min_s\bigcup_{x_1\in X_1}(0,1-x_2)\\&=(0,0).\end{aligned}$$

Thus x^* is a SMREVP with respect to (A_1,A_2).

3.2. **Strong ε-minimax regret equilibria of vector-valued payoffs.**

Lemma 3.5. *Let $i\in\mathcal{N}$ and let X_i be a nonempty compact subset of E_i. Assume that the vector-valued functions $U_i:X\to Y$ and $S_i:X_{-i}\to 2^{X_{-i}}$ satisfy the following conditions:*
(i) $U_i:X\to Y$ is continuous;
(ii) $U_i(x_i,\cdot)$ is downward directed;
(iii) S_i is nonempty, compact, and continuous.
Then, the function $\mathscr{L}_i:X\to V$ is continuous on X, where $\mathscr{L}_i$ is defined as follows:

$$\mathscr{L}_i(x):=Max_sU_i(x_i,S_i(x_{-i})).$$

Proof. For any $i\in\mathcal{N}$, since X_i is compact and both U_i and S_i are continuous, $\bigcup_{y_{-i}\in S_i(x_{-i})}U_i(x_i,y_{-i})$ is compact. Since X is compact, it suffices to verify that $Graph\mathscr{L}_i$ is closed to show that $\mathscr{L}_i$ is continuous. By the definition of $\mathscr{L}_i$, we have

$$Graph\mathscr{L}_i:=\Big\{(x_i,z_i)\in X\times V: z_i=Max_sU_i(x_i,S_i(x_{-i})),\ x_i\in X\Big\}.$$

Now, let $(x_n,z_n)\subset Graph\mathscr{L}_i$ and $(x_n,z_n)\to(x_0,z_0)$. We assume that $(x_0,z_0)\notin Graph\mathscr{L}_i$. First of all, we show that $z_0\in U_i(x_i^0,S_i(x_{-i}^0))$. Since $(x_n,z_n)\subseteq Graph\mathscr{L}_i$, we have $x_n\in X$ and $z_n=$

$Max_s \bigcup_{y^n_{-i} \in S_i(x^n_{-i})} U_i(x^n_i, y^n_{-i})$. Hence, $z_n \in U_i(x^n_i, S_i(x^n_{-i}))$, which yields that there exists $y^n_{-i} \in S_i(x^n_{-i})$ such that $z_n = U_i(x^n_i, y^n_{-i})$. Since S_i is upper semi-continuous, for $y^n_{-i} \in S_i(x^n_{-i})$, without loss of generality, there must exist $y^0_{-i} \in S^0_{-i}$ such that $y^n_{-i} \to y^0_{-i}$. Furthermore, since U_i is continuous and $z_n \to z_0$, we have $z_0 = U_i(x^0_i, y^0_{-i})$. In addition, since $y^0_{-i} \in S^0_{-i}$, we have $z_0 \in U_i(x^0_i, S^0_{-i})$. By the definition of $Graph\mathscr{L}_i$, we have $z_0 \neq Max_s \bigcup_{y^0_{-i} \in S_i(x^0_{-i})} U_i(x^0_i, y^0_{-i})$. Thus, there exusts some $\overline{y_{-i}} \in S_{-i}(x^0_{-i})$ such that

$$U_i(x^0_i, \overline{y_{-i}}) \notin z_0 - S. \tag{3.1}$$

Since S_i is l.s.c., for $\overline{y_{-i}} \in S_i(x^0_{-i})$, there exists $\overline{y^n_{-i}} \in S_i(x^n_{-i})$ such that $\overline{y^n_{-i}} \to \overline{y_{-i}}$. Combining the continuity of U_i and the closedness of S, one has $U_i(x^n_i, \overline{y_{-i}}) \notin z^n - S$. Combining (3.1) with $\overline{y^n_{-i}} \in S_i(x^n_{-i})$, we obtain $z^n \neq Max_s \bigcup_{y^n_{-i} \in S_i(x^n_{-i})} U_i(x^n_i, y^n_{-i})$, which contradicts the assumption. Therefore, $\mathscr{L}_i$ is continuous. □

Theorem 3.6. *Let $i \in \mathscr{N}$, and X_i and A_i be compact subsets of locally convex space E_i. If*
(i) U_i is continuous;
(ii) $-U_i(\cdot, x_{-i})$ is downward directed;
(iii) $U_i(x_i, \cdot)$ is downward directed;
(iv) $-Max_s U_i(x_i, \cdot)$ is downward directed;
(v) X_i is convex;
(vi) $U_i(\cdot, x_{-i})$ is S-quasi-concave;
(vii) S_i is nonempty, compact, and continuous,
then there exists $x^* = (x^*_1, x^*_2, \ldots, x^*_n) \in X$ such that, for any $i \in \mathscr{N}$,

$$Max_s \bigcup_{x_{-i} \in \mathscr{A}^{\varepsilon}_i(x^*_{-i})} \mathscr{R}_i(x^*_i, x_{-i}) = Min_s \bigcup_{x_i \in X_i} Max_s \mathscr{R}_i(x_i, \mathscr{A}^{\varepsilon}_i(x^*_{-i})).$$

Proof. For any $i \in \mathscr{N}$ and $\varepsilon \in [0,1]$, we denote the function $\mathscr{H}_i : X_{-i} \to 2^{X_i}$ by

$$\mathscr{H}_i(x_{-i}) = \Big\{ x_i \in X_i : Max_s \mathscr{R}_i(x_i, A^{\varepsilon}_i(x_{-i}) = Min_s \bigcup_{x_i \in X_i} Max_s \mathscr{R}_i(x_i, \mathscr{A}^{\varepsilon}_i(x_{-i})) \Big\}, \forall x_{-i} \in X_{-i}.$$

From the definition of A^{ε}_i and similar process of Theorem 3.2, we obtain $x_i \in \mathscr{H}_i(x_{-i})$, for all $x_{-i} \in X_{-i}$. Let $a = Min_s \bigcup_{x_i \in X_i} Max_s \mathscr{R}_i(x_i, \mathscr{A}^{\varepsilon}_i(x_{-i}))$. In the following, we prove that, for any $x_{-i} \in X_{-i}$, $\mathscr{H}_i(x_{-i})$ is a closed set. Let $x^n_i \in \mathscr{H}_i(x_{-i})$ and $x^n_i \to x^0_i$. Since $x^n_i \in \mathscr{H}_i(x_{-i})$, $Max_s \mathscr{R}_i(x^n_i, A^{\varepsilon}_i(x_{-i}) = a$. Furthermore, by the compactness of A^{ε}_i,

$$Max_s \mathscr{R}_i(x^n_i, A^{\varepsilon}_i(x_{-i}) \to Max_s \mathscr{R}_i(x^0_i, A^{\varepsilon}_i(x_{-i}).$$

Hence, $Max_s \mathscr{R}_i(x^0_i, A^{\varepsilon}_i(x_{-i}) = a$, i.e. $x^0_i \in \mathscr{H}(x_{-i})$ and $\mathscr{H}_i$ has closed values.

Now, we consider that $\mathscr{H}_i$ has convex-valued on X_{-i}. Suppose $x^1_i, x^2_i \in \mathscr{H}_i(x_{-i})$ and $\lambda \in [0,1]$. From $x^1_i, x^2_i \in \mathscr{H}_i(x_{-i})$, $Max_s \mathscr{R}_i(x^1_i, A^{\varepsilon}_i(x_{-i}) = a$ and $Max_s \mathscr{R}_i(x^2_i, A^{\varepsilon}_i(x_{-i}) = a$. By virtue of the definition, for any $u_{-i} \in A^{\varepsilon}_i(x_{-i})$, $\mathscr{R}_i(x^1_i, u_{-i}) \in a - S$ and $\mathscr{R}_i(x^2_i, u_{-i}) \in a - S$. Notice that

$$\mathscr{R}_i(x^1_i, u_{-i}) = Max_s U_i(X_i, u_{-i}) - U_i(x^1_i, u_{-i}),$$
$$\mathscr{R}_i(x^2_i, u_{-i}) = Max_s U_i(X_i, u_{-i}) - U_i(x^2_i, u_{-i}).$$

We can easily obtain

$$U_i(x^1_i, u_{-i}) \in Max_s U_i(X_i, u_{-i}) - a + S,$$
$$U_i(x^2_i, u_{-i}) \in Max_s U_i(X_i, u_{-i}) - a + S.$$

Since for every $u_{-i} \in X_{-i}$, $U_i(\cdot, u_{-i})$ is S-quasiconcave, for any $\lambda \in [0,1]$,

$$U_i(\lambda x_i^1 + (1-\lambda)x_i^2, u_{-i}) \in Max_s U_i(X_i, u_{-i}) - a + S.$$

For any $\lambda \in [0,1]$ and $u_{-i} \in A_i^{\varepsilon}(x_{-i})$, we obtain that

$$\mathscr{R}_i(\lambda x_i^1 + (1-\lambda)x_i^2, u_{-i}) = Max_s \bigcup_{y_i \in X_i} U_i(y_i, u_{-i}) - U_i(\lambda x_i^1 + (1-\lambda)x_i^2, u_{-i}) \in a - S.$$

Next, we prove that $a \in \mathscr{R}_i(\lambda x_i^1 + (1-\lambda)x_i^2, A^{\varepsilon}_{-i}(x_{-i}))$. By the definition of strong minimal element, because $\lambda x_i^1 + (1-\lambda)x_i^2 \in X_i, \forall \lambda \in [0,1]$, $Max_s \mathscr{R}_i(\lambda x_i^1 + (1-\lambda)x_i^2, A^{\varepsilon}_{-i}(x_{-i})) \in a + S$. Furthermore, $\mathscr{R}_i(\lambda x_i^1 + (1-\lambda)x_i^2, A^{\varepsilon}_{-i}(x_{-i})) \subset a - S$. Suppose that

$$a \notin \mathscr{R}_i(\lambda x_i^1 + (1-\lambda)x_i^2, A^{\varepsilon}_{-i}(x_{-i})).$$

Then

$$Max_s \mathscr{R}_i(\lambda x_i^1 + (1-\lambda)x_i^2, A^{\varepsilon}_{-i}(x_{-i})) \in a + S \setminus \{0\},$$

$$Max_s \mathscr{R}_i(\lambda x_i^1 + (1-\lambda)x_i^2, A^{\varepsilon}_{-i}(x_{-i})) \in a - S \setminus \{0\}.$$

This contradicts the sharpness of S. Thus $a \in \mathscr{R}_i(\lambda x_i^1 + (1-\lambda)x_i^2, A^{\varepsilon}_{-i}(x_{-i}))$. Hence, $\mathscr{H}_i$ is convex-valued.

Now, we prove that $\mathscr{H}_i$ is u.s.c. on X_{-i}. Since X_i is compact, it suffices to show that $\mathscr{H}_i$ is closed. Let $(x^n_{-i}, x^n_i) \in Graph\mathscr{H}_i$ and $(x^n_{-i}, x^n_i) \to (x^0_{-i}, x^0_i)$. According to the definition of $Graph\mathscr{H}_i$, we have

$$Max_s \mathscr{R}_i(x_i^n, A_i^{\varepsilon}(x^n_{-i}) = Min_s \bigcup_{x_i \in X_i} Max_s \mathscr{R}_i(x_i, \mathscr{A}_i^{\varepsilon}(x^n_{-i})).$$

We know that A_i^{ε} is a continuous and compact-valued mapping. Furthermore, based on the assumptions and Lemma 3.1, we have $Max_s \mathscr{R}_i(x_i^n, A_i^{\varepsilon}(x^n_{-i}) \to Max_s \mathscr{R}_i(x_i^0, A_i^{\varepsilon}(x^0_{-i})$, as $n \to \infty$. For any $x_i \in X_i$,

$$Max_s \mathscr{R}_i(x_i, A_i^{\varepsilon}(x^n_{-i}) \to Max_s \mathscr{R}_i(x_i, A_i^{\varepsilon}(x^0_{-i}),\ n \to \infty.$$

Following Lemma 3.5, we can deduce that

$$Min_s \bigcup_{x_i \in X_i} Max_s \mathscr{R}_i(x_i, \mathscr{A}_i^{\varepsilon}(x^n_{-i})) \to Min_s \bigcup_{x_i \in X_i} Max_s \mathscr{R}_i(x_i, \mathscr{A}_i^{\varepsilon}(x^0_{-i})), n \to \infty.$$

Thus, $\mathscr{H}_i$ is closed and u.s.c. on X_{-i}. We define $H : X \to 2^X$ by $H(x) = \prod_{i=1}^n \mathscr{H}_i$. Thus H is u.s.c. set-valued mapping and H has nonempty, closed, and convex values. Due to Theorem 2.10, there exists $x^* \in X$ such that $x^* \in H(x^*)$, which means

$$Max_s \mathscr{R}_i(x_i^*, A_i^{\varepsilon}(x^*_{-i}) = Min_s \bigcup_{x_i \in X_i} Max_s \mathscr{R}_i(x_i, \mathscr{A}_i^{\varepsilon}(x^*_{-i})),\ \forall i \in \mathscr{N}.$$

□

Remark 3.7. *If the payoff functions in Theorem 3.6 are reduced to single-valued, the results of this paper degrade to the case of Theorem 3.6 in [11]. Therefore, the model in this paper is more general than that in [11], and optimal solutions for each objective function are obtained. Hence, the existence results are different.*

4. Stability

Let $\mathscr{F}$ denote the set of all assumptions satisfying Theorem 3.6, let $K(F)$ be the ε-strong minimax regret equilibrium set about F, where $F \in \mathscr{F}$ and $F = (U_1, U_2, \cdots, U_n, A_1^\varepsilon, A_2^\varepsilon, \cdots, A_n^\varepsilon)$.

We define the distance ρ on $\mathscr{F}$ as follows:

$$\rho(F_1, F_2) = \sup_{x \in X} \sum_{i=1}^{n} \|U_i^1(x) - U_i^2(x)\| + \sum_{i=1}^{n} D_i(A_i^{1\varepsilon}, A_i^{2\varepsilon}),$$

where $F_1 = (U_1^1, U_2^1, \cdots, U_n^1, A_1^{1\varepsilon}, A_2^{1\varepsilon}, \cdots, A_n^{1\varepsilon})$, $F_2 = (U_1^2, U_2^2, \cdots, U_n^2, A_1^{2\varepsilon}, A_2^{2\varepsilon}, \cdots, A_n^{2\varepsilon})$ and D_i is the Hausdorff distance. Clearly, the space $(\mathscr{F}, \rho)$ is complete.

Theorem 4.1. *K is compact-valued and u.s.c. on $\mathscr{F}$.*

Proof. First, we prove that K is compact. For any $F \in \mathscr{F}$, let $\{x^n\} \subset K(F)$ be such that $x^n \to x^0$. We assume that $x^0 \notin K(F)$, which shows that there exists some $i \in \mathscr{N}$ such that

$$Max_s \mathscr{R}_i(x_i^0, A_i^\varepsilon(x_{-i}^0)) \neq Min_s \bigcup_{x_i \in X_i} Max_s \mathscr{R}_i(x_i, A_i^\varepsilon(x_{-i}^0)).$$

It shows that there exists $\bar{x}_i \in X_i$ such that $Max_s \mathscr{R}_i(\bar{x}_i, A_i^{0\varepsilon}(x_{-i}^0)) \notin Max_s \mathscr{R}_i(x_i^0, A_i^{0\varepsilon}(x_{-i}^0)) + S$. By the assumptions on U_i^0 and Lemma 3.5, we have

$$Max_s \mathscr{R}_i^0(\bar{x}_i, A_i^\varepsilon(x_{-i}^n) \to Max_s \mathscr{R}_i^0(\bar{x}_i, A_i^\varepsilon(x_{-i}^0)),$$

$$Max_s \mathscr{R}_i^n(\bar{x}_i, A_i^\varepsilon(x_{-i}^n) \to Max_s \mathscr{R}_i^0(x_i^0, A_i^\varepsilon(x_{-i}^0)).$$

Moreover, since S is closed, $Max_s \mathscr{R}_i^0(\bar{x}_i, A_i^\varepsilon(x_{-i}^n) - Max_s \mathscr{R}_i^0(x_i^0, A_i^\varepsilon(x_{-i}^n) \in V \setminus S$. Thus,

$$Max_s \mathscr{R}_i^0(\bar{x}_i, A_i^\varepsilon(x_{-i}^n) \notin Max_s \mathscr{R}_i^0(x_i^n, A_i^\varepsilon(x_{-i}^n) + S.$$

By the concept of strong minimal elements, we deduce that

$$Max_s \mathscr{R}_i^0(\bar{x}_i, A_i^\varepsilon(x_{-i}^n) \neq Max_s \mathscr{R}_i^0(x_i, A_i^\varepsilon(x_{-i}^n)).$$

This shows that $x^n \notin \text{Graph}K$, leading to a contradiction, i.e. $x^0 \in K(F)$.

Next, we prove that K is u.s.c.. It is equivalent to showing that the graph of K is closed. Let $(F^n, X^n) \subset \text{Graph } K$ such that $(F^n, X^n) \to (F^0, X^0)$. We suppose that $(F^0, X^0) \notin \text{Graph } K$, which implies there exists $i \in \mathscr{N}$ such that

$$Max_s \mathscr{R}_i(x_i^0, A_i^\varepsilon(x_{-i}^0)) \neq \min_s \bigcup_{x_i \in X_i} Max_s \mathscr{R}_i(x_i, \mathscr{A}_i^\varepsilon(x_{-i}^0)),$$

which yields that there exists $\bar{x}_i \in X_i$ such that $Max_s \mathscr{R}_i(\bar{x}_i, A_i^{0\varepsilon}(x_{-i}^0)) \notin Max_s \mathscr{R}_i(x_i^0, A_i^{0\varepsilon}(x_{-i}^0)) + S$. By the assumptions on U_i^0 and Lemma 3.5, we obtain that

$$Max_s \mathscr{R}_i^0(\bar{x}_i, A_i^\varepsilon(x_{-i}^n) \to Max_s \mathscr{R}_i^0(\bar{x}_i, A_i^\varepsilon(x_{-i}^0),$$

$$Max_s \mathscr{R}_i^0(x_i^n, A_i^\varepsilon(x_{-i}^n) \to Max_s \mathscr{R}_i^0(x_i^0, A_i^\varepsilon(x_{-i}^0).$$

Furthermore, due to the closedness of S, $Max_s \mathscr{R}_i^0(\bar{x}_i, A_i^\varepsilon(x_{-i}^n) - Max_s \mathscr{R}_i^0(x_i^0, A_i^\varepsilon(x_{-i}^n) \in V \setminus S$. Thus, $Max_s \mathscr{R}_i^0(\bar{x}_i, A_i^\varepsilon(x_{-i}^n) \notin Max_s \mathscr{R}_i^0(x_i^n, A_i^\varepsilon(x_{-i}^n) + S$. By the definition of strong minimal element, one has

$$Max_s \mathscr{R}_i^0(x_i^n, A_i^\varepsilon(x_{-i}^n) \neq Max_s \mathscr{R}_i^0(x_i, A_i^\varepsilon(x_{-i}^n).$$

Therefore, $(F^n, X^n) \notin \text{Graph } K$. This leads to a contradiction. In conclusion, $K : (\mathscr{F}, \rho) \to 2^X$ is u.s.c. and has compact values. $\square$

Lemma 4.2. *[11] K is l.s.c. if and only if for every $F \in \mathscr{F}$ is essential.*

Given that K is u.s.c. on $\mathscr{F}$ and Fort's Lemma, the stability of the vector-valued strong ε-minimax regret equilibrium when both the feasible constraint mapping and the payoff functions are perturbed simultaneously can be obtained as follows.

Theorem 4.3. *There exists a dense residual subset $Z \in \mathscr{F}$ such that, for any $f \in \mathscr{F}$, Z is essential.*

Remark 4.4. *From Theorem 4.1, it follows that, for any $f \in Z$ and $\varepsilon > 0$, if there exists $\rho(f,\bar{f}) < \lambda$, $D_i(K(f),K(\bar{f})) < \varepsilon$. This implies that f determines the generic stability of ASMREVP.*

Let $\mathscr{F}^{'}$ be the set satisfying all the assumptions of Theorem 3.6, and let $K^{'}(F^{'})$ be the set of vector-valued strong minimax regret equilibrium with respect to $F^{'}$. Here, $F^{'} \in \mathscr{F}^{'}$ and $F^{'} = (U_1,U_2,\cdots,U_n,A_1,A_2,\cdots,A_n)$. We define the distance $\rho^{'}$ on $\mathscr{F}^{'}$ as

$$\rho^{'}(F_1^{'},F_2^{'}) = \sup_{x\in X}\sum_{i=1}^{n}\|U_i^1(x) - U_i^2(x)\| + \sum_{i=1}^{n} D_i(A_i^1,A_i^2),$$

where $F_1^{'} = (U_1^1,U_2^1,\cdots,U_n^1,A_1^1,A_2^1,\cdots,A_n^1)$, $F_2^{'} = (U_1^2,U_2^2,\cdots,U_n^2,A_1^2,A_2^2,\cdots,A_n^2)$, and D_i is the Hausdorff distance.

Corollary 4.5. *$K^{'}$ is u.s.c. and has compact values.*

Corollary 4.6. *There exists some residual set $Z^{'} \subset \mathscr{F}^{'}$ such that, for any $f \in \mathscr{F}^{'}$, $Z^{'}$ is essential.*

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