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CONTROLLABILITY RESULTS FOR NONLINEAR NEUTRAL FUZZY INTEGRODIFFERENTIAL SYSTEMS IN FUZZY SEMIGROUPS

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Abstract. In this paper, we study the existence and uniqueness of nonlinear fuzzy neutral integrodifferential equations. We also establish the controllability of nonlinear fuzzy neutral integrodifferential systems. The results are obtained in this paper are mainly based on the fuzzy strongly continuous semigroup theory. An example is provided to illustrate the main results of this paper.

Keywords. Controllability; Fuzzy semigroup; Fuzzy neutral integrodifferential system; Fuzzy number; Fixed point theorem.

1. Introduction

Differential equations arise in many areas of science and technology, specifically whenever a deterministic relation involving some continuously varying quantities (modeled by functions) and their rates of change in space and/or time are known or postulated. This is illustrated in classical mechanics where the motion of a body is described by its position and velocity as the time varies. Newton's laws allow one (given the position, velocity, acceleration and various forces acting on the body) to express these variables dynamically as a differential equation for the unknown position of the body as a function of time. An initial value or initial-boundary value problem for an evolution partial differential equation (an equation whose solutions depend

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on time) can usually be written as an abstract differential equation

$$y'(t) = f(t, y(t)) \tag{1.1}$$

in a suitable function space, the function f describing the action of the equation on the space variables with boundary conditions in the definition of the space or of the domain of f. The similarity of (1.1) with a true ordinary differential equation is only formal (f may not be everywhere defined, bounded or continuous) but gives heuristic insight into the problem, suggests ways to extend results from ordinary to partial differential equations and stresses unification leading to discovery of common threads and economy of thought.

Generally, several systems are mostly related to uncertainty and inaccuracy. The problem of inaccuracy is considered in general an exact science and that of uncertainty is considered as vague or fuzzy and accidental. A differential and integral calculus for fuzzy-set-valued, shortly fuzzy-valued, mappings was developed in recent papers of Dubois and Prade [1, 2, 3] and Puri and Ralescu [4]. In particular, Kaleva [5] researched the fuzzy differential equations, Cauchy problem for continuous fuzzy differential equations was studied by Nieto [6], and Song *et al.* [7] obtained the global solutions. Park and Han [8] studied the existence and uniqueness theorem for a solution of fuzzy Volterra integral equations by using the method of successive approximation. Seikkala [9] proved the existence and uniqueness of the fuzzy solution for the following systems:

$$\begin{cases} x'(t) = f(t, x(t)), \quad t \in [0, b], \\ x(0) = x_0, \end{cases}$$

where f is a continuous mapping from $R^+ \times R$ into R and x_0 is a fuzzy number. Recently, the above concept has been extended t_0 the integrodifferential equations in [10] and Alikharni *et al.* [11] proved the existence of global solutions to nonlinear fuzzy Volterra integrodifferential equations. Balachandran and Duar [12] studied the solutions of perturbed fuzzy integral equations.

Neutral differential equations arise in many areas of applied mathematics and for this reason these equations have received much attention during the last few decades [13, 14, 15]. There are also a number of applications in which the delayed argument occurs in the derivative of the state variable as well as in the independent variable, as in the so called neutral differential difference equations. A neutral functional differential equation is one in which the derivatives of the past history or derivatives of functionals of the past history are involved as well as the present state of the system. A good guide to the literature for neutral functional differential equations is the book by Hale and Verduyn Lunel [16] and the references therein. Neutral integrodifferential equations occur in the study of population dynamics, compartmental systems, viscoelasticity and many other fields of science.

In [17], authors introduced for the first time in the fuzzy literature, the fuzzy semigroups of linear operators to solve fuzzy differential equations. Then, Kaleva [18] used nonlinear iteration semigroups (with exponential formula) to study fuzzy Cauchy problems. Ding and Kandel [19] analyzed a way to combine differential equations with fuzzy sets to form a fuzzy logic systems called a fuzzy dynamical system, which can be regarded to form a fuzzy neutral functional differential equations. The theory of semigroups has many applications, it have been studied extensively in the classic literature in recent years. For the reader, we refer to the interesting book of Pazy [20], where the author applied semigroups to solve ordinary and partial differential equations.

This paper is to investigate the controllability results for the following nonlinear neutral fuzzy integrodifferential system using the method of fuzzy (nonlinear) semigroups:

$$\begin{cases} \frac{d}{dt}\Big[x(t)+h(t,x(t))\Big] = Ax(t)+\int_0^t k(t,s,x(s))ds+f(t,x(t))+u(t),\ t\in J, \\ x(0)=x_0, \end{cases} \tag{1.2}$$

where $x_0 \in \mathscr{E}_N$ and the operator A generates a strongly continuous semigroup $\{S(t), t \geq 0\}$ on $\mathscr{E}_N$ and $J = [0,b]$. $\mathscr{E}_N$ is the set of all upper semicontinuous convex normal fuzzy numbers with bounded α - level intervals, $f : J \times \mathscr{E}_N \to \mathscr{E}_N,\ h : J \times \mathscr{E}_N \to \mathscr{E}_N$ and $k : J \times J \times \mathscr{E}_N \to \mathscr{E}_N$ are nonlinear continuous functions.

2. Preliminaries

2.1. Fuzzy sets and numbers

Let $\mathscr{Q}_k(\mathbb{R}^n)$ denote the family of all nonempty compact convex subset of $\mathbb{R}^n$ and define the addition and scalar multiplication in $\mathscr{Q}_k(\mathbb{R}^n)$ as usual. Let A and B be two nonempty bounded subsets of $\mathbb{R}^n$. The distance between A and B is defined by the Hausdorff metric

$$d(A,B) = \max\left\{ \sup_{a\in A}\inf_{b\in B} \|a-b\|,\ \sup_{b\in B}\inf_{a\in A} \|a-b\| \right\},$$

where $\|\cdot\|$ denotes the usual Euclidean norm in $\mathbb{R}^n$. Then it is clear that $\left(\mathscr{Q}_k(\mathbb{R}^n),\ d\right)$ becomes a complete and separable metric space.

Let x be a point in $\mathbb{R}^n$ and A be a nonempty subsets of $\mathbb{R}^n$. We define the Hausdroff separation of B from A by be $d(x,A) = \inf\{\|x-a\| : a \in A\}$. Now let A and B be nonempty subsets of $\mathbb{R}^n$. We define the Hausdroff separation of B from A by

$$d_H^*(B,A) = \sup\{d(b,A) : b \in B\}.$$

In general,

$$d_H^*(A,B) \neq d_H^*(B,A).$$

We define the Hausdroff distance between nonempty subsets of A and B of $\mathbb{R}^n$ by

$$d_H(A,B) = \max\{d_H^*(A,B), d_H^*(B,A)\}.$$

The supremum metric d_∞ on $\mathscr{E}_N$ is defined by

$$d_\infty(v,w) = \sup\{d_H([v]^\alpha,[w]^\alpha) : \alpha \in (0,1]\},\ \text{for all } v,w \in \mathscr{E}_N,$$

and is obviously metric on $\mathscr{E}_N$. The supremum metric H_1 on $\mathscr{C}(J,\mathscr{E}_N)$ is defined by

$$H_1(x,y) = \sup\{d_\infty(x(t),y(t) : t \in J),\ \text{for all } x,y \in \mathscr{C}(J : \mathscr{E}_N)\}.$$

Let $I = [0,1] \subseteq R$ be a compact interval and denote

$$\mathscr{E}_N = \Big\{v : \mathbb{R}^n \to [0,1] |\ v \text{ satisfies } (i)-(iv) \text{ below}\},$$

where

(i) v is normal i.e. there exists an $x_0 \in \mathbb{R}^n$ such that $v(x_0) = 1$;

(ii) v is fuzzy convex;

(iii) v is upper semicontinuous;

(iv) $[v]^0 = cl\{x \in \mathbb{R}^n :\ v(x) > 0\}$ is compact.

For $0 < \alpha \leq 1$, denote $[v]^\alpha = \{t \in \mathbb{R}^n |\ v(t) \geq \alpha\}$. Then from $(i)-(iv)$, it follows that the $\alpha-$ level set $[v]^\alpha \in \mathscr{Q}_k(\mathbb{R}^n)$, for all $0 \leq \alpha \leq 1$.

According to Zadeh's extension principle, we have addition and scalar multiplication in fuzzy number space $\mathscr{E}_N$ as follows:

$$[v+w]^\alpha = [v]^\alpha + [w]^\alpha;\ \ [kv]^\alpha = k[v]^\alpha,$$

where $v,w \in \mathscr{E}_N,\ k \in \mathbb{R}$ and $0 \leq \alpha \leq 1$. Define a mapping $\mathscr{D} : \mathscr{E}_N \times \mathscr{E}_N \to \mathbb{R}^+$ by

$$\mathscr{D}(v,w) = \sup_{0 \leq \alpha \leq 1} d\Big([v]^\alpha, [w]^\alpha\Big),$$

where d is the Hausdorff metric non empty compact sets in $\mathbb{R}^n$. Then it is easy to see that $\mathscr{D}$ is a metric in $\mathscr{E}_N$. Using the results in [21], we know that

(i) $(\mathscr{E}_N, \mathscr{D})$ is a complete metric space;

(ii) $\mathscr{D}(v+z, z+w) = \mathscr{D}(v,w)$, for all $v,z,w \in \mathscr{E}_N$;

(iii) $\mathscr{D}(kv,kw) = |k|\ \mathscr{D}(v,w)$, for all $v,w \in \mathscr{E}_N,\ k \in \mathbb{R}$.

A fuzzy number a in real line $\mathbb{R}$ is a fuzzy set characterized by a membership function $\chi_a : \mathbb{R} \to [0,1]$. A fuzzy number a is expressed as

$$a = \int_{x \in \mathbb{R}} \frac{\chi_a}{x}$$

with the understanding that $\chi_a(x) \in [0,1]$, represents the grade of membership of x in a and $\int$ denotes the union of $\frac{\chi_a}{x}$.

Result 2.1.1. [22] *Let $\mathscr{E}_N$ be the set of all upper semicontinuous convex normal fuzzy numbers with bounded $\alpha-$ level intervals. This means that if $a \in \mathscr{E}_N$, then $\alpha-$ level set*

$$[a]^\alpha = \{x \in \mathbb{R} : a(x) \geq \alpha, 0 \leq \alpha \leq 1\},$$

is a closed bounded interval, which we denote by $[a]^\alpha = [a_q^\alpha, a_r^\alpha]$ and there exists a $t_0 \in \mathbb{R}$ such that $a(t_0) = 1$.

Result 2.1.2. [22] *Two fuzzy numbers a and b are called equal $a = b$, if $\chi_a(x) = \chi_b(x)$, for all $x \in \mathbb{R}$. It follows that $a = b \Leftrightarrow [a]^\alpha = [b]^\alpha$, for all $\alpha \in (0,1]$.*

Lemma 2.1.3. [9] *Let* $[a_q^\alpha, a_r^\alpha]$, $0 < \alpha \leq 1$, *be a given family of nonempty intervals. If*

$$[a_q^\beta, a_r^\beta] \subset [a_q^\alpha, a_r^\alpha] \text{ for } 0 < \alpha \leq \beta,$$

$$[\lim_{k\to\infty} a_q^{\alpha_k}, \lim_{k\to\infty} a_r^{\alpha_k}] = [a_q^\alpha, a_r^\alpha],$$

whenever (α_k) *is non-decreasing sequence converting to* $\alpha \in (0,1]$, *then the family* $[a_q^\alpha, a_r^\alpha]$, $0 < \alpha \leq 1$, *are the* α *-level sets of a fuzzy number* $a \in \mathscr{E}_N$.

We consider $\mathscr{C}(J,\ \mathscr{E}_N)$ the space of all continuous fuzzy functions defined on $[0,b] \subset \mathbb{R}$ into $\mathscr{E}_N$, where $b > 0$. For $v, w \in \mathscr{C}(J,\ \mathscr{E}_N)$, we define the metric

$$H(v,w) = \sup_{t\in[0,b]} \mathscr{D}\Big(v(t), w(t)\Big).$$

Then $(\mathscr{C}(J,\mathscr{E}_N), H)$ is a complete metric space.

We recall some measurability, integrability properties for fuzzy set-valued mappings (see [5]). Let $\mathscr{I} = [0,1] \subset \mathbb{R}$ be a compact interval.

Definition 2.1.4. [23] Let $F(t)$ be a nonempty subset of $\mathbb{R}^n$. Let $\mathscr{F}$ be the set of all point-valued functions $f : \mathscr{I} \to \mathbb{R}^\ltimes$ such that f is integrable over $\mathscr{I}$ and $f(t) \in F(t)$, for all $t \in \mathscr{I}$. It is denoted by $\int_{\mathscr{I}} F(t)dt$, is defined by the equation $\int_{\mathscr{I}} F(t)dt = \Big\{ \int_{\mathscr{I}} f(t)dt\ :\ f \in \mathscr{F} \Big\}$.

Definition 2.1.5. [5] A mapping $F : \mathscr{I} \to \mathscr{E}_N$ is strongly measurable if, for all $\alpha \in [0,1]$ the set-valued function $F_\alpha : \mathscr{I} \to \mathscr{Q}_k(\mathbb{R}^n)$ defined by $F_\alpha(t) = [F(t)]^\alpha$ is Lebesgue measurable when $\mathscr{Q}_k(\mathbb{R}^n)$ has the topology induced by the Hausdorff metric d.

Definition 2.1.6. [5] A mapping $F : \mathscr{I} \to \mathscr{E}_N$ is called integrably bounded if there exists an integrable function k such that $\|x\| \leq k(t)$, for all $x \in F_0(t)$.

Definition 2.1.7. [21] The integral of a fuzzy mapping $F : [0,1] \to \mathscr{E}_n$ is defined level wise by

$$\begin{aligned}\Big[\int_{\mathscr{I}} F(t)dt\Big]^\alpha &= \Big\{\int_{[0,1]} F_\alpha(t)dt\Big\} \\ &= \Big\{\int_{[0,1]} f(t)dt : f : [0,1] \to \mathbb{R}^n \ \text{ is a measurable selection for } F_\alpha\Big\},\end{aligned}$$

for all $\alpha \in [0,1]$.

It has been proved by Puri and Ralescu [21] that a strongly measurable and integrably bounded mapping $F : \mathscr{I} \to \mathscr{E}_N$ is integrable (i.e.$\int_{\mathscr{I}} F(t)dt \in \mathscr{E}_N$). The concept of a fuzzy integral generalizes the Aumann integral of a set-valued mapping. The following results are proved in [23].

Theorem 2.1.8. *If $F : \mathscr{I} \to \mathscr{E}_N$ is continuous, then it is integrable.*

Theorem 2.1.9. *Let $F, G : \mathscr{I} \to \mathscr{E}_N$ be integrable and $\lambda \in \mathbb{R}$. Then*

(i) $\int_{\mathscr{I}} \big(F(t) + G(t)\big)dt = \int_{\mathscr{I}} F(t)dt + \int_{\mathscr{I}} G(t)dt$;

(ii) $\int_{\mathscr{I}} \lambda F(t)dt = \lambda \int_{\mathscr{I}} F(t)dt$;

(iii) $\mathscr{D}(F,G)$ *is integrable;*

(iv) $\mathscr{D}(\int_{\mathscr{I}} F(t)dt, \int_{\mathscr{I}} G(t)dt) = \int_{\mathscr{I}} \mathscr{D}(F(t),G(t))dt$.

2.2. Fuzzy strongly continuous semigroups

Gal and Gal claimed in [17] that the main difficulty of dealing with fuzzy differential equations is the fact that the space of fuzzy numbers $\mathscr{E}_N$ and the space of fuzzy valued functions are not linear spaces. However, they are complete metric spaces with their metric having nice properties. Fortunately, Brezis and Pazy [24] introduced the concept of semigroups of nonlinear contractions on convex sets.

For the proofs of all the following results about fuzzy nonlinear semigroups enounced in this subsection based on the results about the nonlinear semigroups [20].

Definition 2.2.1. [17] By a fuzzy (one parameter strongly continuous nonlinear) semigroup $\{S(t),\ t \geq 0\}$ of operators from $\mathscr{E}_N$ into itself satisfying the following conditions:

(I) $T(0) = I$, the identity mapping on $\mathscr{E}_N$;

(II) $T(t+s) = T(t)\ T(s)$ for all $t,s \geq 0$;

(III) the function $g : [0,\infty) \to \mathscr{E}_N$, defined by $g(t) = T(t)(x)$ is continuous at $t = 0$, for all $x \in \mathscr{E}_N$ i.e., $\lim_{t\to 0^+} S(t)(x) = x$;

(IV) There exist two constants $M > 0$ and ω such that

$$\mathscr{D}\Big(S(t)x, S(t)y\Big) \leq Me^{\omega t}\mathscr{D}(x,y), \quad \text{for } t \geq 0,\ x,y \in \mathscr{E}_N,$$

$\{S(t),\ t \geq 0\}$ is also called a fuzzy C_0-semigroup. In particular if $M = 1$ and $\omega = 0$, we say that $\{S(t),\ t \geq 0\}$ is a contraction fuzzy semigroup.

Lemma 2.2.2. [17] *Let A be the generator of a semigroup $\{S(t),\ t \geq 0\}$ on $\mathscr{E}_N$, then for all $x \in \mathscr{E}_N$ such that $S(t)x \in \mathscr{D}(A)$, for all $t \geq 0$, then mapping $t \to S(t)$ is differentiable and*

$$\frac{d}{dt}(T(t)x) = AT(t)x = T(t)Ax,\ \forall\, t \geq 0.$$

Remark 2.2.3. [17] In the bounded linear case $\frac{d}{dt}T(t) = AT(t) = T(t)A,\ \forall\, t \geq 0.$ but in the general (fuzzy case) $AT(t) \neq T(t)A$.

Lemma 2.2.4. *If $x(t)$ is an integral solution of (1.2) ($u \equiv 0$), then x is given by*

$$\begin{aligned} x(t) &= S(t)(x_0 + h(0,x_0)) - h(t,x(t)) - \int_0^t AS(t-s)h(s,x(s))ds \\ &\quad + \int_0^t S(t-s)f(s,x(s))ds, \text{ for } t \in J. \end{aligned}$$

Proof. Let $x(t)$ be a solution of (1.2). Define $\xi(s) = S(t-s)x(s)$, we have

$$\begin{aligned} \frac{d\xi(s)}{ds} &= -\frac{dS(t-s)}{ds}x(s) + S(t-s)\frac{dx(s)}{ds} \\ &= -AS(t-s)x(s) + S(t-s)\frac{dx(s)}{ds} \\ &= S(t-s)f(s,x(s)) + \mathscr{N}(x,t), \end{aligned} \tag{2.2.1}$$

where $\mathscr{N}(x,t) = -S(t-s)\dfrac{d}{dt}h(t,x(t))$. Now integrating equation (2.2.1) over the interval $[0,t]$, we have

$$\begin{aligned} \int_0^t \frac{d\xi(s)}{ds} &= \int_0^t S(t-s)f(s,x(s))ds + \int_0^t \mathscr{N}(x,s)ds, \\ \xi(t) - \xi(0) &= \int_0^t S(t-s)f(s,x(s))ds \\ x(t) &= S(t)\big[x_0 + h(0,x(0))\big] - h(t,x(t)) \\ &\quad - \int_0^t AS(t-s)h(s,x(s))ds + \int_0^t S(t-s)f(s,x(s))ds. \end{aligned}$$

Therefore, one has

$$\begin{aligned} x(t) &= S(t)\big[x_0 + h(0,x_0)\big] - h(t,x(t)) - \int_0^t AS(t-s)h(s,x(s))ds \\ &\quad + \int_0^t S(t-s)f(s,x(s))ds. \end{aligned}$$

This completes the proof.

3. Existence and uniqueness of fuzzy solutions

In this section, we consider the existence and uniqueness of the fuzzy solution for nonlinear fuzzy neutral integrodifferential equations of the form

$$\begin{cases} \frac{d}{dt}(x(t)+h(t,x(t))) = Ax(t) + \int_0^t k(t,s,x(s))ds + f(t,x(t)), \ t \in [0,b], \\ x(0) = x_0, \end{cases} \tag{3.1}$$

where A generates a strongly continuous fuzzy semigroup $\{S(t),\ t \geq 0\}$ on $\mathscr{E}_N$, $f : J \times \mathscr{E}_N \to \mathscr{E}_N$, $h : J \times \mathscr{E}_N \to \mathscr{E}_N$ and $k : J \times J \times \mathscr{E}_N \to \mathscr{E}_N$ are nonlinear continuous functions and satisfy a global Lipschitz condition, that is, there exist positive finite constants $\Delta_f, \Delta_k, \Delta_h$ and Δ_{ah} such that

- $(\mathbf{H_0})$ A is the infinitesimal generator of a strongly continuous fuzzy semigroup $\{S(t),\ t \geq 0\}$ on $\mathscr{E}_N$ such that $D(A) = \mathscr{E}_N$.
- $(\mathbf{H_1})$ $d_H([f(s,x_1(s))]^\alpha, [f(s,x_2(s))]^\alpha) \leq \Delta_f d_H([x_1(s))]^\alpha, [x_2(s))]^\alpha)$;
- $(\mathbf{H_2})$ $d_H([k(t,s,x_1(s))]^\alpha, [k(t,s,x_2(s))]^\alpha) \leq \Delta_k d_H([x_1(s))]^\alpha, [x_2(s))]^\alpha)$;
- $(\mathbf{H_3})$ $d_H([h(s,x_1(s))]^\alpha, [h(s,x_2(s))]^\alpha) \leq \Delta_h d_H([x_1(s))]^\alpha, [x_2(s))]^\alpha)$;
- $(\mathbf{H_4})$ $d_H([Ah(s,x_1(s))]^\alpha, [Ah(s,x_2(s))]^\alpha) \leq \Delta_{ah} d_H([x_1(s))]^\alpha, [x_2(s))]^\alpha)$, $\forall\, x_1(t), x_2(t) \in \mathscr{E}_N$.

Let $x \in \mathscr{C}(J,\mathscr{E}_N)$, then $[x'(t)]^\alpha = [(x_q^\alpha)', (x_r^\alpha)'],\ 0 < \alpha \leq 1$. The fuzzy integral $\int_a^b x(t)dt$, $a,b \in I$, is defined by $\int_a^b [x(t)]^\alpha = [\int_a^b x_q^\alpha, \int_a^b x_r^\alpha]$ provided Lebesgue integrals on the right exist.

Theorem 3.1. *Suppose that assumptions* $(\mathbf{H_0}) - (\mathbf{H_4})$ *hold. Then, for every* $x_0 \in \mathscr{E}_N$, *the initial value problem (3.1) without control function has a unique solution* $x \in \mathscr{C}(J,\mathscr{E}_N)$.

Proof. Define a mapping $\Gamma : \mathscr{C}(J,E_N) \to \mathscr{C}(J,E_N)$ by

$$\begin{aligned}(\Gamma x(t)) \quad &= \quad S(t)[x_0 + h(0,x_0)] - h(t,x(t)) - \int_0^t AS(t-s)h(s,x(s))ds \\ &\quad + \int_0^t S(t-s)\Big(\int_0^s k(s,r,x(r))dr\Big)ds + \int_0^t S(t-s)f(s,x(s))ds,\end{aligned}$$

for all $x(t) \in \mathscr{C}(J,E_N),\ t \in J$. Moreover, $S(t)$ is a fuzzy number and

$$[S(t)]^\alpha = [S_q^\alpha, S_r^\alpha] = \Big[exp\Big\{\int_0^t a_q^\alpha(s)\Big\}, exp\Big\{\int_0^t a_r^\alpha(s)\Big\}\Big],$$

and $S_i^\alpha(t), (i=q,r)$ is continuous. That is, there exist a constant $\Delta_s > 0$ such that $\big|S_i^\alpha(t)\big| \leq \Delta_s$, for all $t \in J$. For $x_1,\ x_2 \in \mathscr{C}(J,E_N)$, then we have

$$\begin{aligned}
& d_H\Big([(\Gamma(x_1)(t))]^\alpha, (\Gamma(x_2)(t))]^\alpha\Big) \\
= \;& d_H\Big(\Big[S(t)[x_0+h(0,x_0)] - h(t,x_1(t)) - \int_0^t AS(t-s)h(s,x_1(s))ds \\
& + \int_0^t S(t-s)\Big(\int_0^s k(s,r,x_1(r))dr\Big)ds + \int_0^t S(t-s)f(s,x_1(s))ds\Big]^\alpha, \\
& \Big[S(t)[x_0+h(0,x_0)] - h(t,x_2(t)) - \int_0^t AS(t-s)h(s,x_2(s))ds \\
& + \int_0^t S(t-s)\Big(\int_0^s k(s,r,x_2(r))dr\Big)ds + \int_0^t S(t-s)f(s,x_2(s))ds\Big]^\alpha\Big) \\
\leq \;& \Delta_h d_H([x_1]^\alpha,[x_2]^\alpha) + \Delta_s\Delta_k b\int_0^t d_H\Big(\Big[x_1(s))\Big]^\alpha, \Big[x_2(r)\Big]^\alpha\Big)ds \\
& + \Delta_s\Delta_f \int_0^t d_H\Big(\Big[x_1(s))\Big]^\alpha, \Big[x_2(s))\Big]^\alpha\Big)ds + \Delta_s\Delta_{ah}\int_0^t d_H\Big(\Big[x_1(s)\Big]^\alpha, \Big[x_2(s))\Big]^\alpha\Big)ds \\
\leq \;& \Delta_h d_H([x_1]^\alpha,[x_2]^\alpha) + \big(b\Delta_s\Delta_k + \Delta_s\Delta_f + \Delta_s\Delta_{ah}\big)\int_0^t d_H([x_1]^\alpha,[x_2]^\alpha)ds \\
\leq \;& \Delta_h d_H([x_1]^\alpha,[x_2]^\alpha) + \eta\int_0^t d_H([x_1]^\alpha,[x_2]^\alpha)ds,
\end{aligned}$$

where $\eta = \Delta_s\big(b\Delta_k + \Delta_f + \Delta_{ah}\big)$. Therefore

$$\begin{aligned}
d_\infty((\Gamma x_1, \Gamma x_2) &= \sup_{t\in J} d_H([(\Gamma x_1)]^\alpha, [(\Gamma x_2)]^\alpha) \\
&\leq \Delta_h \sup_{t\in J} d_H([x_1]^\alpha,[x_2]^\alpha) + \sup_{t\in J}\eta\int_0^t d_H([x_1]^\alpha,[x_2]^\alpha)ds \\
&\leq \Delta_h d_\infty([x_1]^\alpha,[x_2]^\alpha) + \eta\int_0^t d_\infty([x_1]^\alpha,[x_2]^\alpha)ds.
\end{aligned}$$

Hence

$$\begin{aligned}
H_1(\Gamma x_1, \Gamma x_2) &= \sup_{t\in J} d_\infty([x_1]^\alpha,[x_2]^\alpha) \\
&\leq \Delta_h \sup_{t\in J} d_\infty([x_1]^\alpha,[x_2]^\alpha) + \eta\sup_{t\in J}\int_0^t d_\infty([x_1]^\alpha,[x_2]^\alpha)ds \\
&\leq (\Delta_h + b\eta)H_1(x_1,x_2).
\end{aligned}$$

We take sufficiently small b, $(\Delta_h + b\eta) < 1$. Hence, Γ is a contraction mapping. By Banach fixed point theorem, nonlinear neutral fuzzy integrodifferential equation has a unique fixed point $x \in \mathscr{C}(J, \mathscr{E}_N)$.

4. Controllability results

The notion of controllability is of great importance in mathematical control theory. Many fundamental problems of control theory such as pole-assignment, stabilizability and optimal control may be solved under the assumption that the system is controllable. It means that it is possible to steer any initial state of the system to any final state in some finite time using an admissible control. During the last few decades, several authors have discussed the existence, uniqueness, and asymptotic behavior of the solution of these systems. Apart from these, the study of controllability and observability properties of a system in control theory is certainly, at present, one of the most active interdisciplinary areas of research. Control theory arises in most modern applications. On the other hand, control theory has remained a discipline where many mathematical ideas and methods have fused to produce a new body of important mathematics. In control theory, one of the most important qualitative aspects of a dynamical system is controllability. As far as the controllability problems associated with finite-dimensional systems modelled by ODEs are concerned, this theory has been extensively studied during the last decades. In the finite-dimensional context, a system is controllable if and only if the algebraic Kalman rank condition is satisfied. According to this property, when a system is controllable for some time, it is controllable for all the time. But this is no longer true in the context of infinite-dimensional systems modelled by PDEs.

In this section, we show the controllability results for the nonlinear neutral fuzzy integrodifferential system (1.2) of the form

$$\begin{aligned} x(t) = S(t)[x_0 + h(0,x_0)] - h(t,x(t)) - \int_0^t AS(t-s)h(s,x(s))ds \\ + \int_0^t S(t-s)\Big(\int_0^s k(s,r,x(r))dr\Big)ds + \int_0^t S(t-s)f(s,x(s))ds \\ + \int_0^t S(t-s)u(s)ds, \qquad t \in J. \end{aligned} \tag{4.1}$$

Definition 4.1. [25, 26, 27] System (1.2) is said to be controllable on the interval $[0,b]$ if for every $x^1 \in X$, there exists a control $u(t) \in \mathscr{L}^2(J,U)$ such that the solution fuzzy solution $x(t)$ of (1.2) satisfies $x(b) = x^1$, that is $[x(b)]^\alpha = [x^1]^\alpha$, where x^1 is a target set.

Definition 4.2. [25, 26, 27] Equation (4.1) is controllable if there exists $u(t)$such that the fuzzy solution $x(t)$ of (5.1) satisfies $x(T) = x^1$, that is $[x(T)]^\alpha = [x^1]^\alpha$, where x^1 is a target set.

We assume that the linear control system with respect to the nonlinear control system (1.2) is controllable. Then

$$\begin{aligned}
x(b) &= S(b)X_0 + \int_0^b S(b-s)u(s)ds = x^1 \quad \text{and} \\
[x(b)]^\alpha &= [S(b)x_0 + \int_0^b S(b-s)u(s)ds]^\alpha \\
&= [S_q^\alpha(b)x_{0q}^\alpha + \int_0^b S_q^\alpha(b-s)u_q^\alpha(s)ds, S_r^\alpha(b)x_{0r}^\alpha + \int_0^b S_r^\alpha(b-s)u_r^\alpha(s)ds] \\
&= [(x^1)_q^\alpha, (x^1)_r^\alpha].
\end{aligned}$$

Defined the fuzzy mapping $\gamma : \widehat{P(R)} \to \mathscr{E}_N$ by

$$\gamma^\alpha(u) = \begin{cases} \int_0^t S(t-s)u(s)ds, & u \subset \overline{\Gamma}_u, \\ 0, & \text{otherwise.} \end{cases}$$

Then there exists $\gamma_i^\alpha (i = q, r)$ such that

$$\begin{aligned}
\gamma_q^\alpha(u_q) &= \int_0^t S_q^\alpha(t-s)u_q(s)ds, \ \ u_q \in [u_q^\alpha, u^1] \\
\gamma_r^\alpha(u_r) &= \int_0^t S_r^\alpha(t-s)u_r(s)ds, \ \ u_r \in [u^1, u_r^\alpha].
\end{aligned}$$

We assume that γ_i's are bijective mappings. Hence the α-set of $u(s)$ are

$$\begin{aligned}
[u(s)]^\alpha &= [u_q^\alpha(s), u_r^\alpha(s)] \\
&= \Big[(\widehat{\gamma}_q^\alpha)^{-1}\big\{(x^1)_q^\alpha - S_q^\alpha(b)[(x_0)_q^\alpha + h_q^\alpha(0,x_0)] + h_q^\alpha(t,x(t)) + \int_0^t AS_q^\alpha(t-s)h_q^\alpha(t,x(s))ds \\
&\quad - \int_0^t S_q^\alpha(t-s)f_q^\alpha(t,x(s))ds - \int_0^t S_q^\alpha(t-s)(\int_0^t k_q^\alpha(\tau,t,x(t))dt)ds\big\}, \\
&\quad (\widehat{\gamma}_r^\alpha)^{-1}\big\{(x^1)_r^\alpha - S_r^\alpha(b)[(x_0)_r^\alpha + h_r^\alpha(0,x_0)] + h_r^\alpha(t,x(t)) + \int_0^t AS_r^\alpha(t-s)h_r^\alpha(t,x(s))ds \\
&\quad - \int_0^t S_r^\alpha(t-s)f_r^\alpha(t,x(s))ds - \int_0^t S_r^\alpha(t-s)(\int_0^t k_r^\alpha(\tau,t,x(t))dt)ds\big\}\Big].
\end{aligned}$$

Then substituting this expression into equation (4.1) yields α- level set of $x(T)$

$$\begin{aligned}
[x(b)]^\alpha &= \Big[S_q^\alpha(b)[(x_0)_q^\alpha + h_q^\alpha(0,x_0)] - h_q^\alpha(t,x(t)) - \int_0^b AS_q^\alpha(b-s)h_q^\alpha(t,x(s))ds \\
&+ \int_0^b S_q^\alpha(b-s)f_q^\alpha(t,x(s))ds + \int_0^b S_q^\alpha(b-s)(\int_0^t k_q^\alpha(t,x(t))dt)ds) \\
&+ \int_0^b S_q^\alpha(b-s)(\widehat{\gamma}_q^\alpha)^{-1}((x^1)_q^\alpha - S_q^\alpha(b)[(x_0)_q^\alpha + h_q^\alpha(0,x_0)] + h_q^\alpha(t,x(t)) \\
&+ \int_q^t AS_q^\alpha(b-s)h_q^\alpha(t,x(s))ds - \int_0^t S_q^\alpha(b-s)f_q^\alpha(t,x(s))ds \\
&- \int_0^b S_q^\alpha(b-s)(\int_0^t k_q^\alpha(t,s,x(s))dt)ds), \\
&S_r^\alpha(b-s)[(x_0)_r^\alpha + h_r^\alpha(0,x_0)] - h_r^\alpha(t,x(t)) - \int_0^b AS_r^\alpha(b-s)h_r^\alpha(t,x(s))ds \\
&+ \int_0^b S_r^\alpha(b-s)f_r^\alpha(t,x(s))ds + \int_0^b S_r^\alpha(t-s)(\int_0^t k_r^\alpha(t,x(t))dt)ds) \\
&+ \int_0^b S_r^\alpha(b-s)(\widehat{\gamma}_r^\alpha)^{-1}((x^1)_r^\alpha - S_r^\alpha(b)[(x_0)_r^\alpha + h_r^\alpha(0,x(0))] \\
&+ h_r^\alpha(t,x(t)) + \int_0^b AS_r^\alpha(b-s)h_r^\alpha(t,x(s))ds \\
&- \int_0^b S_r^\alpha(b-s)f_r^\alpha(t,x(s))ds - \int_0^b S_r^\alpha(b-s)(\int_0^t k_0^\alpha(t,s,x(s))dt)ds)\Big] \\
&= [(x^1)_q^\alpha, (x^1)_r^\alpha] \\
&= [x^1]^\alpha.
\end{aligned}$$

We now set

$$\begin{aligned}
\Omega\, x(t) &= S(t)[x_0 + h(0,x(0))] - h(t,u(t)) - \int_0^t AS(t-s)h(t,x(s))ds \\
&+ \int_0^t S(t-s)f(s,x(s))ds + \int_0^t S(t-s)(\int_0^t k(t,s,x(s))dt)ds \\
&+ \int_0^t S(t-s)\widehat{\gamma}^{-1}\Big(x^1 - S(b)[x_0 + h(0,x(0))] + h(s,u(s)) \\
&+ \int_0^b AS(b-s)h(s,x(s))ds - \int_0^b S(b-s)f(s,x(s))ds \\
&- \int_0^b S(b-s)(\int_0^s k(\tau,s,x(s))d\tau)ds\Big)ds,
\end{aligned}$$

where the fuzzy mapping $\widehat{\gamma}^{-1}$ satisfied above statement. Notice that $\Omega x(T) = x^1$, which means that the control $u(t)$ steers the equation (4.1) from the origin to x^1 in the time b provided we can obtain a fixed point of the nonlinear operator Ω. To prove the controllability results we need the following hypotheses:

$(\mathbf{H_5})$ The system (4.1) is linear $f \equiv 0$ is controllable.

$(\mathbf{H_6})$ $\Delta_h + (\Delta_s(\Delta_f + b\Delta_k + \Delta_{ah}) < 1$.

Theorem 4.3. *Suppose that the hypotheses* $(\mathbf{H_0}) - (\mathbf{H_6})$ *are satisfied. Then system (1.2) is controllable.*

Proof. We can easily check that Ω is continuous mapping from $\mathscr{C}([0,b],\mathscr{E}_N)$ to itself. For $x,y \in \mathscr{C}([0,b],\mathscr{E}_N)$

$$
\begin{aligned}
& d_H([\Omega x(t)]^\alpha, [\Omega y(t)]^\alpha) \\
&= d_H\Big(\Big[S(t)x_0 - h(0,x_0) + h(t,x(t)) \int_0^t S(t-s)h(t,x(s))ds \\
&\quad + \int_0^t S(t-s)f(s,x(s))ds + \int_0^t S(t-s)(\int_0^t k(t,s,x(s))dt)ds \\
&\quad + \int_0^b S(b-s)\widehat{\gamma}^{-1}\Big(x^1 - S(b)[x_0 - h(0,x(0)] - h(t,x(t)) \\
&\quad - \int_0^b S(b-s)h(s,x(s))ds - \int_0^b S(b-s)f(s,x(s))ds - \\
&\quad \int_0^b S(b-s)(\int_0^s k(b,s,x(s))db)ds\Big)ds\Big]^\alpha, \\
&\quad \Big[S(t)x_0 - h(0,x_0) + h(t,y(t)) + \int_0^t S(t-s)h(t,y(s))ds \\
&\quad + \int_0^t S(t-s)f(s,y(s))ds + \int_0^t S(t-s)(\int_0^t k(t,s,y(s))dt)ds \\
&\quad + \int_0^b S(b-s)\widehat{\gamma}^{-1}\Big(x^1 - S(b-s)[x_0 - h(0,x(0)] - h(t,y(t)) \\
&\quad - \int_0^b S(b-s)h(s,x(s))ds - \int_0^b S(b-s)f(s,y(s))ds \\
&\quad - \int_0^b S(b-s)(\int_0^s k(\tau,s,y(s))d\tau)ds\Big)ds\Big]^\alpha\Big)\Big) \\
&\le \Delta_h d_H([x(s)]^\alpha, [y]^\alpha) + (\Delta_s(\Delta_{ah} + \Delta_f + b\Delta_k) \int_0^t d_H([x(s)]^\alpha, [y(s)]^\alpha) \\
&\quad + \Delta_h d_H([x(s)]^\alpha, [y]^\alpha) + (\Delta_s(\Delta_{ah} + \Delta_f + b\Delta_k) \int_0^b d_H([x(s)]^\alpha, [y(s)]^\alpha) \\
&\le 2\Delta_h d_H([x(s)]^\alpha, [y]^\alpha) + \kappa\Big(\int_0^t d_H([x(s)]^\alpha, [y(s)]^\alpha) + \int_0^b d_H([x(s)]^\alpha, [y(s)]^\alpha)\Big),
\end{aligned}
$$

where $\kappa = \Delta_s(\Delta_{ah} + \Delta_f + b\Delta_k)$. Therefore

$$\begin{aligned} d_\infty(\Omega x(t), \Omega y(t)) &= \sup_{\alpha\in[0,1]} d_H([\Omega x(t)]^\alpha, [\Omega y(t)]^\alpha) \\ &\leq 2\Delta_h d_\infty([x(s)]^\alpha, [y]^\alpha) \\ &\quad + \kappa\Big(\int_0^t d_H([x(s)]^\alpha, [y(s)]^\alpha) + \int_0^b d_inftý([x(s)]^\alpha, [y(s)]^\alpha)\Big). \end{aligned}$$

Hence

$$\begin{aligned} H_1(\Omega x(t), \Omega y(t)) &= \sup_{t\in[0,b]} d_H([\Omega x(t)]^\alpha, [\Omega y(t)]^\alpha) \\ &\leq 2(\Delta_h + 2\kappa b)H_1(x,y) \\ &= (2(\Delta_h + \kappa b))H_1(x,y). \end{aligned}$$

By hypotheses ($\mathbf{H_6}$), we take sufficiently small b, Ω is a contraction mapping. By Banach fixed point theorem, system (1.2) has a unique fixed point $x \in \mathscr{C}([0,b], \mathscr{E}_N)$.

5. Examples

Consider the fuzzy solution of the nonlinear fuzzy neutral integrodifferential equation of the form:

$$\begin{cases} \frac{d}{dt}(x(t) - \mathbf{2}tx(t)^2)) = \mathbf{2}x(t) + \mathbf{2}tx(t)^2 + \mathbf{3}tx(t)^2 + u(t), \quad t \in J, \\ x(0) = \mathbf{0} \in E_N, \\ x^1 = \mathbf{2}, \end{cases} \tag{5.1}$$

where x^1 is target set, and the α - level set of $\mathbf{0}$, $\mathbf{2}$ and $\mathbf{3}$ are $[\mathbf{0}]^\alpha = [\alpha - 1, 1 - \alpha]$, for $\alpha \in [0,1]$; $[\mathbf{2}]^\alpha = [\alpha + 1, 3 - \alpha]$, for $\alpha \in [0,1]$ $[\mathbf{3}]^\alpha = [\alpha + 2, 4 - \alpha]$, for $\alpha \in [0,1]$. Let $\int_0^t k(t,s,x(s))ds = \mathbf{2}tx(t)^2, f(t,x(t)) = \mathbf{3}tx(t)^2, h(t,x(t)) = \mathbf{2}tx(t)^2$. Then α - level set of $\int_0^t k(t,s,x(s))ds = \mathbf{2}tx(t)^2$ is

$$\begin{aligned}\Big[\int_0^t k(t,s,x(s))ds\Big]^\alpha &= [\mathbf{2}tx(t)^2]^\alpha\\ &= t[\mathbf{2}]^\alpha[x(t)^2]^\alpha\\ &= t[\alpha+1,3-\alpha][(x_q^\alpha(t))^2,(x_r^\alpha(t))^2]\\ &= t[(\alpha+1)(x_q^\alpha(t))^2,(3-\alpha)(x_r^\alpha(t))^2],\end{aligned}$$

where $[x(t)]^\alpha = [x_q^\alpha(t), x_r^\alpha(t)]$ and $[\mathbf{2}]^\alpha = [\alpha+1, 3-\alpha]$, for $\alpha \in [0,1]$ and the α - level set of $f(t,x(t))$ is $[f(t,x(t))]^\alpha = [\mathbf{3}tx(t)^2]^\alpha = t[(\alpha+2)(x_q^\alpha(t)^2),(4-\alpha)(x_r^\alpha(t))^2]$, where $[x(t)]^\alpha = [x_q^\alpha(t), x_r^\alpha(t)]$ and $[\mathbf{3}]^\alpha = [\alpha+2, 4-\alpha]$, for $\alpha \in [0,1]$ and the α - level set of $h(t,x(t))$ is

$$\begin{aligned}[h(t,x(t))]^\alpha &= [\mathbf{2}tx(t)^2]^\alpha\\ &= t[\alpha+1,3-\alpha][(x_q^\alpha(t))^2,(x_r^\alpha(t))^2]\\ &= t[(\alpha+1)(x_q^\alpha(t))^2,(3-\alpha)(x_r^\alpha(t))^2].\end{aligned}$$

It follows that

$$\begin{aligned}[u(s)]^\alpha &= \Big[u_q^\alpha, u_r^\alpha\Big]\\ &= \Big[\widehat{\gamma}_q^{-1}\Big((\alpha+1) - t(\alpha+1)(x_q^\alpha)^2 - \int_0^b S_q^\alpha(b-s)t(\alpha+1)(x_q^\alpha)^2(s)ds\\ &\quad - \int_0^b S(b-s)t(\alpha+2)(x_q^\alpha)^2(s)ds - \int_0^b S(b-s)t(\alpha+1)(x_q^\alpha)^2(s)ds\Big),\\ &\quad \widehat{\gamma}_r^{-1}\Big((\alpha+1) - t(\alpha+1)(x_q^\alpha)^2 - \int_0^b S_r^\alpha(b-s)t(\alpha+1)(x_r^\alpha)^2(s)ds\\ &\quad - \int_0^b S_r^\alpha(b-s)t(\alpha+2)(x_r^\alpha)^2(s)ds - \int_0^b S_r^\alpha(b-s)t(\alpha+1)(x_r^\alpha)^2(s)ds\Big)\Big].\end{aligned}$$

Then substituting this expression into the integral system with respect to (5.1) yields $\alpha-$ level set of $x(T)$.

$$\begin{aligned}[x(T)]^\alpha = \Big[&S_q^\alpha(t)(\alpha-1)]t(\alpha+1)t(x_q^\alpha)^2(t) + \int_q^t S_q^\alpha(t-s)t(\alpha+1)t(x_q^\alpha)^2(t)ds\\ &+ \int_0^t S_q^\alpha(t-s)t(\alpha+2)(x_q^\alpha)^2)ds + \int_0^t S_q^\alpha(t-s)t(\alpha+1)(x_q^\alpha)^2(t)ds)\\ &+ \int_0^b S_q^\alpha(b-s)(\widehat{\gamma}_q^\alpha)^{-1}\Big((\alpha+1) - t(\alpha+1)(x_q^\alpha)^2(t) - \int_0^b S_q^\alpha(b-s)t(\alpha+1)(x_q^\alpha)^2(t)ds\\ &- \int_0^t S_q^\alpha(t-s)t(\alpha+2)(x_q^\alpha)^2(s)ds - \int_0^t S_q^\alpha(t-s)t(\alpha+1)(x_q^\alpha(s))^2ds)ds\Big),\end{aligned}$$

$$\begin{aligned}
& \Big[S_r^\alpha(b)(1-\alpha)]t(\alpha+1)t(x_r^\alpha)^2(t)+\int_0^b S_r^\alpha(b-s)t(3-\alpha)t(x_r^\alpha)^2(t)ds \\
& +\int_0^t S_r^\alpha(t-s)t(4-\alpha)(x_r^\alpha)^2)ds+\int_0^t S_q^\alpha(t-s)t(3-\alpha)(x_r^\alpha)^2(t)ds) \\
& +\int_0^b S_r^\alpha(b-s)(\widehat{\gamma}_r^\alpha)^{-1}\Big((3-\alpha)-t(\alpha+1)(x_r^\alpha)^2(t)-\int_0^b S_q^\alpha(b-s)t(3-\alpha)(x_r^\alpha)^2(t)ds \\
& -\int_0^b S_r^\alpha(b-s)t(4-\alpha)(x_r^\alpha)^2(s)ds-\int_0^b S_r^\alpha(b-s)t(3-\alpha)(x_r^\alpha(s))^2ds)ds\Big)\Big] \\
= & \; [\alpha+1,3-\alpha] \\
= & \; [\mathbf{2}]^\alpha.
\end{aligned}$$

Then all condition stated in Theorem 4.3 are satisfied, so system (5.1) is controllable on $[0,b]$.

6. Conclusion

This paper proved the sufficient conditions of controllability for the nonlinear neutral fuzzy integrodifferential control system (1.2) in $\mathscr{E}_N$ and there by the concept of fuzzy number in $\mathscr{E}_N$. The future study is to extend this work to find the stability of the given control system with fixed and random impulse nature of conditions.

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