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A NEW GENERAL MEASURE OF NONCOMPACTNESS AND FIXED POINT THEOREM FOR CONDENSING OPERATORS

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Abstract. This paper is concerned with the new generalized Darbo's fixed point theorems (GDFT). A new version of the GDFT to a condensing maps with respect to a measure of noncompactness and control functions is proved. This new theorem generalizes some GDFT, which were recently proved in [R. Arab, H. K. Nashine, N. H. Can, T. T. Binh, Solvability of functional-integral equations (fractional order) using measure of noncompactness, Adv. Difference Equ. 2020 (2020) 12; A. Das, B. Hazarika, P. Kumam, Some new generalization of Darbo's fixed point theorem and its application on integral equations, Mathematics 7 (2019) 214; N. Khodabakhshi, S. M. Vaezpour, Common fixed point theorems via measure of noncompactness, Fixed Point theory 17 (2016) 381-386].

Keywords. Condensing operator; Darbo's fixed point theorem; Measures of noncompactness.

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1. Introduction

The technique of the measure of noncompactness (MNC), which is now under the research spotlight, is a useful tool in proving the existence results to various types of nonlinear differential equations, nonlinear functional integral equations as well as their infinite systems; se, e.g., [2, 8, 10]. The MNC also plays a significant to prove the existence results for equations in abstract spaces such as a scale of Banach spaces; see [9]. The authors studied the problem by reducing to a fixed point problem of a condensing operator with respect to a MNC. The class of condensing operators which are known as operators satisfying the Darbo condition with respect to the Kuratowski MNC is one of generalizations of compact operators. Based on the concept of a arbitrary MNC, various generalized Darbo's fixed point theorems (GDFT) were introduced and investigated recently. Those theorems generalize the Darbo's fixed point theorem which is based on the Kuratowski MNC and control functions and combine three generalized Darbo's

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fixed point theorems which were proved in [3, 5, 7] with the aid of a new version of generalized Darbo's fixed point theorems.

The paper is organized as follows. In Section 2, we introduce a new version of condensing operator with respect to a generalized version of measure of noncompactness and presented some of standard facts on measure of noncompactness. The main results of the paper are presented in Section 3. The last section, Section 4, ends this paper with a concluding remark.

2. Preliminaries and Assumptions

In this section, we summarize the relevant results on locally convex spaces and on the measures of noncompactness which are used in the sequel.

Proposition 2.1. *Let E be a locally convex space whose topology is defined by a countable separating family of seminorms $(p_k)_{k\geq 1}$. Then E is metrizable and in E, and a sequence $\{u_n\}_n$ converges to u if and only if $\lim_{n\to\infty} p_k(u_n - u) = 0$ for all $k \geq 1$.*

Definition 2.2. [1] Let E be a locally convex space, and let $\mathscr{M}$ be a family of all bounded subsets of E. Let $(Q, \leq)$ be an ordered Banach space, and let K be a cone, contained in the positive cone. An operator $\Phi : \mathscr{M} \to K$ is called a measure of noncompactness (MNC for short) if $\Phi(\overline{conv}(\Omega)) = \Phi(\Omega)$ for all $\Omega \in \mathscr{M}$, where $\overline{conv}(\Omega)$ stands for the closed convex hull of Ω.

Definition 2.3. The MNC Φ is said to be

(i) *regular* if it satisfies the following condition: $\Phi(\Omega) = 0_Q$ if and only if Ω is relatively compact.
(ii) *monotone* if, for all $\Omega \in \mathscr{M}$ and $\Omega_1 \subset \Omega_2$, $\Phi(\Omega_1) \leq \Phi(\Omega)$.
(iii) *nonsingular* if, for all $\Omega \in \mathscr{M}$ and $u \in E$, $\Phi(\Omega \cup \{u\}) = \Phi(\Omega)$.

Proposition 2.4. *Let E be a Banach space. Define the Kuratowski measure of noncompactness in E by, for each bounded subset $\Omega \subset E$,*

$$\alpha(\Omega) = \inf\{d > 0 : \Omega \text{ is covered by a finite family of subsets with diameter less than } d\}.$$

Then

(1) α *is regular;*
(2) $\alpha(\Omega_1 \cup \Omega_2) = \max\{\alpha(\Omega_1), \alpha(\Omega_2)\}$;
(3) $\alpha(\lambda\Omega) = |\lambda|\alpha(\Omega)$;
(4) $\alpha(\Omega_1 + \Omega_2) \leq \alpha(\Omega_1) + \alpha(\Omega_2)$;

We next recall the following celebrated results.

Theorem 2.5. *(Darbo [4]) Let D be a nonempty, bounded, closed, and convex subset of a Banach space E. Let $F : D \to D$ be a continuous mapping. Assume that there exists a constant $k \in (0,1)$ such that $\alpha(F(M)) \leq k\alpha(M)$, $M \subset D$. Then F has a fixed point.*

Theorem 2.6. *(Shauder-Tychonoff [6]) Let D be a nonempty and convex subset of a locally convex Hausdorff topological space E. Let $F : D \to D$ be a compact continuous mapping. Then F has a fixed point.*

If E is a normed space, the above theorem reduces to the Schauder theorem.

Definition 2.7. Let Φ be a MNC defined on a family $\mathscr{M}$ of bounded subsets of the locally convex space E, and let T be continuous operator from $D \subset E$ to E such that $FixT := \{u \in D : Tu = u\} \neq \emptyset$ and, for all $\Omega \subset D$, $T(\overline{conv}(\Omega)) \subset \overline{conv}(T(\Omega))$. An operator $F : D \subset E \to E$ is said to be condensing with respect to Φ, T (or $\Phi, T - condensing$) if

(i) $TF = FT$;
(ii) for every bounded subset Ω of D, if $\Phi(T(\Omega)) > 0$, then $\Phi(F(\Omega)) < \Phi(T(\Omega))$.

3. The Results

The following theorem was proved in [9] for $T = I$ and it is extended now to the new setting.

Theorem 3.1. *Let E be a Fréchet space, and let $D \subset E$ be a nonempty, convex, and closed set. Let $F, T : D \to D$ be continuous operators, and let Φ be a regular, monotone, and nonsingular MNC defined on a family $\mathscr{M}$ of bounded subsets of E. If the operator F is $\Phi, T - condensing$ and $F(D)$ is bounded, then F has at least one fixed point in D.*

Proof. Let us choose a point $u \in Fix(T)$ and set $v := Fu \in \overline{conv}(F(D)) \in \mathscr{M}$. Since $v = Fu = F(Tu) = T(Fu) = Tv$, we see that $v \in Fix(T)$. We denote by Σ the class of all closed and convex subsets Ω of D such that

$$\Omega \in \mathscr{M},\ v \in \Omega, F(\Omega) \subset \Omega \text{ and } T(\Omega) \subset \Omega. \tag{3.1}$$

Set

$$B = \bigcap_{\Omega \in \Sigma} \Omega, \ \ K = \overline{conv}\left(F(B) \cup \{v\}\right).$$

We see that $\overline{conv}(F(D)) \in \Sigma$ by condition 2, $\overline{conv}(F(D)) \subset D$, and

$$T(\overline{conv} F(D)) \subset \overline{conv}(T(F(D))) = \overline{conv}(F(T(D))) \subset \overline{conv}(F(D)).$$

This gives $B \subset \overline{conv}(F(D))$. We next conclude from $F(\Omega) \subset \Omega$, $\forall \Omega \in \Sigma$ that $F(B) \subset B$. Hence $K \in \mathscr{M}$ by condition 2.

We now claim $K \in \Sigma$. Indeed, since $v \in B$ and $F(B) \subset B$, it follows that $K \subset B$, which implies $F(K) \subset F(B) \subset K$. Furthermore, $T(\Omega)$ is in Ω for all $\Omega \in \Sigma$, which gives $T(B) \subset B$. Thus $T(F(B)) = F(T(B)) \subset F(B)$, and that

$$\begin{aligned} T(K) &\subset \overline{conv}(T(F(B) \cup \{v\})) = \overline{conv}(T(F(B)) \cup \{Tv\}) \\ &\overline{conv}(F(B) \cup \{v\}) \subset K. \end{aligned}$$

This clearly forces $B \subset K$, and consequently $B = K$. Therefore the monotonicity and nonsingularity of Φ yield that

$$\begin{aligned} \Phi(TB) &= \Phi(TK) = \Phi\left(T(F(B)) \cup \{Tu\}\right) \\ &= \Phi\left(F(T(B))\right) \leq \Phi(F(B)). \end{aligned}$$

Since F is Φ, T-condensing, it follows that $\Phi(T(B)) = 0$. The regularity of Φ yields that $T(B)$ is compact. As $F(B) \subset B$, we have $F(T(B)) = T(F(B)) \subset T(B)$. From the Schauder–Tychonoff theorem, we conclude that there is a fixed point for $F : T(B) \to T(B)$. □

From now on, we assume that E is a Banach space, $D \subset E$ is a nonempty, closed, and closed set. Let $fix(F) = \{u \in D : f(u) = u\}$ and Φ be a regular, monotone and nonsingular MNC defined on a family $\mathscr{M}$ of bounded subsets of E.

The next subsections show the consequences of the (3.1) where we use the identity operator for T in the first and second sequences.

3.1. The first consequence. We recall the following notation which was introduced in [3].

Let us denote by $\Gamma_{G,\beta}$ the set of pairs (G,β), where $G : \mathbb{R}^+ \to \mathbb{R}$ and $\beta : \mathbb{R}^+ \times \mathbb{R}^+ \to [0,1)$ such that

Γ1 for each sequence $t_n \subset \mathbb{R}^+$, $\limsup_{n\to\infty} G(t_n) \geq 0 \Leftrightarrow \limsup_{n\to\infty} t_n \geq 1$;
Γ2 for the sequences $t_n, s_n \subset \mathbb{R}^+$, $\limsup_{n\to\infty} \beta(t_n, s_n) = 1 \Rightarrow \lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n = 0$;
Γ3 for the sequences $t_n, s_n \subset \mathbb{R}^+$, $\sum_{n=1}^{\infty} G(\beta(t_n, s_n)) = -\infty$

Proposition 3.2. *Let $F : D \subset E \to D$ be continuous operator and $(G,\beta) \in \Gamma_{G,\beta}$. Let $S : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ be continuous and strictly increasing mappings. If $M \subset D$ and $\Phi(F(M)) > 0$, then*

$$S(\Phi(F(M)), \varphi(\Phi(F(M)))) \leq S(\Phi(M), \varphi(\Phi(M))) + G(\beta(\Phi(M), \varphi(\Phi(M)))). \tag{3.2}$$

Then F is $\Phi, I-$condensing where I is the identity operator.

Proof. Suppose $M \subset D$ and $\Phi(M) > 0$. If $\Phi(F(M)) = 0$, then $\Phi(F(M)) < \Phi(M)$. If $\Phi(F(M)) > 0$, on account of the properties of β and G, we have $\beta(\Phi(M), \varphi(\Phi(M))) < 1$. Hence

$$G(\beta(\Phi(M), \varphi(\Phi(M)))) < 0,$$

which together with (3.2) implies

$$S(\Phi(F(M)), \varphi(\Phi(F(M)))) < S(\Phi(M), \varphi(\Phi(M))).$$

We conclude from the strictly increasing monotonicity of S, φ that $\Phi(F(M)) < \Phi(M)$. This proved that F is Φ, I - condensing. ☐

The following result is the main result of [3]. In view of the proposition above, we see that it is a corollary of Theorem 3.1.

Corollary 3.3. [3] *Let $F : D \subset E \to D$ be a continuous operator with the restriction that $F(D)$ is bounded. If (3.2) hols for all $\emptyset \neq M \subset D$ and $\Phi(F(M)) > 0$, then $fix(F) \neq \emptyset$.*

In the sequel, we present an application for the corollary above.

Let X be a Banach space, and let $E = C(I, X)$ be the space of all continuous functions $u : [a,b] \to X$ with the sup-norm $\|.\|$. We denote by α the Kuratowski MNC in E. We will prove an existence theorem in E for the following functional equation

$$u(t) = f(u,u)(t) := F(u)(t), \;\; t \in [a,b] \tag{3.3}$$

where $f : D \times D \to D$, D is bounded in E.

Assume that f is contractive in the first variable and compact in the second variable, that is,

A1. for each $u \in E$ there exists a position number $L < 1$ such that

$$\|f(u_1,v) - f(u_2,v)\| \leq L\|u_1 - u_2\|;$$

A2. for each $u \in E$ and bounded subset $M \subset E$, the set $\{f(u,v) | v \in M\}$ is relative compact in E

Theorem 3.4. *Assume that $f : D \times D \to D$, where D is bounded in E. If A 1 and A 2 hold, equation (3.3) has at least one solution in D.*

Proof. We need to prove that inequality (3.2) holds. Let M be bounded subset in E, and let $\gamma > \alpha(M)$. One denotes by $B(u,r)$ the ball centered at u with radius r in E. According to the definition of Kuratowski MNC (Proposition (2.4)), we have $M \subset \cup_{i=1}^{n} B(u_i,\gamma)$. For each $u_i, i = 1,2,\ldots,n$, by assumption A2, the subset $M_i := \{f(u_i,v) | v \in M\}$ is relative compact in E. It follows that, for any $\varepsilon > 0$, there exists a finite covering with radius ε of M_i. Suppose that

$$M_i \subset \bigcup_{j=1}^{m} B(v_j^i,\varepsilon).$$

We now fix $u \in M$ and choose u_i, v_j^i such that $\|u - u_i\| \leq \gamma$ and $\|f(u_i,u) - v_j^i\| \leq \varepsilon$. Thus

$$\begin{aligned}\|F(u) - v_j^i\| &\leq \|f(u,u) - f(u_i,u)\| + \|f(u_i,u) - v_j^i\|\\ &\leq L\|u - u_i\| + \varepsilon \leq L\gamma + \varepsilon.\end{aligned}$$

We further conclude from the arbitrariness of $\gamma > \alpha(M)$ and $\varepsilon > 0$ that $\alpha(F(M)) \leq L\alpha(M)$, or $\ln(\alpha F(M)) \leq \ln(\alpha(M)) + \ln(L)$. By choosing the function $S(t,s) = G(t) = \ln(t), \beta(t,s) = L < 1$, we see inequality (3.2) holds clearly. □

Example 3.5. Consider the following equation $u'(t) = \frac{t}{1+t^2}u(t) + \int_0^t \frac{su(s)}{1+|u(s)|}ds$ for $t \in [0,T]$. Here $f(u,v)(t) = \frac{t}{1+t^2}u(t) + \int_0^t \frac{sv(s)}{1+|v(s)|}ds$. Thus assumptions A 1 and A 2 hold true.

3.2. The second consequence. In this subsection, we use the following notations, which were introduced in [5].

Let us denote by $\Gamma_{G,\eta}$ the set of pairs (G,η), where $G,\eta : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ such that

$\Gamma 4$ $\eta(t,s) < s - t$ for all $t,s > 0$;
$\Gamma 5$ $\max\{a,b\} \leq G(a,b)$ for $a,b \geq 0$;
$\Gamma 6$ G is nondecreasing.

For example $\eta(t,s) = ms - (t/m)$ and $G(a,b) = a + b$, where $m \in (0,1)$.

Corollary 3.6. *Let $F : D \subset E \to D$, where D is bounded, be a continuous operator. Let $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ be a continuous and nondecreasing function. If, for any $M \subset D, \Phi(M) > 0$, there exists $\gamma(M) \in (0,1)$ such that*

$$\eta(G(\Phi(F(M)),\varphi(\Phi(F(M)))),\gamma G(\Phi(M),\varphi(\Phi(M)))) \geq 0, \tag{3.4}$$

then $fix(F) \neq \emptyset$

Proof. If we prove that F is Φ, I - condensing, where I is the identity operator, the assertion follows immediately. For any $M \subset D$ and $\Phi(M) > 0$, if $\Phi(F(M)) = 0$, then the proof is completed. We consider $\Phi(F(M)) > 0$. From $\Gamma 5$, we have $G(\Phi(M),\varphi(\Phi(M))) \geq \Phi(M) > 0$ and $G(\Phi(F(M)),\varphi(\Phi(F(M)))) \geq \Phi(F(M)) > 0$. Thus

$$\begin{aligned}0 &\leq \eta(G(\Phi(F(M)),\varphi(\Phi(F(M)))),\gamma G(\Phi(M),\varphi(\Phi(M))))\\ &< \gamma G(\Phi(M),\varphi(\Phi(M))) - G(\Phi(F(M)),\varphi(\Phi(F(M)))),\end{aligned} \tag{3.5}$$

by (3.4) and $\Gamma 4$. Assume that $\Phi(F(M)) \geq \Phi(M)$. The nondecreasing property of G shows that

$$G(\Phi(F(M)),\varphi(\Phi(F(M)))) \geq G(\Phi(M),\varphi(\Phi(M))).$$

We conclude from (3.5) that $0 < (\gamma - 1)G(\Phi(M), \varphi(\Phi(M)))$, which is a contradiction to $\gamma \in (0,1)$. Hence $\Phi(F(M)) < \Phi(M)$ and the proof is completed. □

Remark 3.7. The theorem still holds if we replace assumption (3.4) by the following assumption: *For any* $0 < a < b < \infty$, *there exists* $0 < \gamma(a,b) < 1$ *such that, for all* $M \subset D$,

$$a \le G(\Phi(M), \varphi(\Phi(M))) \le b \Rightarrow \eta(G(\Phi(F(M)), \varphi(\Phi(F(M)))), \gamma G(\Phi(M), \varphi(\Phi(M)))) \ge 0,$$

which was used in [5].

In fact, for $M \subset D, \Phi(M) > 0$, we set $a = \Phi(M)$ and $b = \max\{\Phi(M), \varphi(\Phi(M))\}$.

3.3. The third consequence.

Proposition 3.8. *Let* $F, T : D \subset E \to D$ *be continuous operators. Let* $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ *be monotone nondecreasing mapping, and let* $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ *be a monotone nonincreasing mapping with* $\varphi(0) = 0, \varphi(t) > 0$ *for* $t > 0$. *Assume that, for any* $M \subset D$, $\Phi(T(M) > 0$ *implies*

$$\psi(\Phi(F(M))) \le \psi(\Phi(T(M))) - \varphi(\Phi(T(M))). \tag{3.6}$$

Then the condition (ii) in the definition (2.7) *holds.*

Proof. Let M be a subset of D such that $\Phi(TM)) > 0$. Suppose that $\Phi(T(M)) \le \Phi(F(M))$. Since ψ is nondecreasing, (3.6) yields that

$$\psi(\Phi(T(M))) \le \psi(\Phi(F(M))) \le \psi(\Phi(T(M))) - \varphi(\Phi(T(M))).$$

It follows that $\varphi(\Phi(T(M))) \le 0$. Hence $\Phi(T(M)) = 0$, which is impossible. □

Theorem 3.9. *Let* D *be a nonempty, closed, and bounded subset of* E, *and let* $F, T : D \to D$ *be two continuous operators such that* (3.6) *holds and*

H1. $TF = FT$;

H2. $FixT := \{u \in D : Tu = u\} \ne \emptyset$ *and for all* $\Omega \subset D$,

$$T(\overline{conv}(\Omega)) \subset \overline{conv}(T(\Omega));$$

Then $Fix(F) = \{u \in D : F(u) = u\}$ *is nonempty, compact and* T, S *have a common fixed point* u_0, *that is,* $F(u_0) = T(u_0) = u_0$.

Proof. From definition (2.7) and the previous proposition, we see that F is condensing with respect to Φ, T. Thus we obtain from theorem (3.1) that $Fix(F)$ is nonempty and closed. We denote by C the $Fix(F)$.

We proceed to show that T has a fixed point in C. If $u \in C$, then $T(u) = T(F(u)) = F(T(u))$ and T maps C to C. If $\Phi(T(C)) = 0$, then $T(C)$ is compact and $T : TC \to TC$ has a fixed point by the Schauder–Tychonoff theorem. If $\Phi(T(C)) > 0$, then (3.6) implies

$$\begin{aligned}\psi(\Phi(C)) = \psi(\Phi(F(C))) &\le \psi(\Phi(T(C))) - \varphi(\Phi(T(C)))\\ &\le \psi(\Phi(C)) - \varphi(\Phi(C)),\end{aligned}$$

which gives $\varphi(\Phi(C)) = 0$. Hence $\Phi(C) = 0$, which is contrary to $\Phi(T(C)) > 0$. □

Proposition 3.10. *Let* $F, T : D \subset E \to D$ *be continuous operators,* $\eta : \mathbb{R}^+ \to \mathbb{R}^+$ *be a monotone nondecreasing mapping, and* $\lim_{n\to\infty} \eta^n(t) = 0$ *for* $t > 0$. *If for any* $M \subset D$, $\Phi(T(M) > 0$ *implies*

$$\Phi(F(M)) \le \eta(\Phi(T(M))), \tag{3.7}$$

then the condition (ii) in the definition (2.7) *holds.*

Proof. Let M be a subset of D such that $\Phi(TM)) > 0$. Suppose that $\Phi(T(M)) \leq \Phi(F(M))$. We conclude from (3.7) that $\Phi(T(M)) \leq \Phi(F(M)) \leq \eta(\Phi(T(M)))$. It follows from the nondecreasing property of η that

$$\Phi(T(M)) \leq \eta(\Phi(T(M))) \leq \eta(\Phi(T(M))), \forall n \in \mathbb{N}^+.$$

From the assumptions on η, $\Phi(T(M)) = 0$, which is impossible. This clearly forces $\Phi(T(M)) > \Phi(F(M))$. □

Theorem 3.11. *Let D be a nonempty, closed, and bounded subset of E, and let $F,T : D \to D$ be two continuous operators such that (H1), (H2), and (3.7) hold. Then $Fix(F) = \{u \in D : F(u) = u\}$ is nonempty, compact, and T,S have a common fixed point u_0, that is, $F(u_0) = T(u_0) = u_0$.*

Proof. Using similar arguments in the proof of theorem (3.9), we need to prove that $\Phi(T(C)) > 0$, where $C = Fix(F)$, is impossible. In fact, if $\Phi(T(C)) > 0$, then (3.7) implies $\Phi(C) = \Phi(F(C)) \leq \eta(\Phi(T(C))) \leq \eta(\Phi(C))$. Furthermore, the nondecreasing property of η yields that

$$\Phi(C) = \Phi(F(C)) \leq \eta(\Phi(C)) \leq \eta^n(\Phi(C)), \ n \in \mathbb{N}^+,$$

which together with the properties of η gives $\Phi(T(C)) \leq \Phi(C) = 0$. This is contrary to $\Phi(T(C)) > 0$. □

Remark 3.12. The two previous theorems were proven in [7] without the condition $Fix(T) \neq \emptyset$ but under the following property for MNC Φ:

H* If (Ω_n) is a sequence of nonempty, closed, and bounded subsets of E such that $\Omega_{n+1} \subset \Omega_n$ for all $n \in \mathbb{N}^+$ and if $\lim_{n\to\infty} \Phi(\Omega_n) = 0$, then the intersection set $\Omega_\infty = \cap_{n=1}^{\infty} \Omega_n$ is nonempty and $\Phi(\Omega_\infty) = 0$.

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