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TYKHONOV WELL-POSEDNESS IN DISCONTINUOUS NON-COOPERATIVE GAMES

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Abstract. In this paper, we consider the Tykhonov well-posedness of discontinuous n-person non-cooperative games. By giving a new sufficient condition, we prove that several classes of discontinuous n-person non-cooperative games have the Tykhonov well-posedness property, which means that, for any approximate solution sequence of these games, we can find a convergent subsequence whose limit point is a Nash equilibrim point. The results of this paper improve the corresponding results in the existing literature.

Keywords. Better-reply secure; Discontinuous n-person non-cooperative game; Pseudocontinuous function; Tykhonov well-posedness.

1. Introduction

The concept of Tykhonov well-posedness introduced by Tykhonov [28] is a kind of stability on optimization problems. Tykhonov well-posedness requires existence and uniqueness of the minimum solution which is continuously dependent on the problem's function value. The other main concept of well-posedness is Hadamard well-posedness [11], which needs that the unique solution is continuously dependent on the problem's data. In 1993, Dontchcv and Zolezzi [8] presented a comprehensive research on these well-posedness noitons in a series of optimization problems and calculus of variations. For more research on these notions, we refer to [16] and the references therein. In addition, many concepts on well-posedness were introduced and studied for various problems recently, such as fixed points [9, 10, 24], variational inequalities [5, 10, 12, 13, 32], equilibrium problems [2, 3, 4, 35], and so on.

In recent years, the well-posedness on Nash equilibria was under the spotlight of research. Patrone [22] gave some results on Tykhonov well-posedness for two-person games, which have

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a unique solution. Chicco [6] used generalized Tykhonov well-posedness to study the two-person games with not only a unique Nash equilibrium and gave the metric characterization of this property. Yu et al. [32] proved that a class of discontinuous games was Tykhonov well-posed and Hadamard well-posed, which was equipped with conditions: (i) $\sum_{i=1}^{n} u_i(s)$ was upper semicontinuous; (ii) for any player i and strategy s_i, $u_i(s_i, \cdot)$ is lower semicontinuous. Yu [33] gave a new method to study various well-posed properties of several problems by using the model of bounded rationality. Scalzo [26] considered more weaker payoff conditions and proved that pseudocontinuous and better-reply secure games had the Hadamard well-posedness. In addition, well-posedness was also studied in multiobjective games (see, e.g., [15, 18, 23, 30]) and games with α-core (see, e.g, [14, 31]) and so on.

On the other hand, the study of discontinuous games is also a hot topic. The main purpose of it is to find more sufficient conditions for the existence of Nash equilibria for those games whose payoff functions are discontinuous. Reny [20] proved the existence of Nash equilibria for games whose payoff functions are better-reply secure. Then Morgan and Scalzo [17] introduced pseudocontinuous games and proved that if a game was pseudocontinuous it must be a better-reply secure game. More sufficient conditions can be found in, for example, [19, 21, 25, 27, 29] and the references therein. As far as we know, most of literature about discontinuous games mainly focuses on the existence and the Hadamard well-posedness of Nash equilibria. Only few researches considered the Tykhonov well-posedness of discontinuous games, however.

Motivated by the works above, we prove that pseudocontinuous and better-reply secure games are Tykhonov well-posed by a new sufficient condition in this paper. In the next section, we review some notions of discontinuous games. The Tykhonov well-posedness of the two classes of discontinuous n-person games is studied in Section 3. Finally, some conclusions are presented in Section 4.

2. Preliminaries

Assume that $G = (S_i, u_i)_{i\in N}$ is a n-person non-cooperative game, where $N = \{1, \cdots, n\}$ represents the all players, S_i, for all $i \in N$, is the set of player i's strategy, $S = \prod_{i\in N} S_i$, and the bounded function $u_i : S \to R$ is player i's payoff function. For all $i \in N$, we denote that $-i = N\backslash\{i\}$. A strategy $s^* \in S$ is a Nash equilibrium (NE) of the n-person non-cooperative game G if, for each player $i \in N$, $u_i(s_i^*, s_{-i}^*) = \max_{s_i\in S_i} u_i(s_i, s_{-i}^*)$. It is obvious that all players cannot make themselves get more benefits by changing their own strategies at the NE.

Let us review two classes of discontinuous n-person non-cooperative games which guarantee the existence of NE.

Definition 2.1. [20] A game $G = (S_i, u_i)_{i\in N}$ is better-reply secure (BRS) iff for any (s^*, u^*) in the closure of the graph of the payoff function u if s^* isn't a NE, then there is some player i, $\bar{s}_i \in S_i$, a positive real number ε, and some open neighborhood $O\left(s_{-i}^*\right)$ of s_{-i}^* such that

$$u_i\left(\bar{s}_i, s'_{-i}\right) > u_i^* + \varepsilon, \forall s'_{-i} \in O\left(s_{-i}^*\right).$$

Definition 2.2. [17] Assume that S is a topological space and $u : S \to R$ is a real valued function. The function u is lower pseudocontinuous (LPC) at $s_0 \in S$ if, for any $s \in S$ such that $u(s) < u(s_0)$, $u(s) < \lim_{z\to s_0} u(z)$; The function u is upper pseudocontinuous (UPC) at s_0 if $u(s) > u(s_0)$, then $u(s) > \lim_{z\to s_0} u(z)$. The function f is pseudocontinuous (PC) at s_0 if it is both

LPC and UPS at s_0. Furthermore a game $G = (S_i, u_i)_{i\in N}$ is PC if the payoff function $u(s) = (u_1(s), u_2(s), \cdots, u_n(s))$ is PC on S. In other words, for each player i, $u_i(s)$ is PC on S.

Remark 2.3. If a game $G = (S_i, u_i)_{i\in N}$ is PC, it must be a BRS game. But the converse is not true (see [17, Proposition 4.1]).

The following is an important proposition of PC functions, which make a crucial role in the proof of Tykhonov well-posedness.

Proposition 2.4. [17] *Let S be a topological space, and let $u : S \to R$ be a real valued PC function. For all $s, z \in S$, if $u(s) < u(z)$, then there exists a positive real number δ_0 such that*

$$\overline{\lim_{s'\to s}} u(s') + \delta_0 < \underline{\lim}_{z'\to z} u(z').$$

Next, we review the continuity of set-valued mapping. Let S and T be Hausdorff topology spaces, and let $K : S \to P_0(T)$ be a set-valued mapping, where $P_0(T)$ is the collection of all non-empty subsets of T. $K(\cdot)$ is upper semicontinuous at s if, for any open set G, $G \supset K(s)$, there exists an open neighborhood $O(s)$ of s such that $G \supset K(s')$ for all $s' \in O(s)$. $K(\cdot)$ is lower semicontinuous at s if, for any open set G, $G \cap K(s) \neq \emptyset$, there exists an open neighborhood $O(s)$ of s such that $G \cap K(s') \neq \emptyset$ for all $s' \in O(s)$. $K(\cdot)$ is continuous at s if it is not only upper semicontinuous but also lower semicontinuous at s.

In the following, we review a definition of discontinuous functions to obtain a sufficient condition of Tykhonov well-posedness.

Definition 2.5. ([1]) Let S be a Hausdorff topology space, and let $u : S \to R$ be a real valued function such that $0 \in u(S)$. We say that $u(\cdot)$ is 0-lower pseudocontinuous (0-LPC) at s if $u(s) > 0$, then $\lim_{s'\to s} u(s') > 0$.

Remark 2.6. It is obvious that if function $u(\cdot)$ is LPC on S and $0 \in u(S)$, then $u(\cdot)$ must be 0-LPC on S. But the inverse may not true. See the following counterexample.

Example 2.7. Let Q be the set of all rational numbers. Consider the function $u : S \to R$, where $S = [0, 1]$, $\forall s \in S$,

$$u(s) = \begin{cases} 0 & s = 0 \\ 1 & s \in (0,1] \cap Q \\ 2 & others \end{cases}.$$

for any $s \in [0, 1]$, we have $\liminf_{s'\to s} s' = 1$ and $\limsup_{s'\to s} s' = 2$. So it is obvious that $u(\cdot)$ is 0-LPC on S, but $u(\cdot)$ is not LPC at any irrational number point in S.

3. Tykhonov Well-Posedness

Let S be a metric space, and let $u : S \to R$ be a real valued function such that $\inf_{s\in S} u(s) > -\infty$ and the optimization problem $\inf_{s\in S} u(s)$ only has a unique solution s_0. We say that the optimization problem $\inf_{s\in S} u(s)$ is Tykhonov well-posed if, for any sequence $\{s_n\}$, where $u(s_n) \to \inf_{s\in S} u(s)$, $s_n \to s_0$. Similarly, the following is the Tykhonov well-posedness in game theory. For a n-person non-cooperative game $G = (S_i, u_i)_{i\in N}$, we define a real valued function $\varphi : S \to R$ as

$$\varphi(s) = \sum_{i=1}^{n} \left[\sup_{w_i\in S_i} u_i(w_i, s_{-i}) - u_i(s_i, s_{-i}) \right]. \tag{3.1}$$

The approximate solution set is denoted by $E(G,\varepsilon) = \{s \in S \,|\, \varphi(s) < \varepsilon\}$, where ε is a positive real number. Let $E(G,0) = E(G)$, which represents the precise solution set. The game G is generalized Tykhonov well-posed if, for any approximate solution sequence $\{s_n\}$, where $s_n \in E(G,\varepsilon_n)$ and $\varepsilon_n \to 0$, there exists a subsequence $\{s_{n_k}\}$ such that $s_{n_k} \to s$, where s is a NE of G $(s \in E(G))$. The game G is (briefly) Tykhonov well-posed if G is generalized Tykhonov well-posed and has a unique NE. Furthermore, we say that a game G has Tykhonov well-posedness property if it is (generalized) Tykhonov well-posed.

We study Tykhonov well-posedness of BRS and PC games in two models, which are denoted as model (A) and model (B). In model (A), we concern a n-person non-cooperative game with fixed feasible strategy set for all players, but in the model (B), we concerns that each player i's feasible strategy set can change with other players' strategies.

The following lemma gives a sufficient condition of Tykhonov well-posedness.

Lemma 3.1. *Assume that $G = (S_i, u_i)_{i\in N}$ is a game with at least a NE, and $S = \prod_{i\in N} S_i$ is compact. If the real valued function $\varphi : S \to R$ defined by equation (1) is 0-LPC on S, then G is (generalized) Tykhonov well-posed.*

Proof. Due to the definition of the Tykhonov well-posedness, for any sequence $\{s_n\}$, where $s_n \in E(G,\varepsilon_n)$ and $\varepsilon_n \to 0$, we just need to prove that there is a subsequence $\{s_{n_k}\}$ whose limit is a NE. Since $s_n \in E(G,\varepsilon_n)$, we have $\varphi(s_n) < \varepsilon_n$. When $\varepsilon_n \to 0$, it is obvious that $\varphi(s_n) \to 0$. Since S is compact, we have a subsequence $\{s_{n_k}\}$ such that $s_{n_k} \to s \in S$ and $\varphi(s_{n_k}) \to 0$. In the following, we prove that $\varphi(s) = 0$, so s is a NE. We argue by contradiction. If $s \notin E(G)$, then there exists a player i and $\bar{s}_i$ such that $u_i(\bar{s}_i, s_{-i}) - u_i(s) > 0$. By equation (3.1), we have

$$\varphi(s) \geq \sup_{w_i\in S_i} u_i(w_i, s_{-i}) - u_i(s_i, s_{-i}) \geq u_i(\bar{s}_i, s_{-i}) - u_i(s),$$

which implies $\varphi(s) > 0$. Since $\varphi(\cdot)$ is 0-LPC, we have $\lim_{n_k\to+\infty} \varphi(s_{n_k}) > 0$. Since $\varphi(s_{n_k}) \to 0$, we reach a contradiction. Therefore s is a NE. The proof is completed. □

3.1. Tykhonov well-posedness for model (A).

Model (A): let $G = (S_i, u_i)_{i\in N}$ be a compact and quasiconcave non-cooperative game, where

(a) G is compact which means that if, for each player $i \in N$, S_i is a nonempty compact subset of X_i, where X_i is a Hausdorff locally convex topological linear space;

(b) G is quasiconcave which means that if S_i is convex for each player i, and $u_i(\cdot, s_{-i})$ is quasiconcave on S_i for $\forall s_{-i} \in S_{-i}$.

In order to prove that BRS games are Tykhonov well-posed for model (A), the following proposition is needed.

Proposition 3.2. *Let X be a topological space. Let S be a nonempty subset of X, and let $cg(u)$ be the closure of the graph of the real valued function $u : S \to R$. Then the following equation holds $\sup_{(s,u')\in cg(u)} \{u'\} = \max_{(s,u')\in cg(u)} \{u'\}$.*

Proof. Let $\sup_{(s,u')\in cg(u)} \{u'\} = c$. We need to prove that $c \in \{u' \,|\, (s,u') \in cg(u)\}$.

(i) If $c = u(s)$, it is obvious that $c \in \{u' \,|\, (s,u') \in cg(u)\}$;

(ii) If $c \neq u(s)$, by the definition of c, we have $c > u(s)$.

Next, we prove that $c \in \{u' \,|\, (s,u') \in cg(u)\}$. In other words, there exists a sequence $\{s_\alpha\}$ such that $s_\alpha \to s$ and c is a limit of $\{u(s_\alpha)\}$.

We only need to prove that, for any open neighborhood $O(s)$ of s and any positive real number ε, there exists a $s' \in O(s)$ such that $|c-u(s')|<\varepsilon$. Since $\sup_{(s,u')\in cg(u)}\{u'\}=c$, we find $u_0 \in \{u' | (s,u') \in cg(u)\}$ such that

$$|c-u_0|<\varepsilon/2. \tag{3.2}$$

Since $u_0 \in \{u' | (s,u') \in cg(u)\}$, then there exists $s_\alpha \to s$ such that $u(s_\alpha)\to u(s)$. Hence, there exists α_0 such that $|u(s_\alpha)-u_0|<\varepsilon/2$ for all $\alpha \succ \alpha_0$. It is obvious that $\{s_\alpha\}_{\alpha\succ\alpha_0}\bigcap O(s)\neq\emptyset$. Then we can find $s' \in O(s)$ such that

$$\left|u\left(s'\right)-u_0\right|<\varepsilon/2. \tag{3.3}$$

We also have the following triangle inequality

$$\left|c-u\left(s'\right)\right|=\left|c-u_0+u_0-u\left(s'\right)\right|\leq|c-u_0|+\left|u\left(s'\right)-u_0\right|. \tag{3.4}$$

Combining (3.2), (3.3), and (3.4), we obtain that $|c-u(s')|<\varepsilon$. The proof is completed. □

Theorem 3.3. *Let $G=(S_i,u_i)_{i\in N}$ be a compact and quasiconcave n-person game. If G is BRS, then G has the Tykhonov well-posedness property.*

Proof. The existence of NE for game $G=(S_i,u_i)_{i\in N}$ was proved by Reny [20], so $0\in\varphi(S)$. We only need to prove that the corresponding real value function $\varphi(s)$ of game G is 0-LPC, which means that if $\varphi(s)>0$,then $\lim_{s'\to s}\varphi(s')>0$ for any sequence $\{s'\}$ (where $s'\to s$).

If $\varphi(s)>0$, in light of (3.1), then there exists a player j such that

$$\sup_{w_j\in S_j} u_j\left(w_j,s_{-j}\right)-u_j\left(s_j,s_{-j}\right)>0.$$

Hence s is not a NE. Since game G is BRS, $\forall u'_i \in \{u'_i | (s,u')\in cg(u)\}$, we can find a player i, $\bar{s}_i\in S_i$, s_{-i}, an open neighborhood $O(s_{-i})$ of s_{-i}, and a positive real number ε such that

$$u_i\left(\bar{s}_i,s'_{-i}\right)>u'_i+\varepsilon, \forall s'_{-i}\in O\left(s'_{-i}\right). \tag{3.5}$$

Since $u'_i\in cg(u_i)$, by Proposition 3.2, we can assume that

$$\sup_{(s,u')\in cg(u)}\left\{u'\right\}=\max_{(s,u'_i)\in cg(u_i)}\left\{u'_i\right\}=t\in cg(u_i).$$

In light of (3.5), if we assume that $u'_i=t$, then $u_i(\bar{s}_i,s'_{-i})>t+\varepsilon$ for all $s'_{-i}\in O(s'_{-i})$. Since $\sup_{x_i\in S_i}u_i(x_i,s'_{-i})\geq u_i(\bar{s}_i,s'_{-i})$, we have $\sup_{w_i\in S_i}u_i(w_i,s'_{-i})>t+\varepsilon$ for all $s'_{-i}\in O(s'_{-i})$. Since the payoff functions of game G are bounded, for any sequence $\{s'\}$, where $s'\to s$, without losing the generality, we can assume that $\{s'\}\subset O(s'_{-i})$. It follows that

$$\varliminf_{s'\to s}\sup_{w_i\in S_i}u_i\left(w_i,s'_{-i}\right)\geq t+\varepsilon. \tag{3.6}$$

In light of (3.1), the following inequality holds:

$$\varliminf_{s'\to s}\varphi\left(s'\right)\geq\varliminf_{s'\to s}\left\{\sup_{w_i\in S_i}u_i\left(w_i,s'_{-i}\right)-u_i\left(s'\right)\right\}\geq\varliminf_{s'\to s}\sup_{w_i\in S_i}u_i\left(w_i,s'_{-i}\right)-\varlimsup_{s'\to s}u_i\left(s'\right). \tag{3.7}$$

We also have that

$$\varlimsup_{s'\to s}u_i\left(s'\right)\leq\sup_{(s,u')\in cg(u)}\left\{u'\right\}=\max_{(s,u'_i)\in cg(u_i)}\left\{u'_i\right\}=t. \tag{3.8}$$

Combining (3.6), (3.7), and (3.8), we deduce that $\lim_{s'\to s}\varphi(s') \geq \varepsilon > 0$. Hence game G is (generalized) Tykhonov well-posed. □

For a n-person non-cooperative game, if it is PC, it must be BRS. In light of this theorem, we have the following result.

Corollary 3.4. *Let $G = (S_i, u_i)_{i\in N}$ be a compact and quasiconcave n-person non-cooperative game. If G is PC on S, then G is (generalized) Tykhonov well-posed.*

3.2. **Tykhonov well-posedness for model (B).**

Model (B): let $G = (S_i, u_i, K_i)_{i\in N}$ be a quasiconcave non-cooperative game, where

(a) S_i is a compact Hausdorff locally convex topological linear space;

(b) G is quasiconcave means that if, for each player i, S_i is convex and $u_i(\cdot, s_{-i})$ is quasiconcave on S_i for any $s_{-i}\in S_{-i}$;

(c) for any player i, the feasible strategy set is denoted by a set-valued mapping $K_i : S_{-i} \to P_0(S_i)$.

The concept of social equilibria for a non-cooperative game was introduced by Debreu [7]. $\bar{s} = (\bar{s}_1, \bar{s}_2, \cdots \bar{s}_n) \in S$ is said to be a social equilibrium of G if, for any player i, we can find $\bar{s}_i \in K_i(\bar{s}_{-i})$ such that $u_i(\bar{s}_i, \bar{s}_{-i}) = \max_{w_i\in K_i(\bar{s}_{-i})} u_i(w_i, \bar{s}_{-i})$.

We define the real valued function $\varphi : S \to R$ as

$$\varphi(s) = \sum_{i=1}^{n}\left[\sup_{w_i\in K_i(s_{-i})} u_i(w_i, s_{-i}) - u_i(s_i, s_{-i})\right]. \tag{3.9}$$

Lemma 3.5. *Assume $G = (S_i, u_i, K_i)_{i\in N}$ is a game with at least a NE, and $S = \prod_{i\in N} S_i$ is compact. If the real valued function $\varphi : S \to R$ defined by equation (10) is 0-LPC on S, then G is (generalized) Tykhonov well-posed.*

Similarly, from the proof of Lemma 3.1, the desired conclusion is easy to obtain, so we omit the proof here.

Theorem 3.6. *Let $G = (S_i, u_i, K_i)_{i\in N}$ be quasiconcave n-person game. If G is a PC game and for any player i, $K_i(\cdot)$ is continuous on S_{-i} and $K_i(s_{-i})$ is compact, then G has the Tykhonov well-posedness property.*

Proof. The existence of social equilibria for game $G = (S_i, u_i, K_i)_{i\in N}$ was proved by Scalzo [26], so $0 \in \varphi(S)$. Similarly in light of Lemma 3.5, we just need to prove that the corresponding real value function $\varphi(s)$ of G is 0-LPC on S. If $\varphi(s) > 0$, we can find from (3.9) a player j such that

$$\sup_{w_j\in K_j(s_{-j})} u_j(w_j, s_{-j}) - u_j(s_j, s_{-j}) > 0.$$

Hence s is not a social equilibrium of G, and then there exists a player i and $\bar{s}_i \in K_i(\bar{s}_{-i})$ such that $u_i(\bar{s}_i, s_{-i}) > u_i(s_i, s_{-i})$. Since the set-valued mapping $K_i(\cdot)$ is lower semicontinuous on S_{-i}. For any sequence $\{s'\}$, where $s' \to s$, we can find a sequence $\{z'_i\}$ such that $z'_i \to \bar{s}_i, z'_i \in K_i(s'_{-i})$. Since G is PC, in light of Proposition 2.4, there exists a positive real number δ_0 such that

$$\underline{\lim}_{(z'_i, s'_{-i})\to(\bar{s}, s_{-i})} u(z'_i, s'_{-i}) > \overline{\lim_{s'\to s}}\, u(s') + \delta_0. \tag{3.10}$$

Since the payoff functions of G are bounded, and $\sup_{w_i \in K_i(s'_{-i})} u_i(w_i, s'_{-i}) > u(z'_i, s'_{-i})$, the following inequality holds:

$$\underline{\lim}_{s' \to s} \sup_{w_i \in K_i(s'_{-i})} u_i\left(w_i, s'_{-i}\right) \geq \underline{\lim}_{(z'_i, s'_{-i}) \to (\bar{s}_i, s_{-i})} u\left(z'_i, s'_{-i}\right). \tag{3.11}$$

From (3.9), we have

$$\underline{\lim}_{s' \to s} \varphi\left(s'\right) \geq \underline{\lim}_{s' \to s} \left\{ \sup_{w_i \in K_i} u_i\left(w_i, s'_{-i}\right) - u_i\left(s'\right) \right\} \geq \underline{\lim}_{s' \to s} \sup_{w_i \in K_i} u_i\left(w_i, s'_{-i}\right) - \overline{\lim}_{s' \to s} u_i\left(s'\right). \tag{3.12}$$

Combining (3.10), (3.11), and (3.12), we deduce that $\underline{\lim}_{s' \to s} \varphi\left(s'\right) > \delta_0 > 0$. Hence the game G is (generalized) Tykhonov well-posed. □

4. Conclusion

By the definition of the 0-lower pseudocontinuity for real valued functions, we pointed that a game was (generalized) Tykhonov well-posed if the real valued function $\varphi : S \to R$ is 0-LPC on S (this condition is weaker than Theorem 8.2.1, given by Yu [34]). We obtained a new sufficient condition of Tykhonov well-posedness, by which we further obtained that

(i) a pseudocontinuous or better-reply secure game is (generalized) Tykhonov well-posed in model (A);

(ii) a pseudocontinuous game is (generalized) Tykhonov well-posed in model (B).

The results of this paper develop the studies of Reny [20] and Morgan and Scalzo [17], who proved that these discontinuous games had the Hadamard well-posedness. In this paper, we proved that these discontinuous games also have the Tykhonov well-posedness, which enriches the study of well-posedness.

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References

[1] L.Q. Anh, N.T. Anh, P.T. Duoc, The connectedness of weakly and strongly efficient solution sets of nonconvex vector equilibrium problems, Appl. Set-Valued Anal. Optim. 4 (2022) 109-127.

[2] L.Q. Anh, T.Q. Duy, P.Q. Khanh, Levitin–Polyak well-posedness for equilibrium problems with the lexicographic order, Positivity 25 (2021) 1323-1349.

[3] D. Aussel, T.C. Thanh Cong, R. Riccardi, Strategic decision in a two-period game using a multi-leader-follower approach. Part 1 – General setting and weighted Nash equilibrium, J. Appl. Numer. Optim. 4 (2022) 67-85.

[4] M. Bianchi, G. Kassay, R. Pini, Well-posed equilibrium problems, Nonlinear Anal. 72 (2010) 460-468.

[5] J. Cen, C. Min, V.T. Nguyen et al., On the well-posedness of differential quasi-variational-hemivariational inequalities, Open Math. 18 (2020) 540-551.

[6] L.P. Chicco, Approximate Solutions and Tikhonov Well-Posedness for Nash Equilibria. In: Giannessi, F., Maugeri, A., Pardalos, P.M. (eds) Equilibrium Problems: Nonsmooth Optimization and Variational Inequality Models. Nonconvex Optimization and Its Applications, vol 58. Springer, Boston, 2001.

[7] G. Debreu, A social equilibrium existence theorem, Proc. Nat. Acad. Sci. 38 (1952) 886-893.
[8] A.L. Dontchcv, T. Zolezzi, Well-posed Optimization Problems, Springer-Verlag, Berlin, 1993.
[9] N.V. Dung, V.T. Le Hang, Well-posedness in the generalized sense of the multivalued fixed point problem, Quaestiones Math. 41 (2018) 799-810.
[10] Y.P. Fang, N.J. Huang, J.C. Yao, Well-posedness of mixed variational inequalities, inclusion problems and fixed point problems, J. Global Optim. 41 (2008) 117-133.
[11] J. Hadamard, Sur les problèmes aux dérivées partielles et leur signification physique, Princeton University Bulletin, 3 (1902) 49-52.
[12] R. Hu, M. Sofonea, On the well-posedness of variational-hemivariational inequalities and associated fixed point problems, J. Nonlinear Var. Anal. 6 (2022) 567-584.
[13] X. Li, F. Xia, Hadamard well-posedness of a general mixed variational inequality in Banach space, J. Global Optim, 56 (2013) 1617-1629.
[14] Y. Li, W. Jia, Existence and well-posedness of the α-core for generalized fuzzy games, Fuzzy Sets and Systems 458 (2023) 108-117.
[15] Z. Lin, J. Yu, On well-posedness of the multiobjective generalized game, Appl. Math. 19 (2004) 327-334.
[16] R. Lucchetti, J. Revalski, Recent Developments in Well-posed Variational Problems, Kluwer Academic Publishers, Dordrecht, 1995.
[17] J. Morgan, V. Scalzo, Pseudocontinuous functions and existence of Nash equilibria, J. Math. Economics 43 (2007) 174-183.
[18] J. Morgan, Approximations and well-posedness in multicriteria games, Ann. Oper. Res. 137 (2005) 257-268.
[19] Y. Mou, W. Jia, Existence and stability of weakly Pareto-Nash equilibria for discontinuous multiobjective games, Appl. Anal. 102 (2023) 4899-4908.
[20] P.J. Reny, On the existence of pure and mixed strategy Nash equilibria in discontinuous games, Econometrica 67 (1999) 1029-1056.
[21] R. Nessah, G. Tian, On the existence of Nash equilibrium in discontinuous games, Economic Theory 61 (2016) 515–540.
[22] F. Patrone, Well-posedness for Nash equilibria and related topics, Recent Developments in Well-Posed Variational Problems, pp. 211-227, 1995.
[23] J.W. Peng, S.Y. Wu, The well-posedness for multiobjective generalized games, J. Optim. Theory Appl. 150 (2011) 416–423.
[24] S. Reich, A.J. Zaslavski, Generic well-posedness of fixed point problems, Vietnam J. Math. 46 (2018) 5-13.
[25] P.J. Reny, Nash equilibrium in discontinuous games, Ann. Rev. Economics 12 (2020) 439-470.
[26] V. Scalzo, Hadamard well-posedness in discontinuous non-cooperative games, J. Math. Anal. Appl. 360 (2009) 697-703.
[27] V. Scalzo, Equilibrium existence in games: slight single deviation property and Ky Fan minimax inequality, J. Math. Economics 82 (2019) 197-201.
[28] A.N. Tikhonov, On the stability of the functional optimization problem, USSR Comput. Math. Math. Phy. 6 (1966) 28-33.
[29] Q. Xiaoling, P. Dingtao, Y. Jian, Berge's maximum theorem to vector-valued functions with some applications, J. Nonlinear Sci. Appl. 10 (2017) 1861-1872.
[30] N.V. Hung, A.A. Keller, Generalized well-posedness for parametric fuzzy generalized multiobjective games, J. Comput. Appl. Math. 422 (2023) 114917.
[31] Z. Yang, D. Meng, Hadamard well-posedness of the α-core, J. Math. Anal. Appl. 452 (2017) 957-969.
[32] J. Yu, H. Yang, C. Yu, Well-posed Ky Fan's point, quasi-variational inequality and Nash equilibrium problems, Nonlinear Anal. 66 (2007) 777-790.
[33] J. Yu, On well-posed problems, Acta Math. Appl. Sinica 34 (2011) 1007-1022.
[34] J. Yu, Bounded rationality and stability of equilibrium in game theory, Science Press, Beijing, 2017.
[35] A.J. Zaslavski, Generic well-posedness for a class of equilibrium problems, J. Inequal. Appl. 2008 (2008) 581917.